

# Knowledge

## 1. THE SUBJECT MATTER AND THE SCIENCE OF LOGIC

Each of the first two chapters has been devoted to one of two fundamental kinds of entity which we have posited in order to make philosophical sense of logic: possible worlds and propositions, respectively. Given these two kinds of entities it is easy to characterize the *subject matter* of logic. The subject matter of logic, we may now say, comprises the modal properties and relations which propositions have as determined by the ways in which their truth-values are distributed across the set of all possible worlds. Propositions *about* these modal properties and relations may, with obvious propriety, be called *propositions of logic* or, more briefly, *logical propositions*.

Now it is clear from the account we have given of propositions that, in a perfectly straightforward sense, the subject matter of logic exists independently of whether or not anyone ever studies it. The facts, or true propositions, of logic await our discovery in just the same sort of way as do the propositions which make up the subject matter of physics, chemistry, biology, psychology, history, and the like.

Propositions have certain modal properties and stand in certain modal relations whether or not human beings ever come to know that they do. (Compare this with the claim that copper can conduct electricity. This was a fact of physics before human beings discovered it.) In short, the subject matter of logic may be — and has here been — described without invoking any concepts concerning the nature and scope of human knowledge (i.e., without invoking any *epistemic* concepts). In this regard it differs not at all from the subject matter of physics, chemistry, and the like.

But the subject matter of logic no more constitutes a *science* of logic than the subject matter of physics constitutes a science of physics. When we talk about any sort of science we are talking about an organized body of known propositions, or, at the very least, well-confirmed believed propositions, *and* a distinctive methodology for obtaining knowledge of the subject matter of that science. It is by virtue of this latter fact — that the methodology it adopts is a part of the very characterization of a science — that we can say, for example, that modern physics is importantly different from ancient or scholastic physics. It is not so much the subject matter which has changed (although it certainly has) but, more important, its methodology.

If we wish to talk about the science of logic we shall have to talk about our knowledge of the subject matter of logic and about how it is possible to have, and how we do have, knowledge of the modal attributes of propositions. We shall have to see how we can, and how we do in practice, have

knowledge of the ways in which the truth-values of propositions are distributed across the set of all possible worlds.<sup>1</sup> This we shall do in chapter 4.

In this chapter we prepare the way by asking the preliminary questions: What, exactly, is it to have knowledge? What is the scope of human knowledge? In what way or ways, can knowledge be acquired? And, finally: Is our knowledge of the subject matter of logic obtainable by methods which are in any way distinctly different from those employed in such sciences as physics and chemistry?

## 2. THE NATURE OF KNOWLEDGE

There are many uses of the verb “to know” and its cognates. We can speak of knowing a person (a tune, a city, etc.); knowing *of* a good doctor (of the answer to a question, etc.); knowing *why* the unemployment rate is rising (why the landslide occurred, etc.); knowing *where* to go for a good meal (where the capital of British Columbia is situated, etc.); knowing *how to* ski (how to get an “A” in Philosophy, etc.); knowing *how* a clock works (how it is possible that the engine stalled, etc.); knowing *that* snow is white (that  $2 + 2 = 4$ , etc.); and so on.

Of these different uses, some but not all can fairly be explicated in terms of the last mentioned, viz. the one in which the verb “knows” takes as its grammatical object an expression referring to a proposition.<sup>2</sup> Sometimes this is conveniently called “knowing that”, but it should be clear that so-called “knowing that” is not confined to cases where the English verb “knows” is followed by a ‘that’-construction referring to a proposition. Each of the following counts as a case of “knowing that”:

(3.1) She knows that it is 2:30 P.M.

(3.2) He knows what she told him.

(3.3) They know whether it is raining in Chicago.

A more apt description of this sort of knowledge is “propositional knowledge”. It is more apt since it does not make appeal to the vagaries of English usage; indeed, it does not make appeal to any linguistic items at all. We prefer, then, to speak of “propositional knowledge” rather than of “knowing that”. (It was, of course, propositional knowledge that we were talking about when — at the very end of chapter 2 — we discussed a Gettier-type possible-worlds parable which could be brought to bear on the traditional analysis of knowledge.)

Our concern in this chapter is solely with propositional knowledge or, as it is sometimes put, our knowledge of propositions.

Unfortunately both the expressions “propositional knowledge” and “knowledge of propositions” suffer a certain ambiguity. Consider the following two sentences:

(3.4) “They know all the axioms of Euclidean geometry”,

and

(3.5) “He knows the proposition expressed by ‘ $2 + 2 = 4$ .’”

1. Our conception of the subject matter and science of logic differs little, if at all, from that of Strawson. In his introduction to *Philosophical Logic* (Oxford, Oxford University Press, 1967, p. 1), he describes logic itself (what we have called the *science* of logic) as “the general theory of the proposition” and speaks of the subject matter of logic as “relations of deducibility and implication between propositions”.

2. It is not at all clear for the case of *knowing how to*. Indeed Gilbert Ryle has even suggested that some cases of *knowing that* are just as plausibly explicated in terms of *knowing how to* as vice versa. See *The Concept of Mind*, London, Hutchinson, 1949, chap. 2.

These sentences are ambiguous between reporting that certain persons (1) are acquainted with certain propositions, and (2) know the truth-values of those propositions. Similarly, if we were to ask the question, “How do we come to have knowledge of propositions in general and of the propositions which are the subject matter of logic in particular?”, our question would be open to two different interpretations.

The interrogative sentence “How do we have knowledge of such and such a proposition?” may be construed in either of two ways: (i) as asking how we come to be acquainted with, or to entertain in our minds, a given proposition; and (ii) as asking how we come to have knowledge of the *truth-value* of a given proposition. Construed in the manner of (i), most persons are likely to answer the question by saying that they had heard the proposition discussed, seen it expressed in writing, or in some other way been prompted by their sense-experience to entertain it. Construed in the manner of (ii), however, the question needs to be answered in a very different sort of way. In order to “have knowledge of a proposition” in sense (ii) it is necessary that one have knowledge of that proposition in sense (i). But it is by no means sufficient. Hence, even if it were the case (as some would hold) that it is only through experience that we can come to be acquainted with propositions, it by no means follows that our knowledge of the truth-values of propositions is possible only through experience.<sup>3</sup> As we shall see later, it is not only conceivable but also seems to be true that some propositions which we come to entertain as a result of experience are *known* to be true not as a result of experience but by reflection or what some philosophers have called “unaided reason”. When, hereinafter, we ask about our knowledge of propositions we are to be understood as asking question (ii). That is, we shall be asking how we can, on occasion, know a proposition to be true (or false). Or, in still other words, we shall be asking how we can, on occasion, know *that* a proposition is true (or false).

Before pursuing that question, however, we need to be a little clearer about the analysis of the notion of propositional knowledge itself. What does it mean to say that a person, *a*, knows that *P*?

Gettier<sup>4</sup> has shown that the traditional analysis of propositional knowledge as justified true belief falls short of providing a set of sufficient conditions. But does it not at least provide a set of necessary conditions?

Even that has been disputed by some philosophers. Let us consider briefly some of the objections that have been raised: first, to the claim that what is known must be true; second, to the claim that what is known must be believed; and third, to the claim that the belief in what is known must be justified.

1. *Is it a necessary condition of the truth of a's knowing that P, that P should be true?*

It is sometimes pointed out that knowledge-claims are often made about propositions which turn out to be false. In the Middle Ages, for instance, most persons claimed to know — on the basis of their own observation and the authority of the Holy Scriptures — that the earth is flat. Do not examples like this show that the truth of a proposition is not after all a necessary condition of knowledge of that proposition? Hardly. What they show is that the truth of a proposition *P* is not a necessary condition of the truth of the proposition that someone has *claimed* to know that *P*. But that is very different from showing that the truth of *P* is not a necessary condition of the truth of the proposition that someone has *claimed truly* to know that *P*, where by “claimed truly to know that *P*” we do *not* mean “claimed

3. Kant was making somewhat the same point when he wrote: “. . . though all our knowledge begins with experience, it does not follow that it all arises out of experience.” *Critique of Pure Reason*, Introduction, B1, Kemp Smith translation, London, Macmillan, 1950.

4. Recall our discussion of Gettier's paper “Is Justified True Belief Knowledge?” at the end of the section on possible-worlds parables, in chapter 2.

sincerely to know that P” but rather “claimed to know that P where the claim to know is itself a true claim (i.e., it is true that one knows that P)”. If the proposition which *a* claims to know is false, we want to say, then *a*’s claim to know, i.e., the proposition that *a* knows that P, is also false.

It will not do to object that persons in the Middle Ages were nevertheless *justified* in believing, on the evidence then available to them, that the earth is flat. For even if this were so — even if, as we sometimes say, “to the best of their knowledge” the earth is flat — there is a gulf between being justified in claiming to know P and its being true that one knows P. And that gulf can only be bridged by the condition that P actually be true.

Nor will it do to point out that many of the knowledge-claims that *we* make might possibly turn out to be false. In the first place, the claim that one cannot have knowledge that P unless P is true must not be construed as implying that P could not possibly be false if it is known to be true. Contingent propositions (all of which are possibly false), as well as necessary ones, may be true — and may be *known* to be true. And secondly, if it does turn out that a proposition which we claim to know to be true is not true but false, then we — like our forebears — are logically obliged to give up the claim to know it. Knowledge-claims are perfectly compatible with human fallibility. If, contrary to everything that we now with justification believe to be the case, it were to turn out that the earth is flat after all, then we, not our forebears, would have to give up one of our knowledge-claims.

But, it may be said, if we allow that certain of our claims to know may be mistaken, does it not follow that we can never *know* that we know anything? The suggestion that it does follow has lured many thinkers — including some contemporary philosophers — into the radically sceptical position of doubting that we can ever know anything at all. But does it follow? In the first place, it may help to remind ourselves that not all our knowledge-claims are as susceptible to doubt as are those on which the sceptic tends to concentrate attention. We can allow that many, or even all, the knowledge-claims made in contemporary physics, for instance, may turn out to be in need of correction. If so, then as they stand they are mistaken. And if what is claimed to be true is not true but false then it does follow that we do not know it to be true. But there is no need to suppose with the sceptic that *all* the propositions that we currently claim to know are false. Why should we not equally suppose some of them to be true? If it is possible that a contingent proposition is false, it is also possible that it is true. And if a proposition *is* true then — provided, of course, that the other necessary conditions for knowing are satisfied — the sceptic’s objection to our claim that we know that proposition to be true deserves to fall on deaf ears. Secondly, even if we were to allow that any given proposition which we claim to know might turn out to be false, it still would not follow that *all* our knowledge-claims might be mistaken. If this latter proposition — that all our knowledge claims might be mistaken — were true, and the sceptic not only believed it to be true but also claimed that he was justified in believing it to be true, then this very proposition would itself constitute a counterexample to the sceptic’s thesis.<sup>5</sup> For it would itself be a proposition which the sceptic knew to be true. In short, if the sceptic tries to press his case by saying that his thesis is true and that he knows it to be true, he refutes himself.

One cannot without self-contradiction claim to know that nothing can be known. The worry about how one can ever *know* that one knows anything turns out to be no more threatening than the original worry about whether one can ever know a proposition to be true. If we claim to know that P and P turns out to be false then it is false that we know that P and false also that we know that we know that P. But it is demonstrable that some of the propositions that people claim to know are *not* false. The proposition that one *cannot* know that nothing can be known is a case in point. And there are many others besides.

One last point deserves mention. How, it might be asked, can the thesis that if *a* knows that P then P is true be reconciled with the fact that we sometimes claim — and claim truly — to know that a

5. Of course, a sceptic may well resist being lured into making this further claim and thus avoid the charge of self-refutation. Sophisticated scepticism is not as easily refuted as the more naive version presented above.

proposition P is false? This worry, though it deserves mention, hardly deserves serious attention. For it stems from a simple confusion. Consider the claim that *a* knows that the proposition

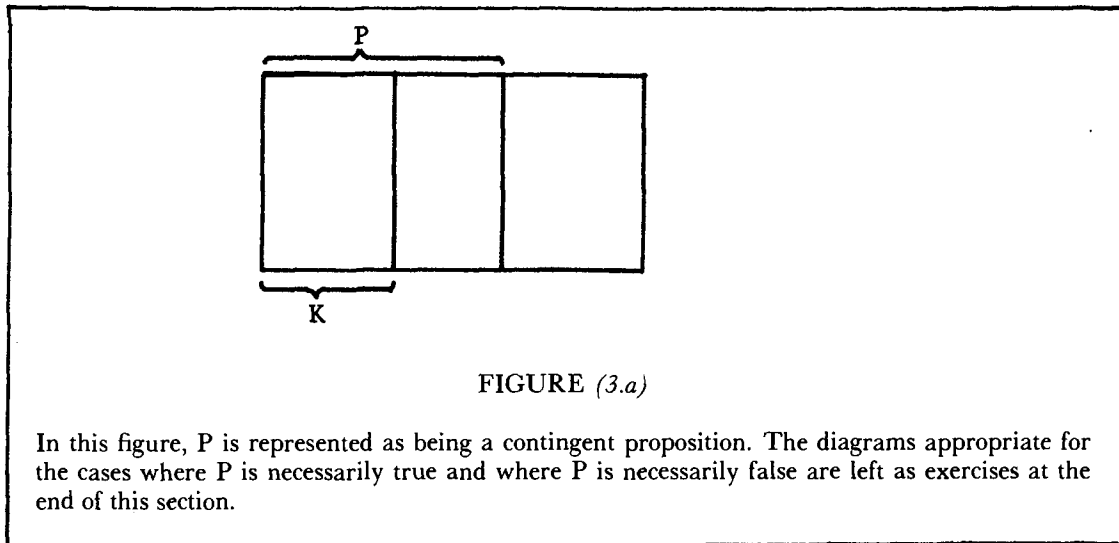
(3.6) Canada is south of Mexico

is false. Then the proposition which *a* knows when *a* knows that (3.6) is false is not the false proposition that Canada is south of Mexico but rather the true proposition that (3.6) is false. That is to say, what *a* knows when *a* knows that (3.6) is false is not (3.6) but

(3.7) The proposition that Canada is south of Mexico is false

which is, of course, true. The traditional analysis which holds that knowledge that P implies the truth of P has nothing to fear from objections like these.

The relation between the proposition that *a* knows that P and the proposition that P is true may be depicted on a worlds-diagram. If we let "P" represent a contingent (knowable)<sup>6</sup> proposition, and let "K" represent the proposition that *a* knows that P, we will have:



Every possible world in which *a* knows that P (i.e., in which K is true) is a world in which P is true; but the converse does not hold. There are possible worlds in which P is true but in which *a* does not know that it is so.

