

Deriving Normal Form from Extensive Form Games

This note describes a procedure for deriving the normal form representation of a two-player extensive form game. The steps are as follows:

1. DETERMINE THE NUMBER OF STRATEGIES FOR EACH PLAYER

The number of (pure) strategies of a player can be calculated by multiplying the number of actions she has at each information set.

EXAMPLE 1: P2 in Figure 1 moves at two information sets,¹ the green one on the left and the red one on the right. At the green information set, P2 has 3 actions (i.e. C, D, and E). At the red information set, she has 2 actions (i.e. F and G). So the number of strategies for P2 in this game is $2 \times 3 = 6$. These six are

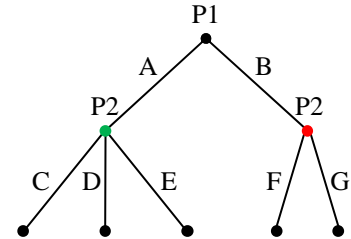


Figure 1

$$\begin{aligned} (A \rightarrow C, B \rightarrow F), & \quad (A \rightarrow D, B \rightarrow F), & \quad (A \rightarrow E, B \rightarrow F), \\ (A \rightarrow C, B \rightarrow G), & \quad (A \rightarrow D, B \rightarrow G), & \quad (A \rightarrow E, B \rightarrow G). \end{aligned}$$

The number of strategies for P1 is just 2, since he moves at only one information set which has two actions.

EXAMPLE 2: In Figure 2, P1 moves at two information sets: the red on top and the green at bottom. There are 2 actions (A and B) for him at the red information set, and 2 (E and F) at the green information set. So the number of strategies for P1 in this game is $2 \times 2 = 4$. These four are

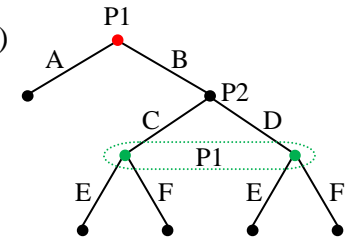


Figure 2

$$(A; B \rightarrow E), \quad (A; B \rightarrow F), \quad (B; B \rightarrow E), \quad (B; B \rightarrow F).$$

P2 only has two strategies in this game, as she only moves at one information set which has two actions.

2. DRAW THE NORMAL FORM GAME.

The normal form game is just a table (or matrix). The number of rows equals the number of P1's strategies, and the number of columns is the number of P2's strategies. So the normal form game corresponding to Figure 1 is a 2-by-6 table; the one for Figure 2 is a 4-by-2 table.

3. FILL IN THE PAYOFF PROFILES.

This step is relatively easy if you understand step 1 well. Consider the example in Figure 3, which is some kind of a combination of the first two figures.

In this game, P1 moves at two information sets: the red and the blue. The number of strategies for him in this game is $3 \times 2 = 6$.

P2 also moves at two information sets: the green and the purple. The number of strategies for her is $2 \times 3 = 6$.

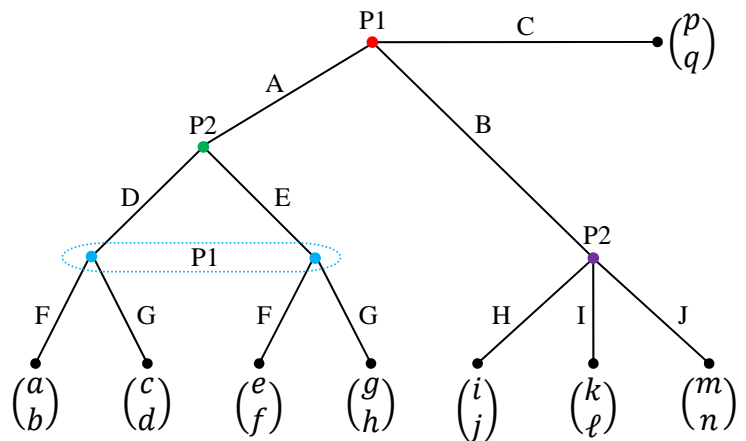


Figure 3

¹ A node that is not "circled" with other nodes is also an information set. Such information sets are *trivial* because you can just circle the node itself.

Hence, we know the normal form is going to be a 6-by-6 table. To simplify on notation, in denoting the strategies, I'll use color to indicate the information set at which an action is to be taken. For example, P1's strategy ($C; A \rightarrow F$) is simply written as **CF**, and likewise, P2's strategy ($A \rightarrow D, B \rightarrow I$) is simply written as **DI**. Then, the normal form representation of the extensive form game in Figure 3 is the following:

P1 \ P2	DH	DI	DJ	EH	EI	EJ
AF	a, b	a, b	a, b	e, f	e, f	e, f
AG	c, d	c, d	c, d	g, h	g, h	g, h
BF	i, j	k, ℓ	m, n	i, j	k, ℓ	m, n
BG	i, j	k, ℓ	m, n	i, j	k, ℓ	m, n
CF	p, q	p, q	p, q	p, q	p, q	p, q
CG	p, q	p, q	p, q	p, q	p, q	p, q

The extensive form representation of a game is particularly useful for finding SPE's. On the other hand, it is easier to spot all the NE's (including the non-subgame perfect ones) in the normal form representation. Moreover, every extensive form game has a **unique** normal form representation.² So knowing how to go from extensive to normal form is a very useful tool in analyzing games.

² The converse the statement, however, is not true: A normal form game will very likely have more than one extensive form representations.