A Study on Hierarchical Floorplans of Order k

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Outline

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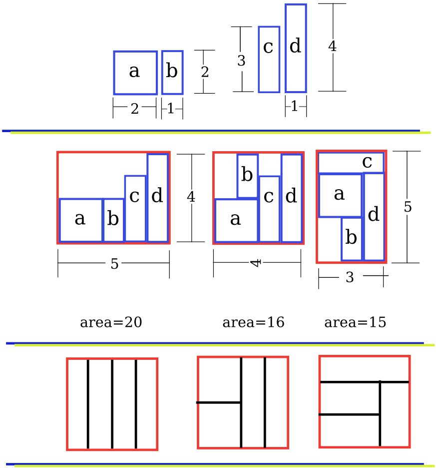
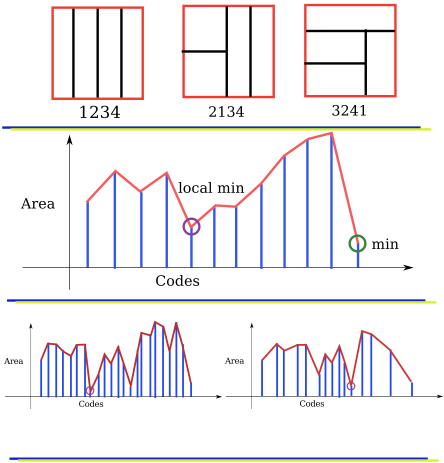
1. Floorplanning
   - Floorplanning Problem
   - Mapping Floorplans to Permutations
   - Sub-Families of Mosaic Floorplans
2. HFO $- k$ Floorplans
   - Characterization
   - Counting
   - Algorithm For Recognition
3. Some interesting properties of Baxter Permutations and Mosaic floorplans
4. Open Problems and Discussions

Birds Eye View of the problem

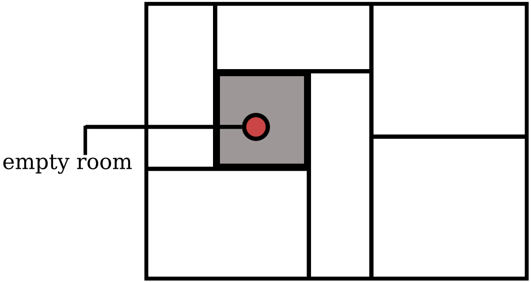




What is a Floorplan ?

- A **rectangular dissection** capturing the relative placement of modules on the chip.
- Only **horizontal** and **vertical line segments** allowed.

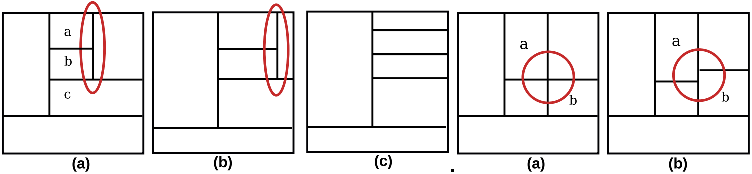
Figure: Example Floorplan



Outline

Mosaic Floorplans

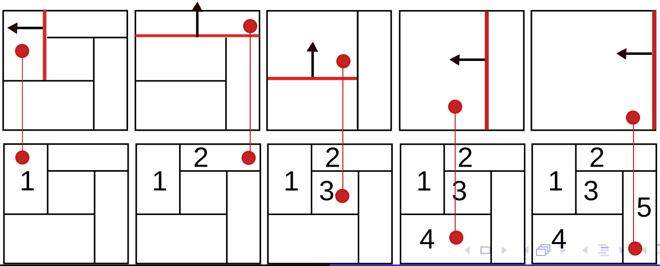
- No empty rooms.
- Topological equivalence on sliding line segments.
- **Non-degenerate Topology** : No degenerate case where two distinct **T** junctions forms a + junction.
- **A room** (basic rectangle) is **above** another if there is a **line segment supporting the bottom of the first room and top of the second room.** The **above relation** is **transitive.**
- Between any two rooms **exactly only one of** $\{A, B, L, R\}$ relations hold.



Algorithm FP2BP[1]

- **Input:** A mosaic floorplan with n rooms **Output:** A permutation of length n.
- **Phase I**
  - Label the rooms in the **top-left** deletion order.
  - Captures **above or to the left** relation.

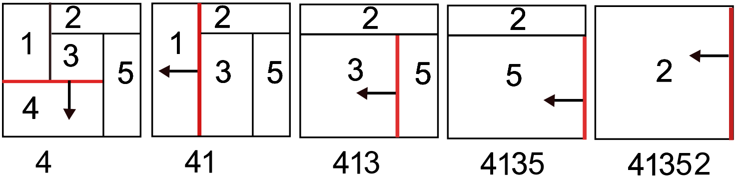
Figure: Labeling Phase



Algorithm FP2BP

- **Phase II**
  - **Input :** A floorplan labeled by the first phase. **Output :** A permutation of length n.
  - Obtain the permutation deleting the **bottom-left room** successively.
  - Captures **below or to the left of** relation.