2. *Is it a necessary condition of a's knowing that P, that a believe that P?*

One argument that has sometimes been advanced against the claim that knowledge implies belief stems from the fact that persons sometimes utter, quite sincerely, sentences such as: "I know it's true; but I just can't believe it" or "I know it's true; but it will take me some time to believe it." Sentences like these may well be uttered in the face of some unexpected momentous or shocking event, e.g., the winning of a large lottery, the outbreak of war, the assassination of a political figure, the death of a

6. We do not assume that all propositions are knowable. See the next section.

close friend or relative. Does that not mean that there are, after all, possible worlds — the actual one among them — in which a person may know that P and yet not believe that P? At least two answers may be given.

One answer is simply to point out that merely saying that something is so does not make it so. Thus, it might be said, from the fact that persons sometimes *claim* to know what they do not believe, it does not follow that what they claim is true or even that what they claim is self-consistent. Indeed, if — as the traditional analysis holds — knowledge does imply belief, then it is no more possible to know what one does not believe than it is possible to square the circle. Some persons have claimed the latter just as some persons have claimed the former. But neither is *really* possible; hence, both these claims are mistaken.

The trouble with this first rejoinder is that it appears to beg the question by simply assuming the truth of that which is being disputed. Further, it is too harsh — because too literalistic — in its construal of what proposition a person asserts when uttering one of the sentences in dispute. The second rejoinder is not subject to these criticisms.

The second rejoinder begins from the recognition that the proposition someone asserts when uttering such a sentence as “I know it; but I just can’t believe it”, is not at all paradoxical or self-contradictory. Are such persons *really* claiming that they don’t believe what they know? A more charitable interpretation is that of supposing that they are indulging in hyperbole when they say “. . . but I just can’t believe it” and that what they are really asserting might more perspicuously be expressed by saying something like: “. . . but I find it astounding that this should have happened”, “. . . but I find it hard to imagine what things will be like now” or “. . . but I find it hard to adjust my thought and action to fit the facts”, or some combination of these or similar expressions. But if this is what is being said, then these putative counterexamples to the thesis that knowing implies believing lose their sting. They are not genuine counterexamples at all. On this interpretation, a person who knows that P also believes that P no matter how difficult he finds it to readjust his expectations and to get his emotions in order.

This second line of defence of the traditional analysis that knowledge implies belief is sometimes itself challenged. Certain persons have argued that both the first objection and the two replies thereto take it for granted that the claim that *a* knows that P is consistent with the claim that *a* believes that P. “But,” these latter persons would object, “careful attention to how we actually *use* the two expressions ‘I know’ and ‘I believe’ would reveal that, far from its being the case that *a*’s knowing that P implies *a*’s believing that P, the claim that *a* knows that P implies that *a* does *not* believe that P.” After all, it would be said, if *a* knows that P then it is wrong for *a*, or anyone else, to say merely that *a* believes that P since this would suggest that *a* does not know it. What are we to make of this objection?

The first point to note is that even if, as is alleged, we do not ordinarily say that we believe that P when we feel entitled to make the stronger claim that we know that P, it by no means follows that believing that P *implies* not knowing that P. It may be true that if *a* merely claims to believe that P other persons will *infer* that *a* doesn’t know that P. But this proves nothing. Inference is not always geared to implication. Not all inference is valid inference. My saying “I believe that P” may *suggest* that I do not know that P; and others may be led to infer the latter. But my saying “I believe that P” does not *imply* that I do not know that P; and anyone who infers the latter from the former has made a deductively invalid inference.

That brings us to a second rejoinder. The objection derives much of its plausibility from failure to distinguish between a person’s merely claiming to believe that P and that person’s claiming to merely believe that P.<sup>7</sup> Now it is clear that if a person, *a*, claims to merely believe that P then what *a* is

7. Failure to make the distinction may be engendered in part by the fact that one and the same ambiguous sentence, viz., “*a* claimed merely to believe that P”, may be used sometimes to express one proposition, sometimes to express the other.

claiming is inconsistent with *a*'s knowing that P. Knowledge that P is incompatible with *mere* belief that P since by "mere belief" we mean "belief which does not qualify as knowledge". *Mere* belief that P is belief which falls short of being knowledge insofar as at least one of the other conditions besides belief which is required for knowledge is not satisfied, e.g., *a* may believe P in the case where P is false; or *a* may believe P, but not be justified in believing P, etc. If, then, *a*'s merely claiming to believe that P implied *a*'s merely believing that P, it would follow that what *a* claims when *a* merely claims to believe that P, viz., that *a* believes that P, would also be inconsistent with *a*'s knowing that P. And if this were the case then the proposition that *a* knows that P would not imply the proposition that *a* believes that P.<sup>8</sup> But does the proposition

(3.8) *a* believes that P

imply the proposition

(3.9) *a* merely believes that P?

Plainly not. To be sure, (3.9) implies (3.8); but that is another matter. We can allow that knowledge does not imply, and indeed is incompatible with, *mere* belief. But we cannot conclude from this that knowledge does not imply, or that it is incompatible with, belief *tout court*.

Much of what we have been saying about the relations between knowledge, belief, and mere belief may profitably be depicted on one worlds-diagram.

- Let "K" = *a* knows that P;
- "B" = *a* believes that P;
- "M" = *a* has a mere belief that P.

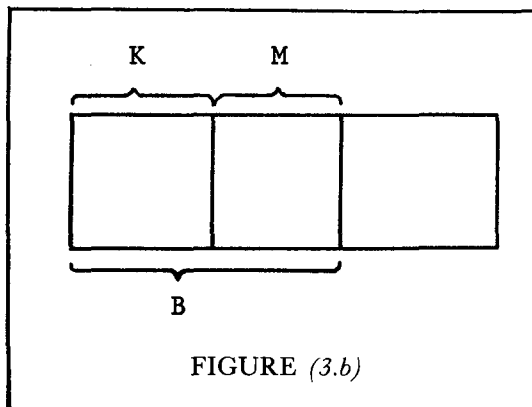


FIGURE (3.b)

8. Unless, of course, all propositions which assert that someone, *a*, knows a proposition, P, were self-contradictory. But they are not.

3. *Is it a necessary condition of a's knowing that P, that a be justified in believing that P?*

One objection to the third condition stipulated in the classical analysis of knowledge stems from the fact that persons sometimes make knowledge-claims about their lucky guesses. For instance, a person *a* may feel quite certain that he will win the next lottery and, when he does win it, say "I just knew I would." The objection to the classical analysis is that here we have a case where a person knows that *P* but where nevertheless only the first two conditions are satisfied while the third is not. How seriously should we take this alleged counterexample?

Once again, not everything that a person says must be taken at face value. Do persons whose hunches or intuitions about the outcomes of lotteries turn out right really mean to claim that they had antecedent *knowledge* of those outcomes? Put the question to such persons and it is likely that their answer will be in the negative and that what they intended was simply that they had a strong or even overpowering conviction about the outcome. Or, if this answer does not emerge at first we might further ask *how* they knew — on what *grounds* their claim to knowledge was being made — and, in all probability, they will then drop the claim to knowledge and settle for an avowal of subjective conviction instead. But subjective conviction is not the same as justification. Once the distinction has been pointed out, most persons will accept the correction. Of course, they didn't *really* know, although the strength of their belief was the same as if they had known. Or if they do not admit this, we feel that they do not fully understand what knowledge is.<sup>9</sup>

More serious difficulties arise when we try to say what it is for a belief to be justified.

What is it for a belief to be justified? Some philosophers have thought that the relation of justification is a relation which can hold only between propositions. Accordingly, they have said that the belief in *P* can be justified only if *P* is either implied by, or is evidentially supported (i.e., probabilified)<sup>10</sup> by, some *other* proposition, *Q*. But if this is one's view of what the relation of justification consists, one is easily led into circularity or an infinite regress. For the very same question about justification which we asked in connection with *P* may also be asked in connection with *Q*. What justifies us in believing *Q*? On the analysis just reported, it can only be some other proposition, *R*. But if *R* is identical with *P*, then our attempt at justification ends in circularity. And if *R* is not identical with *P* then we are embarked on a search for further justifying propositions — a search which must either end in circularity or regress to infinity. In our search for ultimate justification we seem trapped within the infinite domain of propositions with no chance of making a satisfactory independent appeal.

There are two main ways out of this difficulty. One is to point out that the notion of justification need not, indeed should not (on pain of circularity or infinite regress), be identified with that of *ultimate* justification. It is obvious that *a*'s true belief that *P* is not justified by appeal to *Q*, where *Q* either implies or evidentially supports *P*, if *a* believes *Q* to be false, i.e., *a* must believe *Q* to be true if *a* is to justify his belief in *P* by appeal to *Q*. But must the belief that *Q* be itself justified in order that the belief that *P* be justified? Ordinarily, it might be said, we would count the true belief that *P* as a justified true belief provided that it is supported by some other true proposition *Q* — whether or not belief in *Q* is itself justified. Admittedly, the question about *Q*'s justification can in turn be raised. But, it would be insisted, that is *another* question.

Another, more fruitful, approach is to be found in offering an alternative analysis of the relation of justification. Let it be admitted that on many occasions when we ask for the justification of a proposi-

9. This comment is not quite as question-begging as it seems. If a high percentage of English-speakers were to insist that knowledge was nothing more than true belief, we might want to say that the expression "*a* knows that *P*" was ambiguous or that there are *two* senses of "knowledge" according to only one of which is it correct to analyze knowledge as involving justification.

10. A possible-worlds analysis of probability is sketched in chapter 6, section 11.



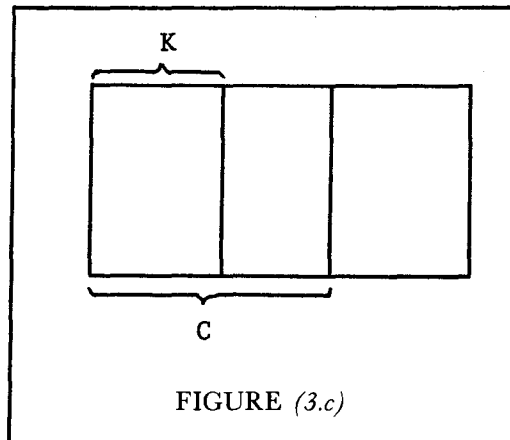
tion, P, our answer consists in appealing to some other proposition, Q. But must all propositional justification be itself propositional? If we were to allow, for instance, that experience itself — not just propositions about experience — could justify belief in the truth of a proposition, the bonds of propositional justification would be broken. Or again if we were to allow that the exercise of reason — not just propositions about the exercise of reason — could justify belief in the truth of a proposition, once more we could evade both circularity and infinite regression.

When, in sections 4 and 5 of this chapter, we investigate various *modes* of knowledge — experiential and ratiocinative, empirical and a priori — we will in effect be adopting this second rejoinder to the objection that justification is in principle impossible to achieve and consequently cannot be a necessary condition of knowledge. For the present, however, we have said enough to defend the traditional view that a person, *a*, can truly be said to know that P only if P is true, *a* believes that P and *a*'s belief that P is justified.

The relation between a person's knowing a proposition P and a person's having a justified true belief that P may easily be depicted on a worlds-diagram.

Let "K" = *a* knows that P,

"C" = *a* has a justified true belief that P.



Before Gettier produced his telling counterexamples (see section 8 in the previous chapter), it was thought that K and C are logically equivalent. Now we know differently: C may be true (i.e., a person may have a justified true belief that P) without K being true (i.e., without that person knowing P). C, it turns out, is 'broader' than K, i.e., C is true in all possible worlds in which K is true and is true in more besides.

In the period since Gettier's paper appeared, philosophers have sought some further condition to conjoin to the classical three so as to make the new analysis narrower; to contract it so that the possible worlds in which it is true are precisely those worlds in which *a* knows that P is true.

#### 4. What might the missing fourth necessary condition for *a*'s knowing that P be?

One of the most promising answers — and the one which we shall adopt here if only for the sake of getting on with the main business of this chapter — is to be found in treating the concept of knowledge as in part a *defeasible* concept. By "a defeasible concept" we mean a concept for which the conditions

of application can be stated only by including at least some *negative* clauses. For instance, it has been argued that the concept of responsibility is a defeasible concept since the ascription of responsibility for an act to a person, *a*, who otherwise satisfies the standardly recognized conditions for being responsible may be 'defeated' by evidence that, for example, *a* was *not* of age, was *not* acting freely, was *not* sane at the time, and so on.<sup>11</sup> In like manner, it has been suggested,<sup>12</sup> the concept of knowledge is defeasible since the ascription of knowledge to a person, *a*, who otherwise satisfies the standardly recognized conditions for knowing that *P*, may be defeated by evidence of a certain kind.

More particularly, the suggestion is that we should ascribe the property of knowing that *P* to a person, *a*, at a specific time *t* provided that (i) *P* is true; (ii) *a* believes at *t* that *P*; (iii) *a*'s belief that *P* is justified at *t*; and further, provided that (iv) *it is not the case* that there is some true disqualifying proposition, *Q*, such that if *a* had believed at *t* that *Q* then *a* would not at *t* have been justified in believing that *P*. When these latter two conditions, (iii) and (iv), are jointly satisfied, we may say that *a*'s belief in *P* is *indefeasibly justified*.

By means of the addition of the fourth, negative, condition we seem to be able to cater for Gettier-type examples. Consider, in its light, the case of the secretary who not only believes and says truly that the time is ten minutes past nine but also is justified in his belief since the clock at which he glances has never been wrong in forty years and gives that time. The Gettier-type objection arises if it happens that, at that precise time on that particular morning, the hitherto trustworthy clock — unbeknown to the secretary — has been at a standstill for twelve hours. For then, we want to say, the secretary's true belief that it is ten past nine is a merely fortuitous one and hence does not qualify as knowledge. It is; we should want to say, a *mere belief*. The inclusion of the fourth condition explains why. The secretary's justified and true belief that the time is ten past nine does not count as knowledge that it is ten past nine because there does exist a true proposition, *viz.*, that the clock has been stopped for some time, such that, were he then to believe *it* to be true, he would not be justified in believing that the time is ten past nine.

On this account, the traditional analysis does provide a set of three necessary conditions for knowledge; and these conditions, when supplemented by the fourth, negative, condition constitute also a set of sufficient conditions. On this account, then, knowledge is to be analyzed as *indefeasibly justified true belief*. Taken together, these four conditions are logically equivalent to *a*'s knowing that *P*.

### EXERCISES

Let "*P*" represent a contingent proposition which is known in some but not all of the possible worlds in which it is true, and

- let
- "*B*" = *a* believes that *P*
  - "*M*" = *a* has a mere belief that *P*
  - "*J*" = *a* has a justified belief that *P*
  - "*T*" = *a* has a true belief that *P*
  - "*C*" = *a* has a justified true belief that *P*.

11. See H.L.A. Hart's paper "The Ascription of Responsibilities and Rights", in *Logic and Language*, edited by A. Flew, Series I, Oxford, Blackwell, 1951.

12. Peter D. Klein, "A Proposed Definition of Propositional Knowledge", *The Journal of Philosophy*, LXVIII, no. 16 (August 1971), pp. 471-482.

