Muscle and Joint Forces

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Chapters 4 and 5 (page 49-115)

Mechanical advantage of a lever

\[ \text{mechanical advantage (MA)} = \frac{\text{moment arm of effort force}}{\text{moment arm of resistance force}} \]

- If MA < 1, the lever amplifies motion.
  Most muscle-joint systems are 1st class or 3rd class levers that amplify motion.
- If MA > 1, then the lever amplifies force

Lecture outline

- components of joint force
- applications of static analysis:
  - mechanics of the elbow during lifting
  - mechanics of the shoulder during lifting
  - mechanics of the hip during single legged stance

Definitions: joint forces

- joint contact force: “bone on bone” or “cartilage on cartilage” force
- ligament force: tensile force in ligament that connects two bones. Typically significant only at extremes of motion.
- joint reaction force: sum of joint contact force + ligament force
- muscle force: tensile force in the muscle. Transmitted to bone via the muscle tendon.

\[ \sum(\text{external forces} + \text{joint reaction force} + \text{muscle force}) = 0 \]
How do I determine the direction of unknown forces?

- Both the magnitude and sense of joint reaction forces $F_{Jx}$ and $F_{Jy}$ are often unknown*
- What if, in setting up the free body diagram, incorrect directions are assumed for $F_{Jx}$ and $F_{Jy}$?
- THIS IS NOT A PROBLEM!
- An incorrect assumption regarding the direction of $F_{Jx}$ or $F_{Jy}$ will be signified by a minus sign in the solution i.e., a minus sign indicates that the force is directed opposite from that originally assumed
- * unlike muscle forces, which are always tensile and directed outward from the border of the FBD

How do I determine the direction of unknown forces? (Cont’d)

(4) in the equation $\Sigma M_O = 0$, refer to the moment generated by a force* as positive if it tends to cause rotation about point $O$ in the defined positive direction (typically CCW)

(5) if, upon solving for $F_{Jx}$ or $F_{Jy}$, one of these forces turns out to be a negative, the true sense of this force is opposite to that assumed in setting up the FBD

*NOTE: we often take moments about the joint centre, where $F_{Jx}$ or $F_{Jx}$ contribute no moment

How do I determine the direction of unknown forces? (Cont’d)

Steps to determining the sense of unknown forces:

1. (1) while creating your FBD, make an educated guess about the sense of $F_{Jx}$ and $F_{Jy}$, and draw these forces acting in the assumed directions
2. (2) when writing out the equation $\Sigma F_x = 0$, insert $F_{Jx}$ as positive if it has been drawn (i.e., assumed) to act in the positive $x$ direction, and negative if it has been drawn to act in the negative $x$ direction
3. (3) same for $F_{Jy}$ in the equation $\Sigma F_y = 0$

Solve for $F_{Jx}$, $F_{Jy}$, and $F_M$ for cases (A) and (B)

(A)

(B)


**Mechanics of the Elbow**

![Diagram](image)

\[ \sum F_y = 0 : \]
\[ -F_J + F_M - W - W_O = 0 \]
\[ F_J = F_M - W - W_O \]  (1)

\[ \sum M_O = 0 : \]
\[ F_M a - W b - W_O c = 0 \]
\[ F_M = W \left( \frac{b}{a} \right) + W_O \left( \frac{c}{a} \right) \]  (2)

Insert (2) into (1):
\[ F_J = W \left( \frac{b}{a} - 1 \right) + W_O \left( \frac{c}{a} - 1 \right) \]

**Remarks - elbow mechanics during support of a hand-held weight**

- the joint reaction force is approximately 8-fold greater than the weight of the held object
- vertical muscle force exceeds vertical joint contact force by an amount \( W + W_O \)
- supporting a load of 67 N (15 lb) requires a muscle force of approximately 650 N (body weight for a 145 lb individual)

![Graph](image)

\[ W = 20 \text{ N} \]
\[ b/a = 3 \]

typically, \( a = 4 \text{ cm}, b = 15 \text{ cm}, c = 35 \text{ cm}, \text{ and } c/a = 8.75 \)
Rotational and stabilizing components of muscle force

- Muscle forces can be resolved into rotational and stabilizing components.
- The relative magnitude of each depends on the angle $\theta$ between the line of action of the muscle force $F_M$ and the long axis of the bone upon which it inserts.
- If $\theta$ is 90 deg, then $F_M$ has only a rotational effect.
- If $\theta$ is less than 90 deg, then $F_M$ has only a rotational and stabilizing effect.
- If $\theta$ is greater than 90 deg, then $F_M$ has a rotational and dislocating effect.

Known: $a = 15$ cm, $b = 30$ cm, $c = 60$ cm, $\theta = 15$ deg, $W = 40$ N, and $W_O = 60$ N

Task: determine $F_M$, $F_J$, and $\beta$
Example problem

Estimate the hip joint reaction force $F_J$ generated during single-legged stance, as a function of body weight $W$. Use the free body diagram shown at right, and assume that $W^2 = \frac{5}{6} W$, $(c/a) = 2.6$, and $\theta = 70$ deg.

Review Questions

- What is the difference between joint contact force and joint reaction force?
- For a “type III lever” joint (e.g., elbow) is $F_J$ greater or smaller than $F_M$? What about for a “type I lever” joint (e.g., hip)?
- What are typical $(c/a)$ ratios for the elbow, shoulder, hip, and spine?
- What is the magnitude of hip reaction force during single legged stance?
- How can I tell whether the direction I initially assume for $F_J$ (in my FBD) is correct?