Figure: Obtaining the permutation



Summary of the algorithm

- **Input :** A **mosaic** floorplan on n rooms **Output :** A **Baxter** permutation of length n corresponding to the floorplan.
- Ackerman et. al.[1] also provide an algorithm to generate a mosaic floorplan from a given Baxter permutation.
- Hence for every mosaic floorplan there is a unique Baxter permutation and vice versa.

Outline

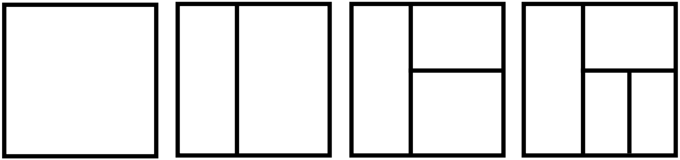
Sub-Families of Mosaic Floorplans

- Slicing $\subset$ HFO $-$ 5 $\subset$ HFO $-$ k $\subset$ Mosaic Floorplans
- Separable Permutations $\subset$? $\subseteq$? $\subset$ Baxter Permutations

Slicing Floorplans

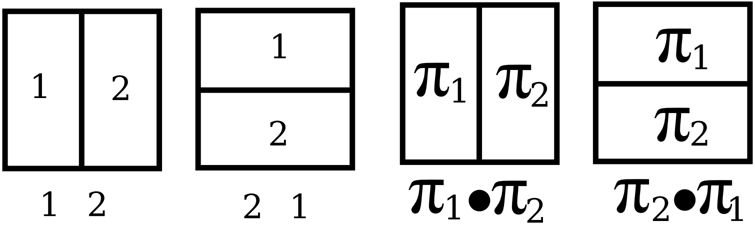
- A Floorplan is Slicing floorplan if it can be obtained from a rectangle by recursively dividing it vertically or horizontally.

Figure: Example of Slicing Floorplan



Connection to Separable Permutations

- The algorithm FP2BP produces **separable permutations** when run on slicing floorplans.

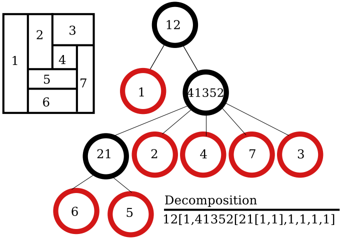


The Hierarchy : HFO — k

- All mosaic floorplans which can be obtained by recursively embedding mosaic floorplans of at most k rooms.
- When $k = 2$ this is the family of slicing floorplans.

Generating Trees of Order k

- Captures the operations done on the basic rectangle to get an HFO – k floorplan.
- Each internal node represents embedding of a Uniquely HFO – l floorplan where $l \leq k$. And such a node will have out-degree l.
- Leaf nodes correspond to rooms in the floorplan and is labeled by the Abe-label of the room.

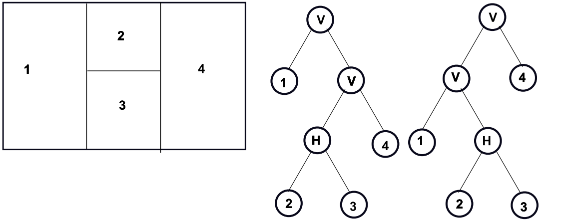


Slicing Floorplans and Skewed Slicing Trees

- A slicing tree is skewed if the left child of a V node cannot be V and left child of an H node cannot be an H node.

Theorem (Wong, Liu)

Slicing floorplans are in bijective correspondence between skewed slicing trees.[3]



Slicing Tree

Skewed Slicing Tree

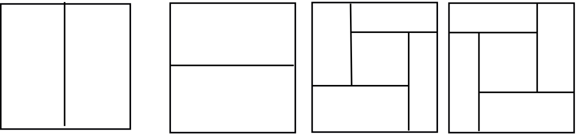
Generating Trees of Order 5

- The internal nodes can be of two types
  - V,H nodes : They are of out-degree 2 and represent the vertical and horizontal cuts respectively
  - RW, LW noes : They are of out-degree 5 and represents the right and left rotating wheels respectively.
- It is a skewed generating tree of order 5 if
  - Left child of a V node cannot be a V node
  - Left child of an H node cannot be an H node

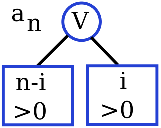
Theorem

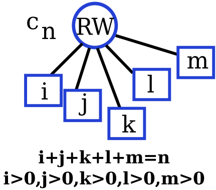
Hierarchical Floorplans of Order 5 *are in bijective correspondence with Skewed Generating trees of Order* 5.