1. On one worlds-diagram, place the three propositions, *B*, *M*, and *J*.
2. On one worlds-diagram, place the three propositions, *B*, *T*, and *J*.
3. On one worlds-diagram, place the three propositions, *B*, *M*, and *T*.
4. On one worlds-diagram, place the three propositions, *P*, *J*, and *T*.
5. On one worlds-diagram, place the three propositions, *P*, *B*, and *M*.
6. On one worlds-diagram, place the three propositions, *C*, *T*, and *M*.
7. If we let "P" now represent a noncontingent true proposition which is known in some but not all possible worlds, which, if any, of the preceding worlds-diagrams will have to be redrawn?
8. If we let "P" now represent a noncontingent false proposition, which, if any, of the worlds-diagrams in exercises 1 – 6 will have to be redrawn?

### 3. THE LIMITS OF HUMAN KNOWLEDGE

One of the incidental conclusions that we tried to establish in section 2 was that, contrary to what the sceptic would have us believe, knowledge of the truth of at least some propositions really is possible. We demonstrated this for the case of the noncontingent proposition that it cannot be known that no one knows nothing: and we suggested that there are countless other propositions, too, some noncontingent and some contingent, which we do in fact know to be true. All of us — even the sceptics — in the conduct of our daily lives, cannot but act upon the presupposition that a great many of the propositions that we *believe* we know to be true are ones that we do in fact *know* to be true: that the words on this page are written in English, that no one can live for long without oxygen, that  $2 + 2 = 4$ , and so on. Suppose, now, that this presupposition is correct. Are there, we may then wish to ask, any *limits* to our knowledge?

#### *The known and the unknown*

One kind of answer would be to draw attention to the distinction between those propositions which are known to be true and those which are not and to say that the boundary between the two fixes the limits of our knowledge.

That there is such a distinction goes without saying. But it would be vacuous and of little interest unless we could further say which propositions are actually known and which are not actually known.

Yet any attempt to say this runs into difficulties. For to say that a proposition is known to be true is ordinarily to say that *at some particular time* it is known to be true. A proposition which is not known to be true at  $t_1$  may come to be known at  $t_2$ . And a proposition which is known at  $t_1$  may be forgotten at  $t_2$ . So we need to specify a temporal parameter and say for what time the limits of the known are supposed to be determined.

This faces us with a dilemma. If we decide on some *particular time* — even the present moment — to the exclusion of any other, then an element of arbitrariness is introduced and the exercise loses any of the *philosophical* interest that it might otherwise have had. It may be interesting from a sociological point of view to know that at such and such a time the limits of human knowledge were so and so. But it is hardly of enduring philosophical significance. For the boundary between the known and the unknown, thus conceived, is wholly subject to the vagaries of human history, and shifts with the passage of time.

If, on the other hand, we try to answer the question by giving an encyclopedic catalogue of all the

true propositions which *at some time or other* are known by someone or other, we run into other difficulties. To be sure, the limits of human knowledge, so conceived, would be permanent and unchanging. But, unless there comes a time later than which no new truths are discovered, the epistemic Eldorado which is here envisaged seems likely forever to elude us. For consider what it would be like to achieve it. We should have to include in our encyclopedia not only all those true propositions which once were and currently are known to be true but also those which, at the time of compiling the encyclopedia, no one yet knew. Suppose, then, that there is some proposition, P, whose truth someone, e.g., an inventor, will first discover at a time,  $t_2$ , which is later than the time,  $t_1$ , at which the encyclopedia is prepared. If P is included in the encyclopedia of propositions known at  $t_1$  then its truth, contrary to our hypothesis, is *not* first discovered at  $t_2$ . And if P is *not* included in the encyclopedia at  $t_1$ , then, contrary to another of our hypotheses, the encyclopedia of all the propositions which are known by someone or other at some time or other is not complete.

How else might the question about the limits of human knowledge be answered?

*The knowable and the unknowable*

A second kind of answer might be given by invoking a distinction between what is knowable and what is unknowable. Such a distinction, if there is one, would not depend, for its philosophical significance, upon our being able to specify which true propositions are in fact knowable and which are unknowable at this particular time or that, or at some time or other. That there should be such a distinction at all, some limit to what it is *possible* for human beings to know, would in itself be a matter of note.

But is there such a distinction? We have already argued that there are at least some true propositions which are in fact known and which are therefore capable of being known. But are there any true propositions which not only are not in fact known — now or at any other time — but also are in principle unknowable? This is the question which we must now face.

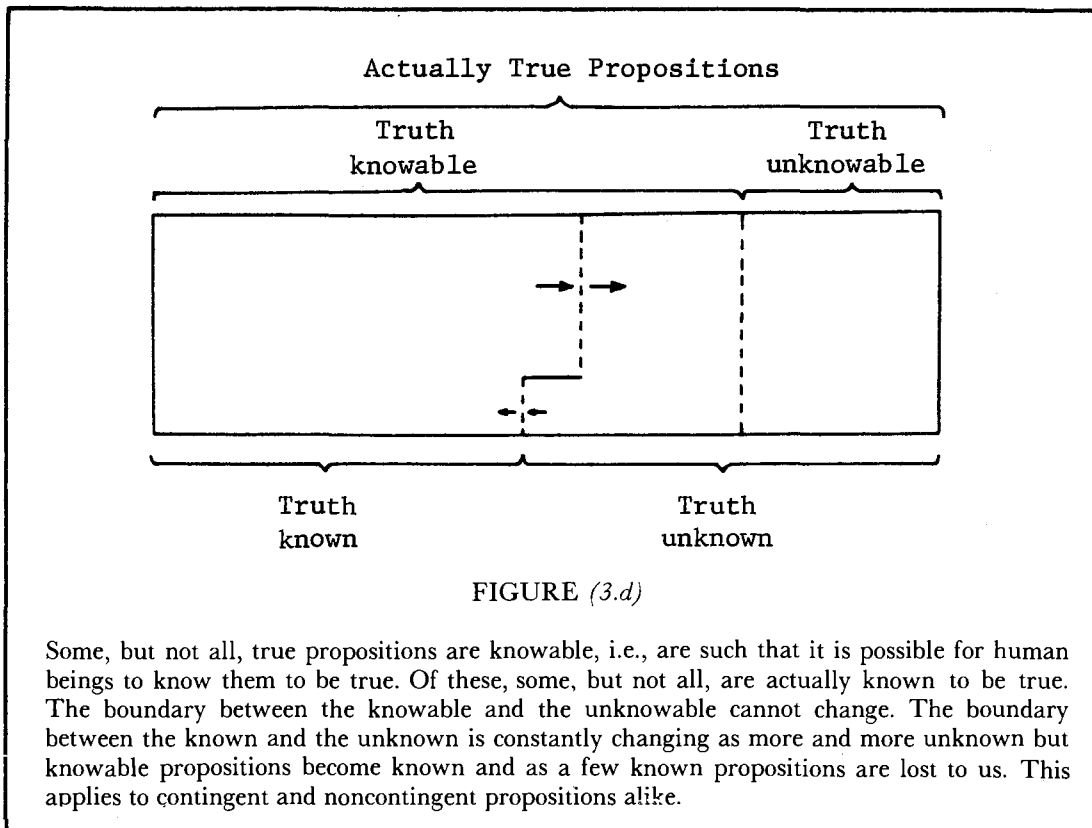
There is one way of construing this question according to which the answer is clearly negative. It is clear, for a start, that to ask whether or not a true proposition, P, is *knowable* is to ask whether or not it is *possible* that P be known. But what do we mean here by “possible”? If we give it a straightforward possible-worlds analysis the question becomes: Is there any possible world in which P is known (by someone or other)? And then, provided that we do not place any restriction on *how* P might come to be known, the answer seems obvious. For any true proposition P, whatever, there is at least one possible world in which someone knows that P. After all, if we place no restrictions on how P may be known, we must allow among our possible worlds ones in which some omniscient god simply *reveals* the truth of P to some human being, *a*, in such circumstances that all the necessary and sufficient conditions for *a*'s knowing that P are satisfied. In this case there can be no proposition which cannot possibly be known, no proposition which is in principle unknowable, and hence no distinction to be drawn between knowable and unknowable propositions.

The trouble with this way of construing the question, however, is that it does not take seriously enough the restrictions under which human beings labor, in the actual world, in their attempts to acquire propositional knowledge. Allowing the possibility that human beings should know any proposition that an omniscient being might know not only destroys any chance of a distinction between what is knowable and what is unknowable but also renders void any distinction between what is knowable and what is simply true. That which it is possible in some purely logical sense for us to know, plainly, ought not to be identified with that which it is *humanly* possible to know.

The objection is a good one. Yet it leaves us with a problem. What account can we give of this notion of humanly possible knowledge?

Let us start again with the notion of a possible world. But this time let us take seriously the facts that in the actual world we human beings possess certain capacities for knowledge-acquisition and lack

others; that our ability successfully to exercise these capacities is at least partly a function of what the world is like; and that any knowledge which merely happens to be bestowed upon us by some god, angel, demon, or whatnot, cannot properly be counted as distinctively human. Then we can say that it is humanly possible for P to be known if and only if there is at least one logically possible world in which (1) P has the same truth-value as P does in the actual world, i.e., P is true; (2) all the laws of nature are identical with those which hold in the actual world; (3) there is a sentient being, *a*, with the very same capacities for knowledge-acquisition as humans have in the actual world; and (4) *a* acquires knowledge that P is true by the unaided exercise of those capacities. In other words, a proposition P will be knowable if either (a) it is known in the actual world by a person exercising ordinary human capacities, or (b) it is known in some other possible world which is very like the actual world in that P is true in both that world and the actual world, and the knower in that world labors under the same physical constraints as are imposed on all of us by the natural laws of the actual world.



This account can be further simplified. As we noted in chapter 1 (pp. 6-7), to talk about a logically possible world in which the laws of nature are identical with those which hold in the actual world is to talk about a *physically* possible world. So we might, alternatively, choose to say that it is humanly possible for P to be known if and only if there is at least one physically possible world in which conditions (1), (3), and (4) obtain.

In the next section we will explore what capacities human beings actually have for knowledge-acquisition. But for the moment it suffices to point out that, on this account of what it is for it to be humanly possible to know P, the distinction between what is humanly knowable and what is not humanly knowable is easily preserved and does not collapse into the distinction between what is true and what is false. On the present account there will, of course, be some actually true propositions which will be known to be true in the actual world or in some other possible world which is very similar to the actual world in the ways just described. These propositions will comprise the class of the humanly knowable true propositions. But in addition (as we shall see later in this chapter), there will be some actually true propositions which are not known to be true, either in the actual world or, again, in other possible worlds of the relevant kind. These propositions will comprise the class of humanly unknowable true propositions. The boundary between these two classes of actually true propositions constitutes, in a philosophically interesting way, the limits of human knowledge.

#### 4. EXPERIENTIAL AND RATIOCINATIVE KNOWLEDGE

What capacities for knowledge-acquisition do human beings actually possess? Most philosophers have said that, as a matter of contingent fact, human beings — as distinct from other conceivable beings such as Martians, angels, gods, and devils — have the capacity to acquire knowledge in only two sorts of way: by some sort of appeal to experience, or by some sort of appeal to reason.

##### *Experiential knowledge*

Experience, we sometimes say, is a great teacher. Kant put it like this:

Experience is therefore our first instruction, and in its progress is so inexhaustible in new information, that in the interconnected lives of all future generations there will never be any lack of new knowledge that can be thus ingathered.<sup>13</sup>

Whether by appeal to our own experience, or to that of others, it seems the pool of human knowledge may be much increased.

But what do we mean here by “experience”? Many philosophers, the so-called Empiricists chief among them, would say that they mean *sense-experience*, i.e., experience through the senses. Now admittedly our senses of sight, hearing, touch, taste, smell, temperature, pain, etc., play a major role in providing us with information about the world around us. But it is surely an open question as to whether there can be other modes of awareness — other modes of experience — besides sense-experience. Telepathy and other forms of so-called extrasensory perception, for instance, cannot be ruled out in advance as possible means of acquiring knowledge even if their credentials have not yet been established. Nor can introspection, meditation, instinct, or even ‘tripping out’, no matter how suspect they have seemed to some. Might not the Yaqui Indians of Mexico have their own way of knowing, as Carlos Castaneda claims in his book *The Teachings of Don Juan: A Yaqui Way of Knowledge*?<sup>14</sup> These are fascinating questions. But we need not delay for their answers here. We can afford to be fairly liberal as to which of these we call a mode of experience. For *whatever* we allow to count as a mode of experience, it still remains a question as to whether, and under what conditions, appeal to that mode of experience justifies our claim to propositional knowledge. Appeals to sense-experience have no guarantee of privileged status in this regard. As philosophers have long

13. *Critique of Pure Reason*, Introduction, A1.

14. Berkeley, University of California Press, 1968.

pointed out, such appeals are by no means infallible guides to knowledge. The deliverances of sense-experience need to be subjected to all sorts of checks before we can justifiably claim that what our senses tell us is the case really is the case. It remains to be seen whether some other mode of experience might on occasion also provide justifying conditions for knowledge-claims. When, therefore, we speak of employing an appeal to experience as a means of acquiring knowledge, we shall leave it open as to which *kinds* of experience may give us knowledge, and in what circumstances.

Nevertheless, appeal to sense-experience provides us with a paradigm. Consider, for instance, our knowledge of the truth of the proposition

(3.10) Krakatoa Island was annihilated by a volcanic eruption in 1883.

