Inverse and forward dynamic models of movement
**Example.** Is the compressive component of joint reaction force \((F_{JC})\) at the L5/S1 vertebrae greater than the maximum safe value of 3.4 kN recommended by NIOSH?

**Answer:** YES, the load is beyond the safe recommended value. \(F_{JC} = 4.38 \text{ N},\) or approx. 6 times body weight.
Newton’s Second Law of Motion

**General:**

\[ \sum F_x = ma_x \quad \sum M_x = I \alpha_x \]
\[ \sum F_y = ma_y \quad \sum M_y = I \alpha_y \]
\[ \sum F_z = ma_z \quad \sum M_z = I \alpha_z \]

**2D:**

\[ \sum F_x = ma_{CG_x} \]
\[ \sum F_y = ma_{CG_y} \]
\[ \sum M_{CG} = I_{CG} \ddot{\theta} \]
Example. Inverted pendulum

\[ \Sigma M_0 = I_0 \ddot{\theta} : \]

\[ mg \frac{l}{2} \sin \theta - T_A = I_0 \ddot{\theta} \]

\[ I_0 \ddot{\theta} - mg \frac{l}{2} \sin \theta = -T_A \]
Example. Mass-spring-damper model

\[ \sum F_x = m\ddot{x} : \]

\[ -b\dot{x} - kx + F(t) = m\ddot{x} \]

\[ m\ddot{x} + b\dot{x} + kx = F(t) \]
Forward and inverse dynamics models

\[ M(\phi)\ddot{\phi} = T^{\text{mus}} + V(\phi, \dot{\phi}) + G(\phi) \]

\[ \ddot{\phi} = M^{-1}(\phi)T^{\text{mus}} + M^{-1}(\phi)V(\phi, \dot{\phi}) + M^{-1}(\phi)G(\phi) \]

\[ T^{\text{mus}} = M(\phi)\ddot{\phi} + V(\phi, \dot{\phi}) + G(\phi) \]

**Inverse dynamics**: estimate joint torques from applied forces and segment motions

**Forward dynamics**: estimate segment motions from joint torques and applied forces
Modeling guidelines

(1) Select the simplest possible model (i.e., the minimum set of variables) capable of:
   a) describing the essential behaviour or dynamics of the system, and
   b) addressing the research questions of interest.

(2) Ensure that the model parameters and initial conditions can be characterized (or measured through experiments).

(3) Ensure the model equations can be solved.
Inverse dynamics models: estimating joint torque from motion and external force

(a) measure joint angular positions $\phi_i$ during movement

(b) differentiate joint positions $\phi_i$ to get velocities and accelerations

(c) multiply joint accelerations by inertia matrix $M(\phi)$ to estimate joint torque components $T_i$ due to inertial forces; add torque components due to gravity and contact forces (not shown) to estimate total joint torque

(d) use optimization routine to estimate individual muscle forces $F_i^m$, if required
**Example: Ankle moments during gait.**

Consider the free body diagram at right, which shows forces and accelerations applied to the foot during the terminal stance phase of gait.

- **Parameters that are directly measured:** \((F_{\text{APP}})_X, (F_{\text{APP}})_Y, c, d\)
- **Parameters that are estimated from anthropometry:** \(m_F, b\)
- **Parameters that are calculated from numerical differentiation:** \(a_X, a_Y, \alpha\)
- **Parameters that are calculated from inverse dynamics:** \(R_X, R_Y, M_A\)
Ankle moments during gait (cont)

(b) Determine the equations for calculating $R_X$, $R_Y$, and $M_A$.

\[
\sum F_X = m_F a_X
\]
\[
R_X + (F_{APP})_X = m_F a_X
\]
\[
R_X = m_F a_X - (F_{APP})_X
\]

\[
\sum F_Y = m_F a_Y
\]
\[
-R_Y - m_F g + (F_{APP})_Y = m_F a_Y
\]
\[
R_Y = (F_{APP})_Y - m_F g - m_F a_Y
\]

\[
\sum M_0 = I_0 \alpha
\]
\[
-M_A - m_F g \cdot b + (F_{APP})_Y \cdot c
\]
\[
+ (F_{APP})_X \cdot d = I_0 \alpha
\]
\[
M_A = (F_{APP})_Y \cdot c + (F_{APP})_X \cdot d - m_F g \cdot b - I_0 \alpha
\]
Inverse dynamics models: advantages and disadvantages

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>• allow estimate of net muscle moments acting at joints from experimental data</td>
<td>• cannot determine individual muscle contributions (without further optimization steps)</td>
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<tr>
<td>• estimated moments are “quasi-experimental data” and therefore “correct”</td>
<td>• simulations usually restricted to experimentally measured conditions (no “what-if” type exploration)</td>
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<td>• provides hints regarding the role of muscle groups in coordinating movement and energy exchanges</td>
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Moment of Inertia

The moment of inertia $I$ of an object about a specific axis is

$$I = \sum m_i(r_i)^2 = \int r^2 \, dm$$

where $r$ is the distance from the mass $m_i$ (or mass element $dm$) to the axis of interest. For the object shown at left:

$$I_X = \sum m_i(y_i)^2 = \int y^2 \, dm$$

$$I_Y = \sum m_i(x_i)^2 = \int x^2 \, dm$$

The moment of inertia reflects that a body has an inertia for turning that depends not only on the masses, but also on how far away they are from the axis of rotation.