A Recurrence Relation for number of Hierarchical Floorplans of Order 5 with n rooms



a_n b_n c_n d_n





$$
\begin{aligned}
t_n &= a_n + b_n + c_n + d_n \\
a_n &= \Sigma_{i=1}^{n-1} \left(t_{n-i}\left(b_i + c_i + d_i\right)\right) \\
b_n &= \Sigma_{i=1}^{n-1} \left(t_{n-i}\left(a_i + c_i + d_i\right)\right) \\
c_n &= \Sigma_{\{i,j,k,l,m \geq 1 | i+j+k+l+m=n\}} t_i t_j t_k t_l t_m \\
d_n &= \Sigma_{\{i,j,k,l,m \geq 1 | i+j+k+l+m=n\}} t_i t_j t_k t_l t_m
\end{aligned}
$$

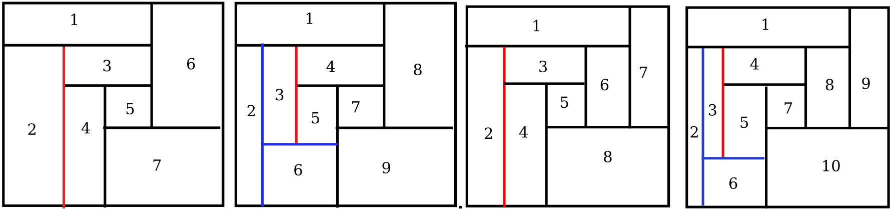
Outline

HFO − k Floorplans-An Infinite Hierarchy

Theorem

For any $k \geq 7$, $HFO - k \setminus HFO - (k-1) \neq \phi$.

Figure: Geometric Proof - Infinite Hierarchy



Simple Permutations

- $\pi \in S_n$ is simple iff there is **no non-trivial** set of consecutive elements which has all the numbers between the minimum and maximum of those set of elements.

Figure: Example Simple Permutation

[5 3] 6 1 4 2 — 5 [3 6] 1 4 2 — 5 3 [6 1] 4 2 — 5 3 6 [1 4] 2 — 5 3 6 1 [4 2]

[5 3 6] 1 4 2 — 5 [3 6 1] 4 2 — 5 3 [6 1 4] 2 — 5 3 6 [1 4 2]

[5 3 6 1] 4 2 — 5 [3 6 1 4] 2 — 5 3 [6 1 4 2]

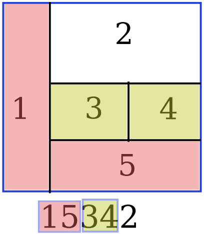
[5 3 6 1 4] 2 — 5 [3 6 1 4 2]

Figure: Example of a permutation which is **not simple**

5 [3 1 2] 4 6

Uniquely HFO − k permutations

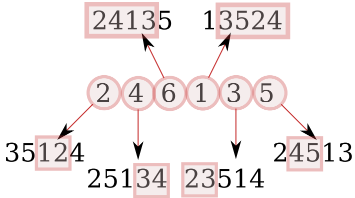
- It is a permutation corresponding to an HFO − k floorplan with k rooms such that it is not an $HFO - j$ floorplan for any $j < k$.
- A permutation π is Uniquely HFO − k if and only if it is a Baxter permutation of length k which is also a simple permutation.
- If it is not simple there is a block which is a set of rooms in an envoloping rectangle (3,4 in the diagram)



Exceptionally Simple Permutations

- It is a simple permutation π such that no single point deletion yields a simple permutation.

Figure: Example of an exceptionally simple permutation



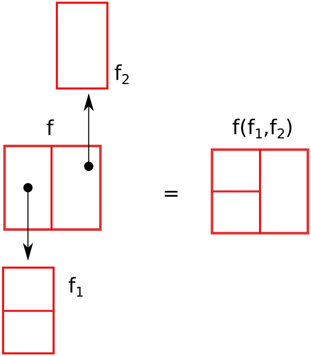
Characterization of HFO − k

Theorem

Permutations Corresponding to HFO − k floorplans are Baxter permutations which avoid patterns which are Simple Permutation of length $k+1$ or Exceptionally Simple permutations of length $k+2$.