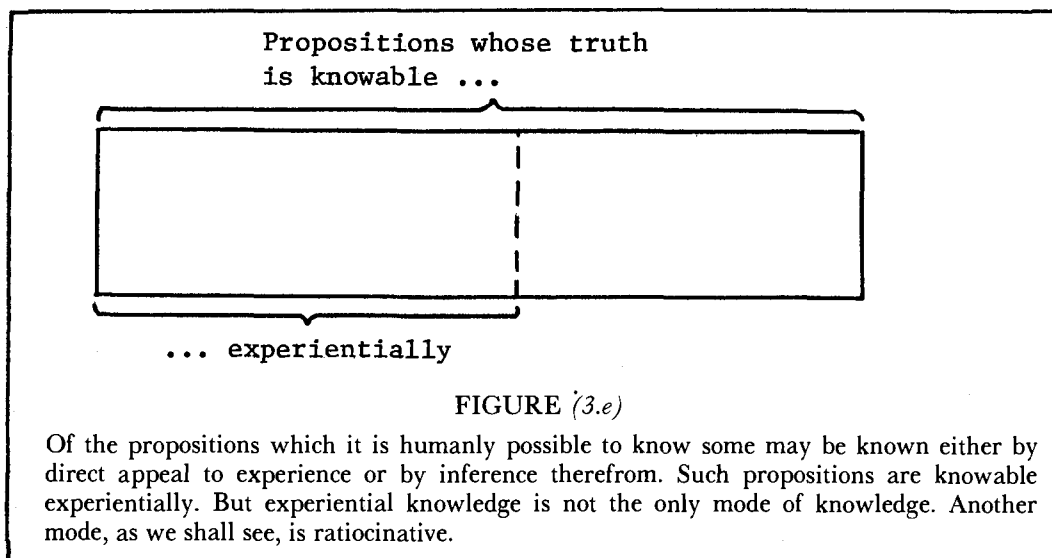
How might we come to know the truth of such a proposition? For most of us today the answer will be that we read about it in a history or geography book, or perhaps a treatise on geology; that we were taught about it at school; that we heard about it from someone else; that we came across reports of it in old newspapers; that we heard about it on a radio or television program. For those of an earlier generation the answer might be that they read about it in newspapers of the time; that they heard about it from eyewitnesses; that they had seen for themselves the physical evidence of its aftermath as for year after year high-altitude dust caused brilliant sunsets around the world; or even that they themselves had witnessed the gigantic eruption from a ship far out at sea. Now all these answers, philosophers have said, reduce to one: *experience*. They reduce to experience in the sense that the evidential support for the truth of (3.10) is sooner or later traceable back to what someone or other experienced or became aware of. If we ourselves had seen the flash, heard the blast, felt the surge of the onrushing tidal wave, smelt the acrid fumes, and sailed into a sea of floating pumice and searing lava toward what had once been a luxuriant tropical island, then our evidence for the truth of (3.10) would have been *direct*. But if, by way of contrast, we had merely read in the newspaper about the eruption, listened to the testimony of reliable witnesses, or noted the condition of the stratosphere, then our evidence for the truth of (3.10) would not have been direct but *inferential*. In either case, however, the source and warrant for our claims to know that (3.10) is true would be — we shall say — *experiential*.

The notion of experiential knowledge, then, may be defined thus:

“P is knowable experientially” =<sub>df</sub> “It is humanly possible to know P either by direct appeal to experience or by valid inference from propositions one or more of which is known experientially.”<sup>15</sup>

Just as we can afford to be undogmatic about what is to count as experience, so too we can be liberal over the question as to precisely where, if at all, to draw the line between direct experience and inference from experience. *Prima facie* there is need for such a distinction. It seems natural to mark it, as we have done in the Krakatoa case, by counting as direct the experiences of so-called “eyewitnesses” and as inferential the experiences of those others whose grounds for claiming to know that (3.10) is true include reliance on the trustworthiness of eyewitnesses, assumptions about stratospheric conditions, geological evidence, and whatnot. But one might want to draw the line elsewhere. Some would want to say that direct experience is limited to how things look, sound, feel, etc. (in a word, how things ‘seem’) to be, and that any claim about how things really are, which goes beyond direct experience, relies on inference. For present purposes, however, we can leave room for philosophical maneuver on this point. And, for that matter, we can even accommodate those who would deny that there is any distinction to be drawn at all. In defining experiential knowledge as knowledge gained

15. In chapter 4, section 4, we distinguish between two kinds of valid inference: deductively valid and inductively valid inference. The above definition encompasses both kinds.



either by direct appeal to experience *or* by inference from experience, we are not committing ourselves as to whether *both* need be involved. We think it natural to say that both sometimes are involved. But that is a point that need not be pressed.

#### *Ratiocinative knowledge*

With but a few exceptions, most philosophers have agreed that experience is not our sole teacher.<sup>16</sup> Our own capacity for understanding and reasoning seems, when it is exercised, to be another.

It is easy to give examples of propositional knowledge which is apparently acquired in this other way. Consider, for instance, the proposition

- (3.11) Either Krakatoa Island was annihilated by a volcanic eruption in 1883 or it is not the case that Krakatoa Island was annihilated by a volcanic eruption in 1883.

Now (3.11), like proposition (1.5) considered earlier, is a proposition which is expressed by asserting of two contradictories,

- (3.10) Krakatoa Island was annihilated by a volcanic eruption in 1883,

and

- (3.12) It is not the case that Krakatoa Island was annihilated by a volcanic eruption in 1883,

that one or the other is true. And hence proposition (3.11), like proposition (1.5), is necessarily true. How do we know? Well, we have only to reflect on the sort of reasoning we went through in

16. John Stuart Mill is one of the exceptions. He was an Empiricist insofar as he insisted that we can have knowledge of the actual world only by appeal to experience. But whereas many other Empiricists allowed that some knowledge, e.g., of mathematical and logical truths, is possible independent of experience, Mill did not. He was, we may say, a Radical Empiricist.



connection with (1.5). We saw then (on pp. 16–17) that since one member of a contradictory pair is true in all those possible worlds in which the other is false, and the other member of the contradictory pair is true in all the remaining possible worlds, then in each possible world one or other of the contradictory pair is true; so that any proposition — such as (1.5) or (3.11) — which is expressed by our asserting that one or the other of a contradictory pair of propositions is true cannot itself fail to be true. Now the interesting thing about all such propositions is that we can show, and hence know, them to be true, indeed to be necessarily true, solely by reasoning them out for ourselves. We do not need at any point to appeal to that sort of experiential evidence that seemed to be needed in the case of proposition (3.10). It does not matter for the purposes of our determining the truth-value of (3.11) whether (3.10) is true or false, or whether (3.12) is true or false, let alone whether (3.10) and (3.12) are individually *known* to be true or false by experience or by any other means. For we can know that (3.11) is true, and know that it must be true, without knowing of (3.10) and (3.12) which is true and which is false. The source and warrant for our knowledge of the truth of (3.11) is not experiential, but, as we shall say, *ratiocinative*. It is to be found within ourselves, in our own powers of understanding and reasoning.

There was a time when philosophers were wont to try to explain our ratiocinative knowledge of propositions like (3.11) by invoking the notion of *self-evidence*. But such an approach is notoriously unhelpful. If a person says that the truth of a proposition is self-evident, and means simply that the truth of that proposition is evident to him or herself, we are being told more about the person than about the proposition. Self-evidence, if it is taken to be a mere measure of subjective certainty, can vary from person to person and hence affords no explanation of how knowledge-claims are to be justified. Yet if, on the other hand, a person says that the truth of a proposition is self-evident, and means simply that the truth of that proposition is evident from the nature of the proposition itself, we are being told very little — except perhaps that knowledge-claims about its truth stand in no need of appeal to experience. We are not being told *how* knowledge-claims about its truth are to be supported. Talk of self-evidence, thus construed, is itself in need of explanation.

What, then, is involved when we acquire knowledge by the exercise of our own powers of understanding and reasoning? Once more Kant comes to our aid when he points out

A great, perhaps the greatest, part of the business of our reason consists in the analysis of the concepts which we already have. . . .<sup>17</sup>

Note that Kant does not say that the *only* business of reason consists in analysis of concepts. He certainly would want to insist that at least part of the business of reason consists in making deductively valid *inferences* from the results of conceptual analysis. For instance, he explains that in giving a demonstration of the properties of the isosceles triangle, a mathematician uses reason “not to inspect what he discerned either in the figure, or in the bare concept of it . . . but to bring out what was necessarily implied in the concepts that he himself had formed [of the isosceles triangle] . . .”<sup>18</sup>

This talk of analyzing concepts and of making inferences therefrom certainly seems more informative than mere talk of self-evidence. And it seems especially apt when we try to explain how our reasoning concerning the proposition (3.11) yielded knowledge of that proposition’s truth. For a start, we analyzed that proposition itself and noted that it asserts of a pair of contradictories that one or the other is true. Now the concept involved when we say of one or other of a pair of propositions that at least one is true is — as we saw earlier (chapter 1, section 3) — the concept of *disjunction*. In short, we analyzed the proposition as involving the disjunction of contradictories. Next, we analyzed these

17. *Critique of Pure Reason*, Introduction, B9.

18. *Critique of Pure Reason*, Preface to 2nd edition, Bxii.

concepts themselves and the way in which they are articulated in proposition (3.11). And then, reflecting upon the results of this simple analysis, we inferred that the proposition is true — indeed that it is necessarily true.

In the light of this we propose the following definition of ratiocinative knowledge:

“P is knowable ratiocinatively” =<sub>df</sub> “It is humanly possible to know P by appeal to reason (e.g., by analysis of concepts) or by valid inference from propositions which themselves are known by appeal to reason.”

Note that we do not here commit ourselves to the view that knowledge acquired by analysis of concepts or by inference therefrom exhausts all modes of ratiocinative knowledge. We cite them only as examples — paradigm examples — of what it is to appeal to reason. Thus we leave open the question whether there might not be some other modes of ratiocinative reasoning. Again we cite the examples of alleged ESP, intuition, etc. If they were to count as genuine modes of knowledge, it would still be unclear whether they were experiential, ratiocinative, or something else. In due course we will examine two further kinds of reasoning which some philosophers have suggested are also modes of ratiocinative reasoning: transcendental argumentation and baptismal reference-fixing. For the present it will simplify our discussion if we restrict it to the paradigm cases of ratiocination, viz., analysis of concepts and inference therefrom. And just as we can afford to be liberal about where to draw the line between direct experience and inference therefrom, so, too, we can allow ourselves to be liberal about where to draw the line between analysis of concepts and inference from the results of such analysis.

Sometimes — we are inclined to say — we can know a proposition to be true by analysis of the concepts involved with little or no inference therefrom. To take one of Kant’s examples, the proposition

(3.13) All bodies are extended

is one which we can know to be true — indeed know to be *analytically true*,<sup>19</sup> as Kant put it — simply by virtue of the fact that in analyzing the concept of *body* we come across the concept of extension as covertly contained therein. Do we, in such a case, *infer* that the proposition, the constituent concepts of which we have analyzed, is true? If so, the inference is of the most trivial kind. It is, as Kant put it, “thought through identity”;<sup>20</sup> or, as we shall later describe it, it is an inference which may be justified by appeal to the rule of inference known as that of Simplification.<sup>21</sup>

In other cases the inferential element seems to be more at a premium. Consider, for example, the proposition

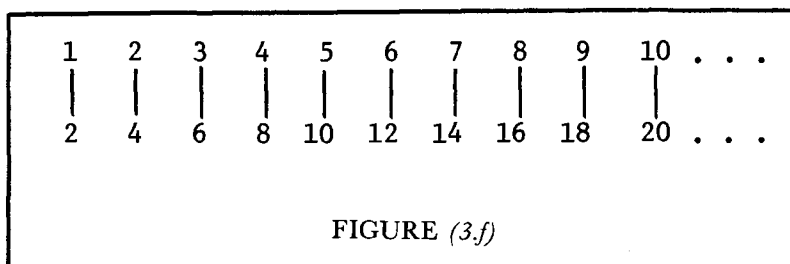
(3.14) There are as many even numbers as there are both even and odd numbers.

19. In this book we shall make little use — except in chapter 4 — of the notion of analytically true (or analytically false) propositions. Such a notion involves an amalgamation of the purely logical (or modal) notion of necessarily true (or necessarily false) propositions with the epistemic notion of how we can acquire knowledge of the truth (or falsity) of certain propositions.

20. *Critique of Pure Reason*, Introduction, B10.

21. See chapter 4, section 4. Essentially the point is that the proposition, P1, that something is a body, implies the proposition, P2, that that thing is extended and . . . and . . . , etc., (where the blanks are to be filled in by predicates naming other properties the concepts of which are involved in the analysis of the concept of *body*), and that the proposition, P3, that that thing is extended, may be validly inferred in accordance with the Rule of Simplification from the proposition P2.

We do in fact know this proposition to be true. But how? Talk of self-evidence is distinctly unhelpful here, especially if it is construed as mere subjective conviction. For, far from seeming to be self-evidently true, this proposition, upon first acquaintance, strikes most persons as self-evidently false. Nor does simple analysis of concepts seem to suffice to establish its truth. The truth of (3.14) can be established ratiocinatively, but only by analysis of the concepts involved — those of *even number*, *odd number*, and *as many as*, in particular — together with a good deal of inferential reasoning. In simplified form that inferential reasoning consists in the demonstration that each member of the series of all integers can be put into a one-to-one correspondence with each member of the series of even integers in the following way:



(3.15) To every number, even or odd, there corresponds a unique even number, its double; to each even number there corresponds a unique number in the series of all integers, viz., its half. But this is just to say that there are exactly as many even numbers as there are even and odd numbers together.

The conclusion we have just reached was by way of ratiocination. We made no appeal to experience whatever in determining that (3.14) is, after all, true.<sup>22</sup> Our reasoning involved both analysis of concepts and inference therefrom.

There are, it should be noted, some propositions the knowledge of whose truth, if it is humanly possible at all, can be acquired only by an enormous investment in inferential reasoning. The proofs of many theorems in formal logic and pure mathematics certainly call for a great deal more than simple analytical understanding of the concepts involved. And in some cases the amount of investment in analysis and inference that seems to be called for, in order that we should know whether a proposition is true or false, may turn out to be entirely beyond the intellectual resources of mere human beings. As a case in point consider the famous, but as yet unproved, proposition of arithmetic known as Goldbach's Conjecture, viz.,

(3.16) Every even number greater than two is the sum of two primes.

Most persons know what "even number" means (that is, they understand the concept of being-an-even-number). And most persons know what "prime number" means (they understand the

22. It is important to recognize that establishing the conclusion that there are as many even numbers as even and odd numbers together was via the argument of (3.15) and not via the figure (3.f). The function of the figure is purely heuristic; insofar as it is incomplete it cannot establish the conclusion we have reached. It serves merely to illustrate some, but not all, of what is being asserted in the ensuing argument (3.15).

concept of what it is for a number to be prime): a number is prime if it is divisible without remainder only by the number one and by itself. Goldbach's Conjecture is easily understood. In fact we understand it well enough to be able to test it on the first few of an infinite number of cases, thus:

Four is the sum of two and two.

Six is the sum of three and three.

Eight is the sum of three and five.

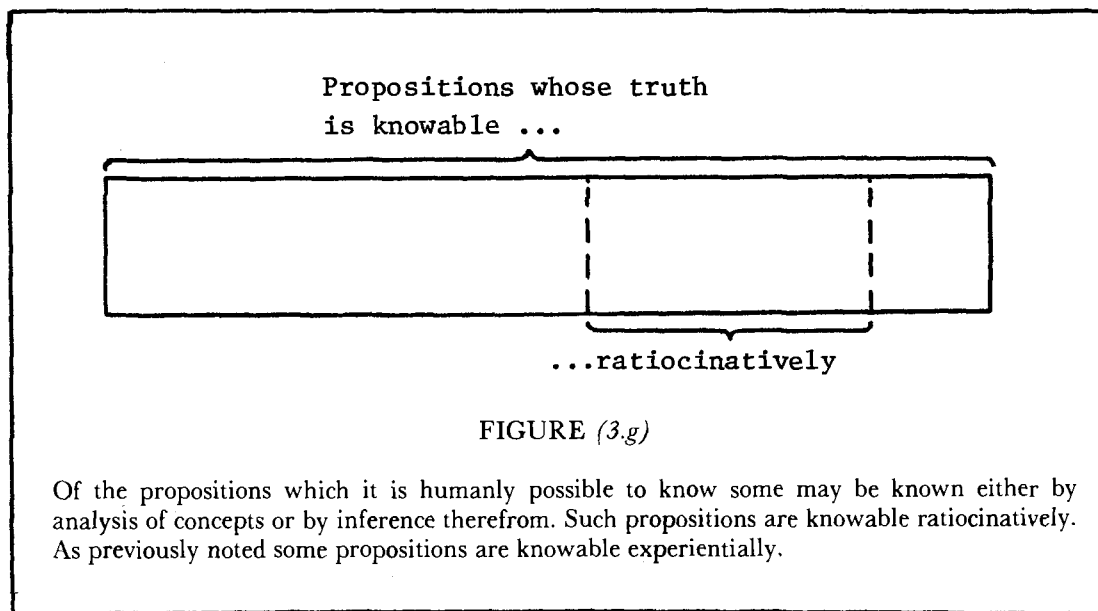
Ten is the sum of three and seven.