Clearly, $I_x > I_y$ in this example

Moment of inertia has units of $[\text{kg}][\text{m}^2]$
The Parallel Axis Theorem

If the moment of inertia about an axis passing through the center of gravity is known, the moment of inertia about a second, parallel axis is given by:

\[ I = I_{CG} + m_{TOTAL}(r_{CG})^2 \]

where
\[ I_{CG} = \text{MOI about axis passing through the CG} \]
\[ m_{TOTAL} = \text{total mass of the object} \]
\[ r_{CG} = \text{perpendicular distance between axes} \]
**Example.** Moment of inertia

Show that a narrow rod with uniformly-distributed mass (often used to model body segments) has (a) a moment of inertia about its center of gravity equal to $I_{CG} = \frac{1}{12} ml^2$, and (b) a moment of inertia about its end equal to $I_0 = \frac{1}{3} ml^2$. 
Forward dynamics models: estimating motion from muscle force (or joint torque)

(a) CNS sends neuromuscular excitation signals $EMG^i$ to muscle

(c) forces $F^i$ produce joint torques $T_{i\text{mus}}$ depending on muscle moment arms

(b) muscle acts to low pass filters the $EMG^i$ signals to produce muscle forces $F^i$

(d) joint torques (multiplied by inertia matrix inverse) result in angular accelerations, and subsequent changes in joint angular velocities and positions
**Example:** limb acceleration by soleus and gastrocnemius

\[
\begin{bmatrix}
\ddot{\phi}_{\text{ankle}} \\
\ddot{\phi}_{\text{knee}} \\
\ddot{\phi}_{\text{hip}}
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_{11} & \cdots \\
\alpha_{21} & \cdots \\
\alpha_{31} & \cdots
\end{bmatrix}
\begin{bmatrix}
T_{\text{SOL}}^{\text{ankle}} \\
0 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
\ddot{\phi}_{\text{ankle}} \\
\ddot{\phi}_{\text{knee}} \\
\ddot{\phi}_{\text{hip}}
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_{11} & \alpha_{12} & \cdots \\
\alpha_{21} & \alpha_{22} & \cdots \\
\alpha_{31} & \alpha_{32} & \cdots
\end{bmatrix}
\begin{bmatrix}
T_{\text{GAS}}^{\text{ankle}} \\
T_{\text{GAS}}^{\text{knee}} \\
0
\end{bmatrix}
\]

*Zajac and Gordon, 1989*
Example: Maximum height jumping

Research questions:
(1) to achieve maximum propulsion during jumping, should limb muscles be fully excited?
(2) if so, which ones and when?
(3) what is the role of uniarticular and biarticular leg muscles?

Pandy and Zajac, 1991
Jumping model

- Model has four segments (feet, shins, thighs, HAT) and eight muscle groups (5 uniarticular and 3 biarticular; OPF = “other plantarflexors”)

- Muscles have Hill-type force-velocity characteristics, user-defined shortening speed and PCSA
Why does a countermovement enhance jump height?

- Vertical velocity is increased by increasing the duration and magnitude of acceleration (i.e., vertical GRF) during the upward propulsion phase.
- The countermovement increases acceleration by (a) allowing increased stretching and energy storage in tendons, (b) generating increased force in (eccentrically-contracting) muscle, and (c) allowing muscles increased time to develop force.
What is the role of individual muscles in jumping?

- Simulations of optimal height jumping indicate that the majority of propulsive power comes from uniarticular extensor muscles (GMAX, VAS, uniarticular plantarflexors (UPF))
- In contrast, biarticular muscles (GAS, HAMS, RF) act to fine-tune coordination just before take-off
- The real value of the model is to indicate how performance is affected by specific changes to individual muscles, or groups of muscles
# Forward dynamics models: advantages and disadvantages

<table>
<thead>
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<th>Advantages</th>
<th>Disadvantages</th>
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<tbody>
<tr>
<td>• model can predict the effect on system performance of variations in individual parameters (e.g., individual muscle strengths, timing of activation, etc.)</td>
<td>• experimental data are required to validate model’s predictive ability</td>
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<tr>
<td>• valuable design tool</td>
<td>• input parameters may be difficult to estimate (e.g., muscle properties)</td>
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<tr>
<td>• theoretically, provides a means for examining role of individual muscles in coordinating movement</td>
<td>• model is often formulated to work under highly restrictive conditions (e.g. both feet flat on floor)</td>
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<td></td>
<td>• complex models may be difficult to program, and results may be difficult to interpret</td>
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**Example: mechanics of backward falls**

Research questions:

1. How is kinetic energy at impact during a backward fall affected by muscle contractions during descent?

2. How is ability to reduce impact energy affected by variations in parameters representative of strength, flexibility, and reaction time?

*Sandler and Robinovitch, 2001*
Modeling backward falls

- $KE_v$
- $W_{tot}$
- $KE_{rot}$
- $\Delta PE$
- $KE_h$

Graphs showing energy (J) vs. strength factor (percent) for one-link, two-link, and three-link systems.
Modeling backward falls

- Model shows that, while the change in PE during the fall is nearly 300 J, only about 100 J ends up as vertical KE at impact.

- Strong individuals can “transform” about 100 J from vertical KE to horizontal KE.

- Even with 80% declines in strength, individuals can reduce vertical KE at impact by about 50%.

- This reflects that joint energy absorption depends more on technique (lots of joint rotation) than strength.