Characterization of HFO − k Main Idea

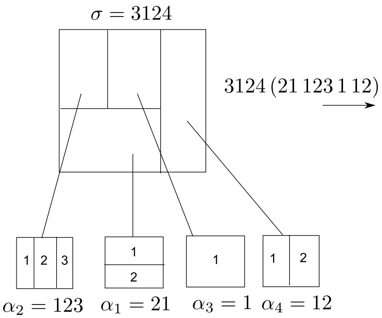
- Injection :



Characterization of HFO − k Main Idea

- Inflation : 2 1 [132, 1432] — 2(132) 1(1432) — (576) (1432)

Characterization of HFO — k Main Idea



Characterization of HFO − k[proof sketch]

- If it is mosaic(Baxter) and it is not HFO − k then it is HFO − j for some $j > k$.
- Let j be the smallest such that π is $HFO - j$. Then in the generating tree corresponding to π there will be an internal node whose sub-trees forms a pattern from Uniquely HFO − j permutation.
- Any Uniquely HFO − j permutation is also a simple permutation. Hence it contains a pattern from simple permutations of length j.

Theorem (M.H Albert, M.D Atkinson)

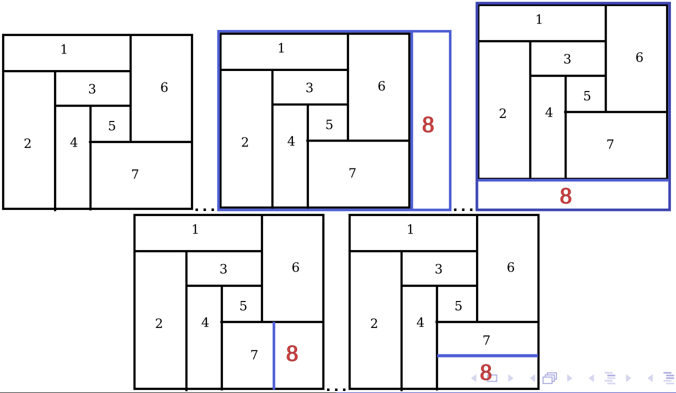
[2]If π is a simple permutation (of length j) then there is a one point deletion that is also simple (of length $j - 1$) or π is exceptional in which case it has a two point deletion which is simple (of length $j - 2$).

Outline

A Lowerbound

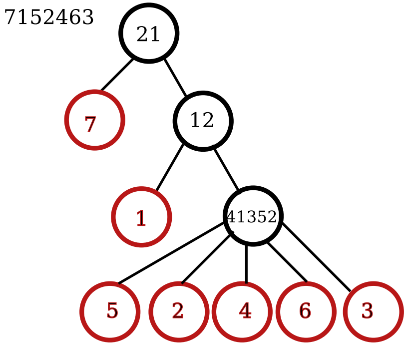
Theorem

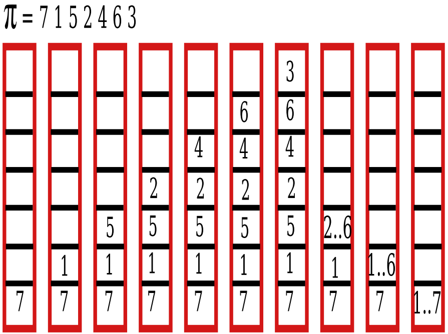
There are at least $O(3^n)$ $HFO - k$ permutations of length n which are not $HFO - j$ for $j < k$ for any $n > k$.



Outline

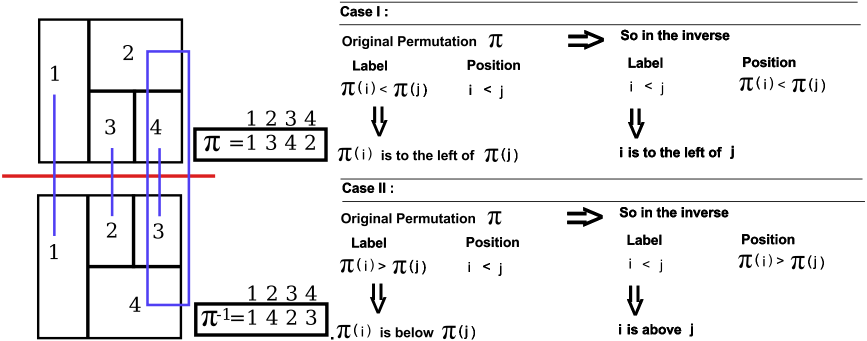
Algorithm for Recognition of HFO – k





Closure Under Inversion

- **Vertically flip the floorplan** corresponding to π to obtain floorplan corresponding to π^{-1}.



Summary

- Characterized Permutations corresponding to Abe-label of HFO – k permutations.
- Proved that there are exponentially more HFO – k floorplans than there are HFO – $(k-1)$ floorplans with n rooms for any $n > k$, $k \geq 7$.
- Gave a linear time algorithm for recognition of HFO – k and suggested moves for combinatorial optimization.
- Constructed an $O(n^2 \log n)$ algorithm to find out for what minimum k a given mosaic floorplan is HFO – k.
- Gave a poly-time algorithm for generating the count of HFO – k floorplans for any n.
- Open Questions
    - Exact count of HFO – k.

Thank You

Questions ?

References

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