(Also, ten is the sum of five and five.)

Twelve is the sum of five and seven.

(and so on and on and on)

In this case, it is clear that we know what proposition is being expressed by the sentence, "Every even number greater than two is the sum of two primes." And in virtue of knowing this, we know what concepts are involved. But our familiarity with these concepts, and our ability to apply them in the above five examples, does not (apparently) suffice to allow us to know whether the Conjecture is true or false. For, as we have observed, proposition (3.16) is — at the time of writing — unproved. Moreover, for all we know, it may turn out to be unprovable by any being having the capacities for knowledge-acquisition which we human beings have. Of course, we do not *now* know whether or not it will eventually succumb to our attempts to prove it. Maybe it will. In this case it will be known



ratiocinatively. But then, again, maybe it will not. In that case it may well be one of those propositions whose truth is not known because its truth is *unknowable*. At present we simply do not know which.

#### *Appendix to Section 4*

Finally we might ask ourselves about the status of traditional so-called ‘pencil and paper’ operations and, in modern times, the use of computational aids such as electronic calculators and the like. How do they figure in the scheme of things? Is the knowledge they furnish to be construed as experiential or ratiocinative? To answer this question, let’s back up a bit.

Clearly if a person adds together a series of numbers ‘in his head’, as it were, then surely we must construe his knowledge as ratiocinative. But what if he were to do just the same thing but write the appropriate numerals on a sheet of paper as he proceeded? That simple fact hardly ought to cause us to want to change our account of the *kind* of knowledge obtained. His markings on the paper are most naturally construed as just the expression, or public display, of his ratiocinative reasoning. They should be regarded, clearly, as simply an extension or an aid to his reasoning; they should hardly be considered ‘an appeal to experience’. Should our account, however, change when a person comes to rely on a self-running calculator to perform calculations which obviously transcend his ability to ‘do in his head’? For example, no one is capable of calculating in his head the twenty-two thousandth digit in the decimal expansion of  $\pi$ . A person must use several pencils and a great deal of paper, and expend many hours in the task, or he must rely on a computer. If he does the latter, shall we say that he has appealed to experience? Even in this latter case we shall not do so. A person who learns the twenty-two thousandth digit in the decimal expansion of  $\pi$  by reading the printout of a computer is entitled to claim that his knowledge is obtained ratiocinatively. Why? Because computers are designed to carry through mathematical computations in a mechanical fashion each of whose steps is a mechanical (or electrical) analogue of a computational step which is sanctioned by a ratiocinatively warranted rule of inference. The computer, in calculating the twenty-two thousandth digit in the decimal expansion of  $\pi$ , does nothing that a human being *could* not do ratiocinatively using pencil and paper and given enough time and enough helpers to detect any errors he might make. In short, the conclusion may be put this way: in the twentieth century we have arrived at a state of technology sufficiently advanced that we are able to have our machines carry on some of our ratiocinative reasoning for us. And the knowledge they furnish us, when it proceeds from ratiocinatively known truths in accordance with ratiocinatively warranted inference rules,<sup>23</sup> may itself properly be termed “ratiocinative”.

## 5. EMPIRICAL AND A PRIORI KNOWLEDGE

In their discussions of the possible modes of human knowledge many philosophers have felt the need for a further distinction: that between *a priori* knowledge and *empirical* knowledge.

Kant was one of these. Indeed it is his use of the terms “a priori” and “empirical” which most philosophers today take themselves to be following. Now Kant thought that the ratiocinative/experiential distinction was an important one. He drew it along the same sorts of lines as we have: indeed, his talk of knowledge “through reason’s own resources” and of knowledge “a posteriori” conforms precisely to the definitions we have given of “ratiocinative knowledge” and “experiential knowledge”, respectively. But he also thought it important to draw a distinction between a priori and empirical knowledge.

23. For a discussion of the justifying of inference rules, see chapter 4, section 4.

*Definitions of “empirical” and “a priori”*

Kant’s definition runs as follows:

We shall understand by *a priori* knowledge, not knowledge independent of this or that experience, but knowledge absolutely independent of all experience. Opposed to it is empirical knowledge, which is knowledge possible only *a posteriori*, that is, through experience.<sup>24</sup>

Note that empirical knowledge is here, and elsewhere, defined — *not* as knowledge which is possible a posteriori — but rather, as knowledge which is possible *only* a posteriori, i.e., *only* experientially.<sup>25</sup> That is to say, it is defined as knowledge which it is not humanly possible to acquire without appeal to experience. A priori knowledge, by way of contrast, is defined as knowledge which is absolutely independent of all experience; that is to say, it is defined as knowledge which it *is* humanly possible to acquire without appeal to experience.

Following Kant, and sharpening up his definitions so as to highlight the contrast between the empirical/a priori distinction and the experiential/ratiocinative distinction, let us say:

“P is knowable empirically” =<sub>df</sub> “It is humanly possible to know P *only* experientially”,

and let us also say:

“P is knowable a priori” =<sub>df</sub> “It is humanly possible to know P *other than* experientially.”

What is essential to our (and Kant’s) definition of “empirical” is that a proposition may be said to be knowable empirically only if an appeal to experience is *necessary* in order for us to acquire knowledge of its truth. And what is essential to our (and Kant’s) definition of “a priori” is that a proposition may be said to be knowable a priori if an appeal to experience is *not* necessary in order for us to acquire knowledge of its truth. By way of contrast, our definition of “experiential” (and Kant’s of “a posteriori”) merely picks out a way in which it is *possible* for us to acquire knowledge of a proposition’s truth. And our definition of “ratiocinative” (like Kant’s expression “through reason’s own resources”) merely picks out another way in which it is *possible* for us to acquire knowledge of a proposition’s truth. The latter definitions, unlike the former, have nothing whatever to say about what is necessary or not necessary for human knowledge.

Unfortunately, the difference between these two sets of definitions is rather subtle. Indeed, it is so subtle that sometimes philosophers who subscribe to the above explicit definitions of “empirical knowledge” slip into using the term “empirical” when they should use the term “experiential”. They slip into saying that something is knowable empirically when it can be known by appeal to experience, i.e., is knowable experientially. If we remember that something is knowable empirically when it can be known *only* by appeal to experience, i.e., when it can be known *only* experientially, we should be able to avoid this mistake. It might also help if we remember that the terms “experiential” and “ratio-

24. *Critique of Pure Reason*, Introduction, B2–3.

25. Kantian scholars might like to note that, throughout the *Critique of Pure Reason*, Kant preserves the distinction between the empirical and the a posteriori — in the way we are drawing it — with complete scrupulousness.

cinative” are definable independently of one another, in terms of whether it is experience or reason (respectively) which makes knowledge possible, whereas the terms “empirical” and “a priori” are *interdefinable*: a priori knowledge is knowledge for which experience is *not* necessary, i.e., a priori knowledge is *nonempirical* knowledge, while empirical knowledge is knowledge for which experience *is* necessary, i.e., empirical knowledge is knowledge which is *not* a priori.

We have here two epistemic distinctions: two distinctions between ways of acquiring knowledge of what is knowable — two distinctions having to do with the *epistemic status* of those items which are knowable, viz., propositions.

But why, it may be asked, have we, like Kant and many other philosophers, thought it *important* to introduce the second classification of modes of knowledge? Why should we not remain content with the first and try to say everything we want to say in terms of it? The answer has to do with the standard requirements of philosophical taxonomy.

There are two properties that we usually demand of a satisfactory classificatory scheme, viz., (1) that it be exhaustive in the sense of covering all the cases to be classified and (2) that it be exclusive in the sense of covering each case only once. The taxonomic scheme used by biologists, for instance, is drawn up in such a way as to satisfy both these requirements. It is, or aims to be, exhaustive insofar as it includes all species so far known. And it is, or aims to be, exclusive insofar as no individual is counted as belonging to more than one species. Admittedly, the job of the taxonomist in biology seems never to be complete. But that is because the frontiers of biological knowledge are forever expanding. A new species is discovered and the classificatory scheme is expanded to include it; exhaustiveness is thus preserved. Or a borderline case between two recognized species is discovered and the boundaries between species are adjusted to avoid overlap: the problematic individuals are assigned to one species or the other, or an entirely new species is defined; exclusiveness is thus preserved.

Changes in the taxonomy of what is known might be expected so long as our knowledge is increasing. But changes in the taxonomy of what is humanly knowable can hardly be expected on these grounds. Indeed, since the humanly knowable comprises the totality of what is known at any time by the exercise of human capacities in all physically possible worlds, its boundaries can neither increase nor decrease but timelessly remain the same. Any classificatory scheme for talking about the humanly knowable might be expected, therefore, to satisfy both the requirements of exhaustiveness and exclusiveness.

This is where the first of our two epistemic distinctions — that between experiential and ratiocinative ways of knowing what is knowable — lets us down. It is only dubiously exhaustive and certainly is not exclusive.

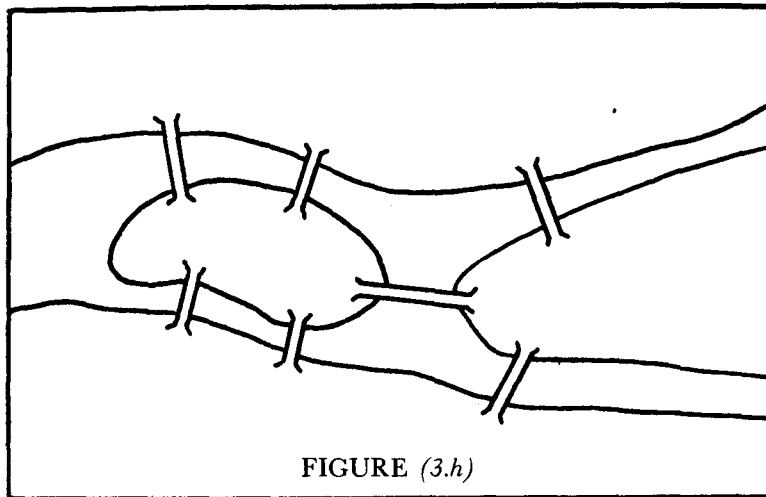
*The nonexhaustiveness and nonexclusiveness of the experiential/ratiocinative distinction*

In the first place it is highly questionable whether it is exhaustive. Might there not be other ways for human beings to acquire knowledge besides experience and reason? We have already seen that unless we define “experience” extremely broadly, cases of alleged telepathy, precognition, meditation, inspiration, and whatnot, will demand a hearing as apparent exceptions. And there is no guarantee against other counterexamples, apparent or real, being produced in the future.

In the second place the distinction is certainly not an exclusive one. The truth-value of some propositions can be known both experientially and ratiocinatively. Consider the proposition,

- (3.17) There is no route by which one can cross over all seven bridges of Königsberg without recrossing at least one bridge.

The seven bridges of Königsberg (now Kaliningrad) posed a unique problem to the burghers. There were two islands in the river which passed through the town. Seven bridges connected the islands and the banks of the river in the fashion shown in the following illustration.



The question arose whether there is a route by which one can cross each of the bridges without crossing any bridge twice. It doesn't take very long to convince oneself that there is no such route. Presumably the burghers tried the test with their feet. The townspeople of Königsberg, having actually tried numerous times to cross all the bridges just once, could be said to have *experiential* knowledge that proposition (3.17) is true.

In the early eighteenth century, the mathematician Leonhard Euler, in a famous paper,<sup>26</sup> was able to prove mathematically that proposition (3.17) is true. With Euler we have come to have *ratiocinative* knowledge of the truth of (3.17). What was first learned experientially outdoors by tramping around the banks of the river Pregel (Pregolya) was later relearned by the powers of pure reason (presumably) in the comfort of Euler's study where he merely carefully and ingeniously thought about the problem.

It thus emerges that some truths, that about the bridges of Königsberg being one, are knowable both experientially *and* ratiocinatively. We have already said that the experiential/ratiocinative distinction is (probably) not exhaustive. And now we have just shown that it surely is not exclusive. It thus lacks both the properties we would like in a classificatory scheme: exhaustiveness and exclusiveness.

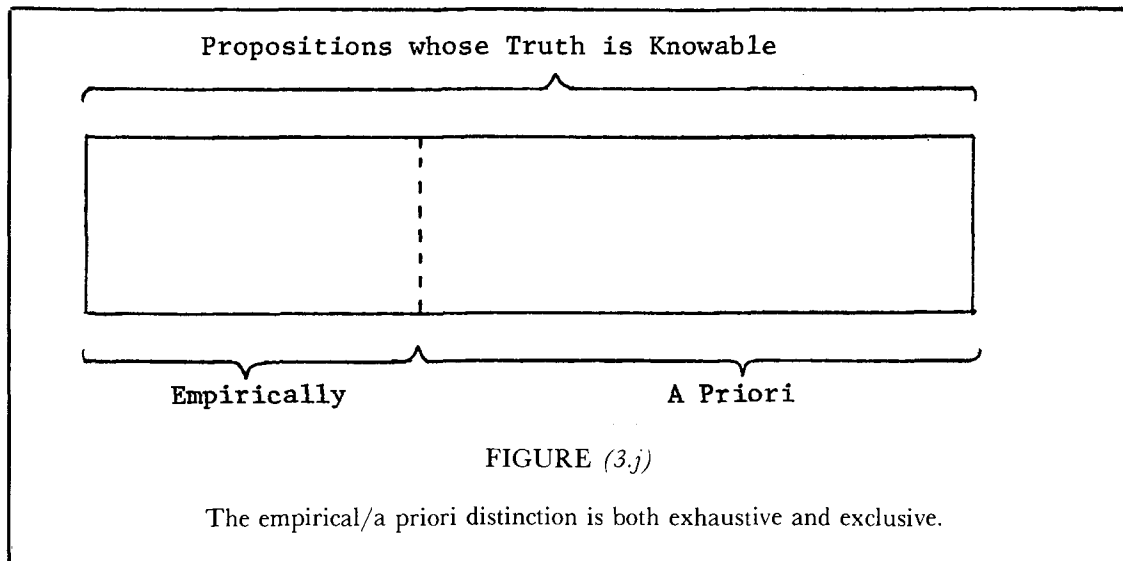
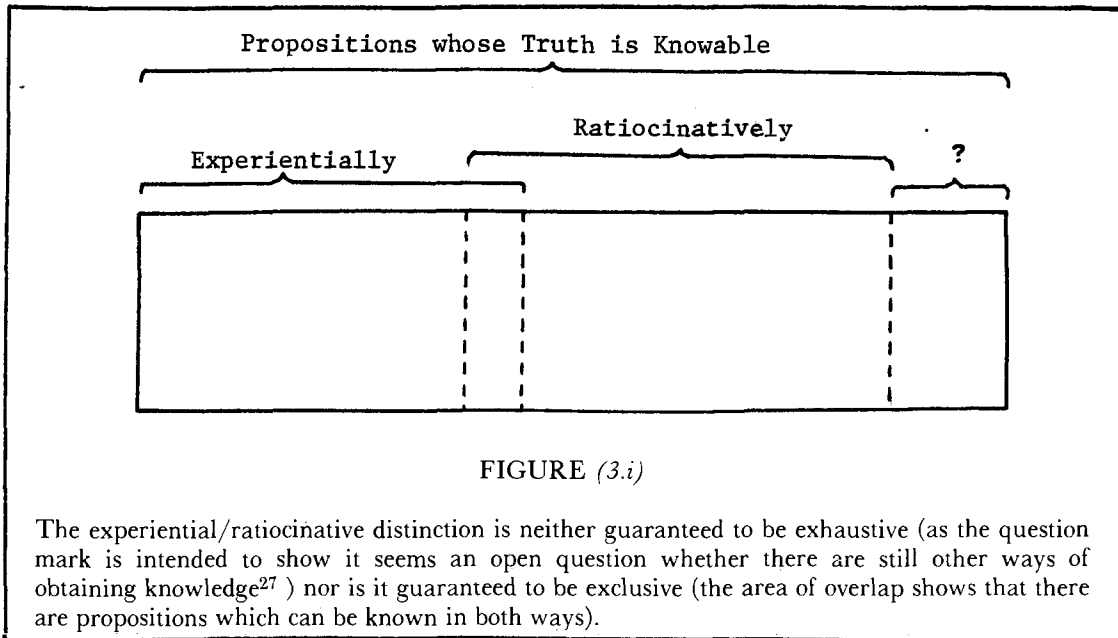
*The exhaustiveness and exclusiveness of the empirical/a priori distinction*

By way of contrast, the empirical/a priori distinction has both properties. In the first place, it is exhaustive for the class of knowable propositions. To be sure there are probably infinitely many propositions whose truth cannot be known by any finite sentient being. (And remember [section 3], the knowledge possessed by an omniscient God is hardly relevant in an examination of *human* knowledge.) But for any proposition which is or can be known through human faculties, we can confidently assert that it is empirical, since it is necessary to appeal to experience in order to know it, or a priori, since it is *not* necessary to appeal to experience in order to know it.

The proposition which asserts of a given knowable proposition, P, that it is empirical is a contradictory of the proposition which asserts of that very same knowable proposition, viz., P, that it is a priori. Little wonder, then, that the empirical/a priori distinction is exhaustive for the class of

26. Leonhard Euler, "The Seven Bridges of Königsberg", in *The World of Mathematics*, ed. James R. Newman, New York, Simon and Schuster, 1956, vol. I, pp. 573-580. This paper is easy reading and is enthusiastically recommended.





27. For the purposes of simplifying both our discussions and various subsequent figures, we will proceed as if propositions knowable by some means other than experience or reason could not also be known in one (or both) of these more familiar ways. Of course, strictly speaking, we are not entitled to this assumption. In figure (3.i), the area enclosed by the bracket bearing the question mark as a label should overlap *each* of the areas bearing the labels “experientially” and “ratiocinatively”. This refinement, or correction, will be made later in figure (3.m). Here we shall ignore it, but do not wish to suggest that it cannot nor need not eventually be made.



could be known both experientially and ratiocinatively. But this does not mean that it can be known empirically. For this proposition would be knowable empirically only if it were *necessary* to appeal to experience in order to know it to be true. But if, as is the case, it can also be known by ratiocinative means, it follows that it is not knowable *only* by appeal to experience, and hence it is not knowable empirically, but rather is knowable a priori. In short, if a proposition can be known to be true by ratiocinative means then, whether or not it can also be known experientially, it is knowable a priori and is *not* knowable empirically.

It follows from what we have said that the two epistemic distinctions are related to one another as in figure (3.k).

*Is a priori knowledge certain?*

There is a widespread custom of referring to a priori knowledge as “certain”. But is this correct? Is a priori knowledge certain? Is it more certain than empirical knowledge?

Before we can answer these questions we must become aware that they are ambiguous. What exactly is being asserted when someone says “a priori knowledge is certain”? This claim could be understood in either of two very different ways: (1) it could mean that all propositions which are knowable a priori are themselves certain, in the sense of being noncontingently true; or (2) it could mean that a priori reasoning — one of the *ways* in which *we come to know* some truths — is itself certain, in the sense of being infallible. We must be very careful not to run together these two different claims and, more especially, not to think that subscribing to one of them logically commits us to subscribing to the other.<sup>28</sup>

In the next section of this chapter we devote some considerable time to answering the first question. That is, we shall ask whether a priori reasoning can ever suffice to give us knowledge of contingent as well as — as we have already seen — at least some noncontingent propositions.

But in the present subsection, let us examine the other proposition which someone might be asserting in saying “a priori reasoning is certain”, viz., that a priori reasoning is infallible.

Is a priori reasoning infallible? The answer is obvious: No. In order to show this one need look no further than the fact that two persons, each reasoning a priori from the same set of propositions, may come to different answers to a problem. It is all too common to find persons getting different results when, for example, they add up long columns of figures, or when they try to prove theorems in algebra and geometry.

In short, to seek an answer to a problem a priori, or to try to ascertain the truth-value of a proposition a priori, is merely to adopt a way of trying to find an answer. It is simply a way of seeking knowledge, a way which need take no recourse to experience. But it is not an invariably *reliable* path to knowledge; it is not infallible. Persons can be mistaken in their a priori reasoning just as they can be mistaken in an empirical approach to finding out the truth-values of propositions. Not only do we sometimes misperceive the physical world and get some of our beliefs about contingent matters wrong; we may also falter in our reasoning and get some of our beliefs about noncontingent matters wrong. For example, many persons have believed that they have found a priori proofs of Goldbach’s Conjecture. But none of these proofs has withstood careful scrutiny. Other persons, also adopting a priori methods, have always managed to find fatal flaws in the proofs so far advanced.

Does the possibility that one can falter in one’s a priori reasoning discredit it? Not at all — no more

28. The particular kind of ambiguity being examined here has been called by some philosophers “the product/process ambiguity”. John Hospers (*An Introduction to Philosophical Analysis*, second edition, Englewood Cliffs, N.J., Prentice-Hall, 1967, p. 15) illustrates it with the sentence, “They went to look at the construction.” The sentence is ambiguous between expressing the propositions (1) that they went to look at the *thing* being constructed, and (2) that they went to watch the *activity* of constructing that thing. In like manner we are distinguishing knowledge as product and process.

so than the fact that one can falter in the application of empirical methods invalidates them. The possibility of making an error, of being confused or of misremembering, infects both of our ways of knowing, the empirical and the a priori alike. But this is not to say that these methods cannot yield knowledge. It is only to say that they are not infallible guides to truth. The possibility of making errors — perceptual, ratiocinative, or whatever — ought not to deter us from the pursuit of knowledge. Although we have no guarantee that either empirical methods or a priori methods will furnish us knowledge, we can be quite sure that knowledge will elude us if we ignore them.

Once one understands that the a priori mode of seeking knowledge is not infallible, how, we might ask, does one go about checking for mistakes on a given occasion of its use? The answer is: in just the same sorts of ways we try to find whatever mistakes there might be in a particular instance of the use of empirical methods. First and foremost, we recheck the process carefully. Then, if we wish still further corroboration, we might repeat the process, i.e., do it over again from the beginning. Also we might enlist the aid of other persons, asking them to go through the process themselves, and then comparing our results with theirs. And finally, we might make our results public, holding them up for scrutiny to a wider audience, and hoping that if there is a mistake, the joint effort of many persons will reveal it.

How many of these additional tests we will in fact invoke will depend on a variety of extra-logical matters, e.g., such things as the intrinsic difficulty of the reasoning and its importance to us. It is not, after all, a difficult chain of reasoning which allows persons to figure out that every square has the same number of interior angles as it does sides. One hardly needs the corroboration of other persons for this piece of reasoning, and it would be absurd to think that because a priori methods are fallible that this particular result of their application is in any way doubtful. To point out that a priori modes of reasoning are fallible is *not* to endorse scepticism or in any way to suggest that everything which we believe ourselves to know a priori is doubtful. If by no other means, then by *experience* we have learned that much of our a priori reasoning is corroborated by further testing. As a result, even in advance of testing, we are entitled to be confident that on many occasions when we reason a priori we do it correctly.

## 6. EPISTEMIC AND MODAL STATUS CONSIDERED TOGETHER

In the course of introducing each of the two sets of epistemic distinctions we have had occasion to refer to propositions — some of them contingent, the others noncontingent — which are paradigmatic examples of items of knowledge that can be known in one way or another: experientially, empirically (only experientially), ratiocinatively, or a priori. Thus, for instance, we cited the contingent proposition (3.10) as a typical example of something that can be known to be true experientially, and the noncontingent propositions (3.11), (3.13), and (3.14) as typical examples of something that can be known to be true ratiocinatively. But it by no means follows from anything we have said that *all* contingent propositions are knowable *only* experientially (i.e., empirically). Nor does it follow from anything we have said that *all* noncontingent propositions are knowable ratiocinatively, let alone that they are knowable *only* ratiocinatively, hence *only* nonexperientially. Might there not be some contingent propositions belonging to epistemic categories other than the experientially knowable? And might there not be some noncontingent propositions belonging to epistemic categories other than the ratiocinatively knowable? So far we have left these and related questions largely unexamined. It is time for us now to explore, in a quite systematic way, how each of the four *epistemic* categories can be combined with the two *modal* categories of contingency and noncontingency.

Having sorted out the various combinations and interrelations possible among the four epistemic categories, and having pictured these various interrelations on figure (3.k), we must now proceed to add to that figure the modal distinction between the contingent and the noncontingent. (At the same

time we will re-introduce the category of unknowable propositions.) Doing so will give us figure (3.1).<sup>29</sup> On it we notice that there is a total of *ten* different classifications of propositions. This is not to say, however, that propositions of each of these ten different kinds actually exist. It is an important philosophical question to see just how many, and which, of these categories are in fact instanced by propositions.

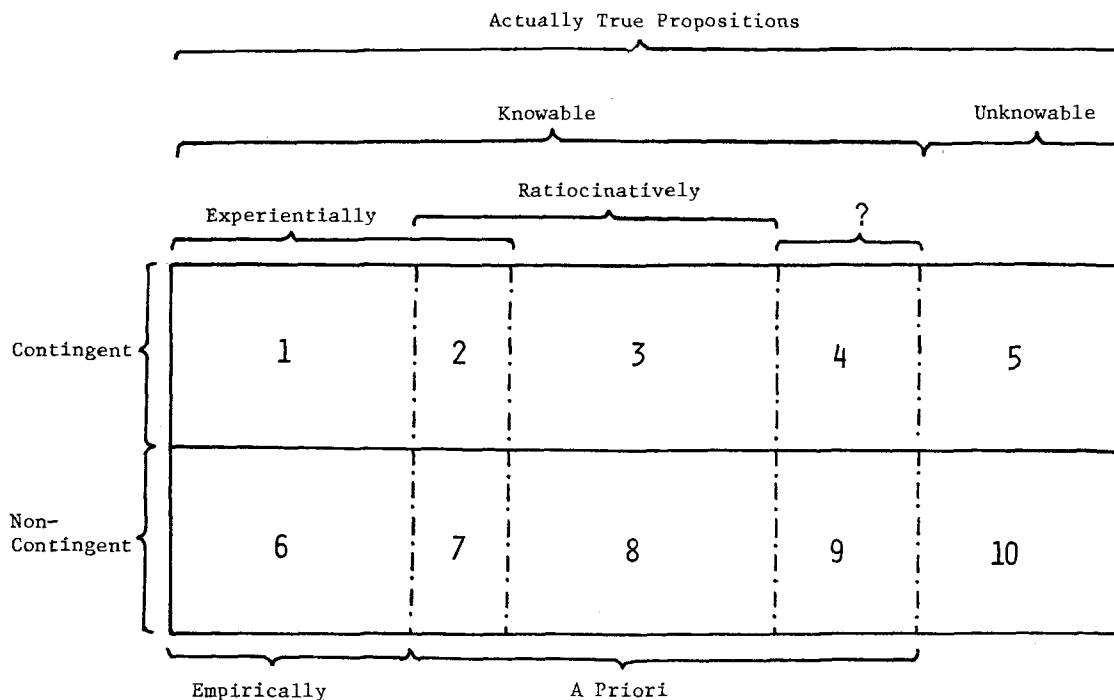


FIGURE (3.1)

1. Are there any contingent propositions which are knowable empirically?

It is quite uncontroversial that there are numerous contingent propositions which are knowable experientially. For example, (1.3) and (3.10) clearly are such. But the question arises whether these, or any other contingent propositions which are knowable experientially are knowable *only* experientially. The question comes down to this: Is there any contingent proposition which is knowable experientially and which is *not* knowable by any other means? The received opinion by the vast majority of philosophers is that among the contingent propositions which are knowable experientially, at least some are *not* knowable by any other means. (The question whether perhaps none is knowable by any other means is treated below [questions, 2, 3, and 4]. In the meantime we will examine the widely held thesis that at least some are knowable exclusively, or solely, by experiential means.) The aforementioned propositions, (1.3) and (3.10), seem to be of just this sort. We know that the U.S. entered World War I in 1917 (i.e., we know (1.3) ); we know that Krakatoa

29. Remember, we made a simplifying assumption in figure (3.1) regarding the area labeled with the question mark.

Island was annihilated by a volcanic eruption in 1883 (i.e., we know (3.10)). In each case our knowledge is experiential. Moreover it seems to be unavoidably experiential. It is very hard to see how it would be possible to have ratiocinative knowledge or any other kind of knowledge of facts such as these. How, for example, in our having mastered the concepts which figure in propositions (7.3) and (3.10) could we come to know specific facts of world history? How indeed? The conclusion we seem driven to is that there is a class of contingent propositions which are knowable only by an appeal to experience, that is, empirically.

2. *Are there any contingent propositions which are knowable both experientially and ratiocinatively (and ipso facto, a priori)?*

Category 2, viz., contingent propositions knowable both experientially and ratiocinatively, constitutes the first of the three most problematic categories in this scheme, specifically, all those categories (2 through 4) of *contingent* propositions which are knowable a priori.

It is a logically necessary condition of there being any members of Category 2 (and of Categories 3 and 4 as well) that there should be some contingent propositions which are knowable a priori.

Are there, then, any contingent propositions whose truth-values are knowable a priori? Although it looks harmless enough, the answer to this question is far from uncontroversial. Indeed it is no exaggeration to say that it effectively divides most philosophers into two camps — those who say “Yes” and those who say “No” — and that controversy still rages between them.<sup>30</sup>

In these pages we shall not align ourselves with one camp or the other. We shall content ourselves merely with a report of the debate. If history is a good guide in these matters, we can expect the controversy to continue for quite some time.

During the past two hundred years or so, the question whether there are any contingent propositions knowable a priori has often (indeed fashionably) been expressed in Kant's terms: Are there any synthetic a priori judgments?<sup>31</sup> By “synthetic” Kant meant roughly what we mean by “contingent”: in any event, he contrasted synthetic judgments with analytic ones and plainly thought that the latter exhausted the class of noncontingent propositions, “analytic” being the term which he chose to explicate the notion of noncontingency. But well before Kant adopted this near-standard way of formulating it, the question — however couched — was seen to be of vital significance: the viability of a whole method of philosophical inquiry turned upon it. After all, ancient Greek mathematicians (such as Euclid) and philosophers (such as Plato and Aristotle) had argued that ratiocinative methods, well suited to the armchair or the ivory tower, sufficed to give us knowledge of certain truths of mathematics and logic: such truths, they claimed, can be known a priori. But the truths of mathematics and logic — on the account we are giving — are all noncontingent. They are true in the actual world, to be sure; but so too are they true in all possible worlds. Consequently these a priori knowable truths tell us nothing *distinctive* about the actual world. They give us no information which would enable us to distinguish our world, the actual world, from possible but non-actual worlds. And so the question naturally enough arises as to whether armchair methods or ivory-tower modes of inquiry can ever suffice to establish the truth or falsity of propositions which might be thought to lie within a natural science — physics or chemistry or biology, for example — propositions which would tell us something about the actual world as distinct from other possible worlds: in a word, contingent propositions. This is what has traditionally been at issue when philosophers have asked, “Are there any synthetic a priori

30. Some, like W.V.O. Quine, belong to neither camp since they would deny the very distinctions between contingent and noncontingent propositions, on the one hand, and between empirical and a priori knowledge, on the other. They reject the very terms of the dispute as confused.

31. *Critique of Pure Reason*, Introduction, B19.

propositions?” Or, as we might more aptly put it, “Are there any contingent propositions *knowable a priori*?”<sup>32</sup>

Few philosophers of repute have ever seriously maintained that we can have a priori knowledge of such contingent matters as the structure of the heavens, or the shape of the earth, or the natural history of ungulates — matters which plainly have to do with distinctive features of the actual world and, for that matter, with highly localized features of that part of the actual world which we happen to occupy.<sup>33</sup> Knowledge of matters such as these, it has usually been allowed, can be achieved only by someone or other getting up and about in the world and experiencing, through observation, experiment, or whatnot, the distinctive features which it happens to have.

However, many reputable philosophers have argued at length that there nevertheless do exist some propositions of a highly general kind — metaphysical propositions, as they are usually called — which, by pure unaided reason, can be known to be true; propositions which, by virtue of the fact that they need not be true in other possible worlds, are contingent; and which, by virtue of the fact that we can argue for their truth without ever stirring from our armchairs or abandoning the seclusion of our ivory towers, can be known a priori.

If there are such propositions, then — as Kant pointed out — a genuine science of metaphysics is possible. For metaphysics, on his conception, is just that field of inquiry which seeks by a priori means to discover the most general truths about the nature of the universe — truths so general and profound as to lie beyond the purview of physics or any other such experientially based science. According to Kant, there are such propositions and a science of metaphysics is thereby shown to be possible. Thus, for instance, he constructed what he called a “transcendental argument” (roughly an argument which transcends, because it does not require any appeal to, experience) for saying that the proposition,

(3.18) Every event has a cause

is both synthetic [contingent] and knowable a priori. It is synthetic, he claimed, because unlike the analytic [noncontingent] mathematical proposition,

(3.19)  $(a+b) > a$  [which he reads as “the whole is greater than its part”]

no amount of analysis of concepts could show (3.18) to be true.<sup>34</sup> And even though (3.18) is avowedly not knowable analytically, it is nonetheless knowable a priori, he claimed, since his transcendental argument establishes a priori that the universality of causal connections in the actual world is a necessary condition of our having any experiential knowledge of this world. In other words, Kant

32. It is misleading to talk of propositions being a priori or empirical. Strictly speaking, it is the modes of *knowing* propositions, not the propositions themselves, which may be spoken of as being a priori or empirical.

33. History is replete, however, with cases of obscurantists who have thought that their astronomical or geographical theories, for instance, could be justified, a priori, on the grounds that they ‘stood to reason’ or could be validly inferred from ‘revealed truth’; and the story (probably apochryphal) still persists of the medieval churchmen who excommunicated a priest for challenging their belief that the number of teeth in a jackass’ mouth should be determined piously and (in a manner of speaking) a priori by consulting the Church’s ‘infallible’ teachings rather than impiously and experientially by opening its mouth and counting.

34. Kant discusses these examples in the *Critique of Pure Reason*, Introduction, B5 and B17 respectively. He seems to be saying that (3.18) is contingent by virtue of its being not analytically knowable. This account of contingency does not jibe with the one we offered earlier. We should prefer to say that (3.18) is contingent, not because it cannot be known analytically, but rather because it is true in some possible worlds and false in all the others. Simply put, we define “contingency” as a logical property and not as an epistemological one. Be that as it may, it remains just as real a problem for us as for Kant whether (3.18) is contingent and knowable a priori.

argued that one couldn't have knowledge of anything unless proposition (3.18) were true; and since we do have knowledge of at least some things, he concluded both that (3.18) is true and that it can be known to be true without appeal to experience.

Other philosophers, before Kant and since, have also argued for the respectability of metaphysics as a genuine mode of inquiry, and many have put forward their own candidates for the status of contingent a priori truths. Indeed a number of prominent philosophers today, of whom Peter Strawson is perhaps the foremost example, have employed arguments strikingly like Kant's to similar ends.<sup>35</sup> Transcendental arguments are enjoying a current vogue; and defenders of the thesis that there are contingent propositions which are knowable a priori are not hard to find.

Ranged against them, however, are hosts of philosophers who deny that a priori knowledge of contingent propositions is possible. They believe that those contingent propositions which can be known are knowable only by an appeal to experience, and that there is no a priori knowledge of contingent truths. To the extent that a priori knowledge is possible, it is alleged to be restricted exclusively to noncontingent propositions.<sup>36</sup> Hence, they maintain, a science of a priori metaphysics of the kind Kant envisaged is impossible in principle. Far from being the grandest of all forms of human inquiry, metaphysics — they would say — is a conceptual fraud foisted on us by those who cannot get their thinking straight.

An antimetaphysical bent of mind characterizes many of those philosophers who, aptly, have come to be called "Empiricists". The Empiricist tradition in philosophy includes among its foremost representatives persons such as the eighteenth-century Scottish philosopher David Hume (in reaction to whose antimetaphysical diatribes Kant was provoked to write his *Critique of Pure Reason*), and the contemporary English philosopher Sir Alfred Ayer (who became one of the principal spokesmen in the 1930s in the English-speaking world of the philosophy known as Logical Positivism or Logical Empiricism).<sup>37</sup>

In line with their contention that there are no contingent propositions which are knowable a priori, it is characteristic of Empiricists, whenever confronted with a proposition which is putatively both contingent and knowable a priori, to argue that either that proposition is not contingent or it is not knowable a priori. Thus, for example, when confronted with Kant's claim that the proposition

(3.18) Every event has a cause

is both contingent and knowable a priori, some Empiricists, as we might expect, have replied that it is not contingent, while others have preferred to argue that it is not knowable a priori. Or again, against those philosophers who would argue that Euclidean geometry provides us a priori with contingent information about the actual world, typically Empiricists have replied that to the extent that Euclidean

35. See, for instance, Strawson's *Individuals: An Essay in Descriptive Metaphysics*, London, Methuen, 1959.

36. A few philosophers of this persuasion take an even more radical stance: they allege that no proposition, regardless of its modal status, can be known a priori. The principal expositor of this stronger thesis was the English philosopher, John Stuart Mill. For Mill, all knowledge was empirical. It should be clear, however, from what has already been said in this book, that we have rejected Mill's thesis. We have already given examples, e.g., (3.11) and (3.13), of propositions which can be known a priori. See Mill's *A System of Logic, Ratiocinative and Inductive*, (1843), 8th ed., London, Longmans Green, 1965, and *Examination of Sir William Hamilton's Philosophy*, London, Longmans Green, 1865.

37. See especially Hume's *A Treatise of Human Nature* (1739), ed. L.A. Selby-Bigge, Oxford, Clarendon Press, 1964; *An Enquiry Concerning Human Understanding* (1745), ed. Eric Steinberg, Indianapolis, Hackett Publishing Company, 1977; Ayer's *Language, Truth and Logic*, London, 1936, 2nd ed., 1946; *The Foundations of Empirical Knowledge*, London, Macmillan, 1940; and *The Problem of Knowledge*, London, Macmillan, 1956.



geometry provides knowledge a priori, all its results are noncontingent, and to the extent that it provides any contingent information, that information can be known only by appeal to experience, i.e., empirically.<sup>38</sup>

In more recent years the dispute over the question whether there are any contingent propositions which are knowable a priori has taken on renewed interest and vitality as a result of the contemporary renaissance in logic itself which we reported in chapter 1. Lately there have appeared arguments in support of the existence of a priori knowledge of contingent propositions, arguments which, however, do not also defend the existence of an a priori metaphysics of the Kantian, or even of the Strawsonian kind.

Using the phrase “stick S” as a description of the well-known glass-encased platinum-iridium bar in Paris which originally served as the standard by means of which meter lengths were conventionally determined, Saul Kripke has argued that the proposition

(3.20) The length of stick S at time  $t_0$  is one meter

is both contingent and knowable independently of experience, i.e., is knowable a priori.<sup>39</sup> Kripke’s claim turns in part on a distinction which he, in effect, draws between two kinds of definition: those which fix the *meaning* by giving synonyms and those which fix *reference*; but it can be understood well enough without going deeply into that.<sup>40</sup> Thus, in the case of (3.20), for instance, he argues first that the proposition is contingent: it is contingent, he claims, because although the length that we call “one meter” is the same in all possible worlds, the length of stick S, which happened to be one meter at  $t_0$  in the actual world, could have been different (i.e., is different in at least some other possible worlds), so that the proposition (3.20), although true, is contingent; nonetheless, it is capable of being known a priori, he argues secondly, because those persons who laid it down as a reference-fixing definition that the term “one meter” should refer to whatever length it was which stick S had at time  $t_0$  would not need to get out any measuring tapes or in any other way try experientially to determine S’s length, but rather, simply as a consequence of their own stipulations, could know (3.20) to be true.

38. For detailed discussions of these controversies see R.D. Bradley’s (1) “The Causal Principle”, *Canadian Journal of Philosophy*, vol. IV, no.1 (Sept. 1974); and (2) “Geometry and Necessary Truth”, *Philosophical Review*, vol. 73 (1964). The latter paper is available in the *Bobbs-Merrill Reprint Series in Philosophy*.

39. Saul A. Kripke, “Naming and Necessity”, in Harman and Davidson, eds., *Semantics of Natural Language*, Dordrecht, D. Reidel, 1972; especially pp. 274–5, 279, and 346–7.

40. Suffice it to say that, if Kripke is right, not all propositions which are knowable a priori are necessarily true. The proposition

(3.21) All sisters are female,

which is expressed by the sentence,

(3.22) “All sisters are female”,

all of whose terms are ‘meaning-defined’ is necessarily true and knowable a priori. But the true proposition expressed by the sentence,

(3.23) “The length of stick S at time  $t_0$  is one meter”

where “one meter” is, as we might say, ‘referentially-defined’, which is (allegedly) also knowable a priori, is not necessary: it is contingent.

Whether Kripke's contributions to the perennial debate can survive criticism remains to be seen. But this much is already clear about his arguments: they offer little if any solace (and were not intended to offer solace) to those who have staked the possibility of a Kantian-type metaphysics on the outcome to the question whether any contingent propositions exist which are knowable a priori. For there is nothing at all grand, but on the contrary something rather trite, about the examples which Kripke espouses. If there are such contingent propositions whose truth-values can be known a priori as it were — that is, as a result of our having stipulated references for certain of the words we use — such propositions would hardly seem to be on a par with the profundities that metaphysicians have traditionally aspired to promote.

Keeping in mind this broad background of debate concerning the existence of contingent propositions knowable a priori, let us now return to its first instance, that concerning Category 2. Note that even if one were to argue that there are some contingent propositions which are knowable a priori, it would not follow that there are any contingent propositions which are knowable ratiocinatively, and still less that there are any contingent propositions knowable both ratiocinatively and experientially.

To argue that some contingent propositions are knowable ratiocinatively requires a stronger argument than one which purports to establish that some contingent propositions are knowable a priori. For ratiocinative knowledge may be but one mode of a priori knowledge. So the question whether there are any contingent propositions knowable ratiocinatively must be independently addressed.

Note that neither Kant nor Kripke argues that one could have knowledge of contingent propositions by means of the *paradigm* kinds of ratiocination, viz., analysis of concepts and inference therefrom. Philosophers like Kant and Kripke, who wish to maintain the possibility of a priori knowledge of contingent propositions, have been driven to propose *exotic* kinds of apriority: transcendental arguments in the case of Kant, and baptismal reference-fixing definitions in the case of Kripke. (In this section we have merely presented these other alleged modes of apriority. Question 4 will give us the opportunity to evaluate these candidates by pursuing the questions whether there are any nonratiocinative but a priori means of gaining knowledge. There we shall examine the credentials of Kant's and Kripke's candidates to see whether they might be better classified as nonratiocinative a priori modes. Our conclusion there will be that not only are they not ratiocinative; they are not a priori, either. But this is to anticipate.)

If one were to construe either Kant's (3.18) or Kripke's (3.20) as being examples of contingent propositions knowable by ratiocination (albeit exotic ratiocination), might either of these two propositions also serve as an example of Category 2, i.e., a contingent proposition knowable both ratiocinatively and experientially?

Kripke's example (3.20) would seem to be the more promising of the two.<sup>41</sup> Indeed

(3.20) The length of stick S at time  $t_0$  is one meter

would appear to be about as good a candidate for Category 2 as one is likely to find.

Now there seems to be no good reason whatever to deny that (3.20) could be known to be true experientially. We have only to suppose that accurate records were kept of the temperature of S at  $t_0$  (they were); that people subsequent to  $t_0$  adopted other means of determining whether something is a meter long (they did); and that the laws of nature pertaining to the relationship between length and temperature have been invariant since  $t_0$  (we have good reason to suppose that they have been). It then becomes easy to envisage someone finding out, *by appeal to the actual experience* of measuring S today, that (3.20) is true, i.e., that S had a certain length at some particular previous time. It would then

41. It has proven notoriously difficult to argue convincingly that (3.18) can be *known* experientially.

follow, that *if* (3.20) is not only, as Kripke claims, knowable a priori, but is, more specifically, knowable ratiocinatively, then at least some contingent propositions are knowable both ratiocinatively and experientially.

None of this, of course, answers the question whether there *are* any contingent propositions which are knowable both ratiocinatively and experientially. If Kripke's examples can be correctly construed as instances of ratiocinatively knowable propositions, then there are; but before concluding that they can be, one would do well to bear in mind that an entire school of philosophy, the Empiricist, adamantly denies the existence of such a class of propositions. Going even further, the Empiricists deny that there are any contingent propositions knowable a priori, whether by ratiocination in its most familiar forms, whether by exotic kinds of ratiocination, or whether by nonexperiential, nonratiocinative means. Empiricists, that is, insist that not only Category 2 but also the subsequent Categories, 3 and 4, are empty.

3. *Are there any contingent propositions which are knowable ratiocinatively (and ipso facto a priori) but which are not knowable experientially?*

Like Question 2, this one turns partly on the answer we give to the logically prior question, viz., whether there are any contingent propositions knowable a priori. If there are no contingent propositions which are knowable a priori then it follows immediately that there are no contingent propositions which are knowable *only* ratiocinatively and not experientially.

But suppose that there are at least some propositions which are contingent and knowable ratiocinatively. Might not some of them be knowable *only* in ways which make no appeal to experience?

Some of the claims made within classical metaphysics would seem to be likely candidates. Many of these propositions do *not* seem to be knowable experientially. If, then, one were to accept the claim that they are contingent and knowable ratiocinatively, one would be satisfied that there are contingent propositions which are knowable only a priori. What might some of the candidates be for such a classification? The following stand out.

(3.24) Nothing can be created out of nothing.

(3.25) There is nothing in an effect which was not present in its cause.

(3.26) Physical things are composed of invisible, weightless, intangible substrata in which the properties of things inhere and cohere.

(3.27) Only those physical things exist which are perceived.

(3.28) There is only one physical space.

(3.29) If a property were universal (e.g., if every physical thing had *precisely* the same temperature), then that feature of the world would be undetectable.

It is characteristic of all these propositions that, at one time or another, metaphysicians have taken some pains to argue that they are *not* knowable experientially. Indeed, it is a principal tenet of metaphysics that its findings should 'transcend' experience, that knowledge of their truth-values should lie outside the capabilities of mere experience.

Again, as with Category 2, immediately preceding, Empiricists will take issue. Empiricists, we can be sure, will claim that none of these propositions is both contingent and knowable a priori, and consequently that none can be both contingent and knowable only ratiocinatively.

4. *Are there any contingent propositions which are knowable by other than experiential or ratiocinative means?*

As we said before, when discussing the nonexhaustiveness of the experiential/ratiocinative distinction, unless we define "experience" and "ratiocination" so as to cover every conceivable alleged case of knowledge being acquired by telepathy, precognition, meditation, intuition, etc., it will remain an open question whether there might not be modes of knowledge-acquisition other than the experiential and the ratiocinative.

Fortunately we have no need here to debate any exotic questions about the credentials of ESP and the like, let alone to decide whether any of the propositions supposedly known by such means fall outside the provinces of experience and reason. For there are other candidates, already better known to us, whose credentials we have still to examine. One is the proposition

(3.18) Every event has a cause

which, according to Kant, can be known a priori, hence without any appeal to experience, but which at the same time cannot be known by analysis of concepts or inference therefrom. The other is the proposition

(3.20) The length of stick S at time  $t_0$  is one meter

which, according to Kripke, also can be known a priori, but again not by analysis of concepts or inference therefrom. The former is said, by Kant and some others, to be knowable by transcendental argument; the latter is said, by Kripke and some others, to be knowable by a means which we earlier described as baptismal reference-fixing.

Let us start with Kant. In raising the question whether

(3.18) Every event has a cause

might be knowable by transcendental argument one presupposes that it is knowable. Yet this presupposition has not passed unchallenged. Some philosophers — especially Logical Positivists — have argued that it is both unverifiable and unfalsifiable by any means whatever and, in the light of their theory, have concluded that it is unknowable. Other philosophers have directed their attacks on the very idea of a transcendental argument, arguing: (1) that transcendental arguments do not do what they are supposed to do since they are either invalid or unsound; or (2) that such arguments make a covert appeal to a now-suspect principle of meaningfulness — the so-called Verificationist Principle which was favored by Logical Positivists and which still infects the arguments of many nonpositivist thinkers.<sup>42</sup>

The criticism which we would urge against the claim that (3.18) is knowable a priori by a transcendental argument is that, even if transcendental arguments were acceptable in other respects, they still would not show that their conclusions can be known a priori since the premises from which

42. For more on the Verificationist Principle see the following discussion of Question 5. For more on the claim that transcendental arguments involve an appeal to verificationism, see: Barry Stroud, "Transcendental Arguments", *Journal of Philosophy*, vol. LXV, no. 9 (May 2, 1968), pp. 241–254; W.B. Stine, "Transcendental Arguments", *Metaphilosophy*, vol. 3, no. 1 (Jan. 1972), pp. 43–52; R. Rorty, Symposium: "Verificationism and Transcendental Arguments", *Nous*, vol. 5, no. 1 (Feb. 1971), pp. 3–14.

they begin can only be known experientially. Let us suppose, for instance, that Kant's transcendental 'proof' of (3.18) does indeed show that the truth of (3.18) is a necessary condition of our having the knowledge and experience which we do in fact have. Even so, we cannot conclude that (3.18) can be known a priori to be true. For the proposition that we have both knowledge and experience is something which itself is known not a priori but only by experience. Accordingly, on both Kant's and our own account of what it is for knowledge to be gained empirically, it follows that (3.18) can be known (if at all) *only* experientially, i.e., empirically, not a priori. We must not be led into supposing that the conclusion to an argument is knowable a priori just because the reasoning from premises to conclusion is purely a matter of a priori ratiocination. The issue, in such a case, is rather whether the premises themselves are knowable a priori or empirically. And in the case before us — as, we suggest, in all other purported examples of conclusions established by transcendental arguments — the premises are plainly empirical. Seen in this perspective the claim of propositions like (3.18) to fill the gap envisaged by Question 4 seems wholly mistaken.

The situation is not altogether different in the case of Kripke's example, (3.20). Kripke claims that (3.20) is knowable a priori by "someone who has fixed the metric system by reference to stick S". Of any person who institutes the convention (or set of conventions) embodied in the metric system by stipulating that the term "meter" shall designate rigidly, i.e., in all possible worlds, the length which S happens contingently to have at  $t_0$  in the actual world, Kripke claims that "he knows automatically, without further investigation, that S is one meter long."<sup>43</sup>

It is clear that the very same argument can be pressed into service for saying that any and every baptismal act of reference-fixing provides the baptiser with a priori knowledge. Not only is a priori knowledge automatically at the command of those who set up the metric system of length or, changing the example to another of Kripke's, to those who set up the Celsius system of temperature by stipulating that "100°" should refer to the temperature at which water boils at sea level; it is automatically provided to all those who give names to their children, their cheeses, or their chickens. Indeed, according to this argument, a priori knowledge can be created by fiat — by the simple device of naming anything at all. In assigning a name, say "Zsa Zsa", to an item,  $x$ , one automatically (and hence, according to Kripke, a priori) knows that the contingent proposition that  $x$  is Zsa Zsa is true.

It is interesting to note how Kripke's suggestion fits into the philosophical tradition.

In one respect Kripke, like all philosophers of genius, is bucking the tradition. Whereas earlier philosophers — especially the Rationalists — thought of a priori knowledge as the proper preserve of persons of intellect, reasoning power, and insight, Kripke has argued that it is within the grasp of anyone who can name anything — presumably even the babe in his cot systematically naming his toys.<sup>44</sup> This does not mean that Kripke is wrong. But it does mean that if Kripke is right then Kant in his quest for an answer to the question "How are synthetic (contingent) a priori judgments possible?", overlooked the obvious. He had no need for the elaborate and difficult transcendental arguments by which he sought, in his *Critique of Pure Reason*, to determine what are the "pure concepts of understanding" which are presupposed by experience and hence not "given" in experience. He needed only, like Kripke, to observe what goes on in any act of naming.

43. "Naming and Necessity", p. 275.

44. Presumably, also, it was within the grasp of our prelinguistic forebears at the moment when — according to most theories — they laid the foundations of language by assigning names to things. If so, Kripke's claim can be recruited in favor of our earlier contention that our prelinguistic ancestors could believe certain propositions to be true even though there were, by hypothesis, no sentences to express them. If, by the single act of assigning names, they came to have a priori knowledge, then *a fortiori* they had beliefs. See chapter 2, pp. 78–79.

In another respect, Kripke is within the tradition. Like Kant and many others he views a priori knowledge — at least insofar as it is of contingent propositions — as ‘maker’s’ knowledge, knowledge which we in a sense make or create. In effect he gives a new twist to the theory — once popular with the Logical Positivists and early so-called Linguistic Philosophers — that truth of certain kinds can literally be created by convention. The twist lies in the fact that whereas conventionalists like Ayer, Carnap, Hahn, and the early Strawson<sup>45</sup> tried to explain the apriority of *necessary* propositions in terms of “definitions of symbols”, “rules of language”, or “conventions of meaning”, Kripke tries to explain the alleged apriority of certain *contingent* propositions in terms of reference-fixing definitions, acts of baptism, or conventions of naming.

But can the mere act of naming ever really create a priori knowledge? Admittedly, the proud parents who name (whether in a baptismal ceremony or less formally) their first-born daughter “Zsa Zsa” know “automatically, without further investigation”, the truth of the proposition that their first-born daughter is Zsa Zsa. But does that mean that they have a priori knowledge in the sense of “a priori” that we have been discussing? The question is by no means a simple or clear one and perhaps must await developments in the semantic theory of naming and in philosophy of psychology before it can be settled definitively. Nevertheless we venture a few simple remarks by way of furthering the present discussion.

Consider, more closely, cases in which it seems natural to say, with Kripke, that in bestowing a name on an object one knows automatically, without further investigation, what the object is (i.e., is named). The parents say, “Let’s call her ‘Zsa Zsa’ ”; or the French academicians say, “We hereby call the length of this stick ‘one meter’ ” (in French, of course). *How* do they, at the time of so saying, know the truth of the relevant propositions? Some philosophers would be inclined to object that the question doesn’t properly arise since the word “name”, like the words “promise”, “guarantee”, and “apologize” — all of which may sensibly be prefixed by “I hereby . . . ” — functions in sentences like the above, not to report *that* one is doing something, e.g., that one is naming, but merely to do or perform that very thing, e.g., perform the act of naming.<sup>46</sup> The objection, however, is misconceived. The fact that one is not, in performing the act of naming, *asserting* that one is so doing, let alone asserting that one *knows* that one is so doing, does not preclude us from asking whether, at that time, one in fact knows what one is doing. Nor does it preclude us from asking *how* one knows what one is doing, or what *follows* from the fact that one knows what one is doing.

But if these questions are permissible, their answers are obvious. Zsa Zsa’s parents know the truth of the proposition that their daughter is Zsa Zsa because that proposition is inferred “automatically” from another proposition which they know to be true, viz., that they gave her that very name. And they know the latter proposition to be true because *they attended to to what they were doing at the time*. Likewise with the French academicians. On this analysis, however, the propositions which were supposed to be known a priori turn out to have their “source and warrant” (as Kant would put it) in experience — the experience of attending to what one is doing in the very act of naming and,

45. See A.J. Ayer, *Language, Truth and Logic*, 1st edition, London, Gollancz, 1936; Rudolf Carnap, *The Logical Syntax of Language*, New York, Harcourt, Brace & World, 1937; Hans Hahn, “Logic, Mathematics and Knowledge of Nature”, 1933, trans. and published in Ayer, ed., *Logical Positivism*, Glencoe, Ill., Free Press, 1959; P.F. Strawson, “Necessary Propositions and Entailment Statements”, *Mind*, vol. 57 (1948), pp. 184–200. For trenchant criticisms see W.C. Kneale, “Are Necessary Truths True by Convention?” *Proceedings of the Aristotelian Society*, Suppl. vol. 21 (1947); Arthur Pap, *Semantics and Necessary Truth*, New Haven, Yale University Press, 1958, chap. 7; and W.V.O. Quine, “Truth by Convention”, reprinted in *Readings in Philosophical Analysis*, ed. Feigl and Sellars, New York, Appleton-Century-Crofts, 1949.

46. These expressions, when so used, are usually called “performatives” following J.L. Austin. See his “Other Minds”, reprinted in A.G.N. Flew, ed., *Logic and Language*, 2nd series, Oxford, Blackwell, 1953, pp. 123–158.

moreover, having experiential cognizance of the very thing named. To restate the point we made earlier in connection with Kant's claim that (3.18) can be known a priori, we must not conclude that a proposition is knowable a priori just because it can be shown to follow, by a piece of a priori reasoning, from some other proposition.

In conclusion, then, we can agree with Kripke that those who fix the reference of the term "meter" could, at the time of fixing it, know the truth of

(3.20) The length of stick S at  $t_0$  is one meter

"automatically, without further investigation". But we should want to insist that this does not mean that (3.20) is knowable a priori. We might allow, perhaps, that the *inference* from the fact that they bestowed the name "meter" on the length which the stick had at that time was made "automatically". And we might allow, too, that once this inference has been made no "further investigation" — no further appeal to experience — is needed. But this is only because, on the account we are suggesting, all the requisite appeal to experience has already been made.

If we are right, neither the Kantian nor the Kripkean examples permit us confidently to assert that there are propositions which are knowable by means other than experience or ratiocination. Indeed, in retrospect, our analysis suggests that the prospects of finding any genuine examples of contingent a priori propositions are even more forlorn than we earlier — in our discussion of Questions 2 and 3 — were prepared to allow.

5. *Are there any contingent propositions which are unknowable?*

This question has an interesting history in modern philosophy. In the 1930s and 40s the Logical Positivists argued that the claim that there are contingent propositions which are unknowable is nonsense. They took the hallmark of a contingent proposition to be its testability, and rejected as meaningless any sentence which did not express something testable. This was the very point of their Verificationist Principle.<sup>47</sup>

Now there are many objections that can be made to the Positivists' thesis. One is that it confuses sentences with what sentences express. Although sentences might be nonsensical, propositions never are. But more to the point is the fact that on the account given in these pages of what a contingent proposition is, namely a proposition which is true in some possible worlds and false in some others, there is good reason to think that some contingent propositions are unknowable. Indeed we can even give an example of a contingent proposition which is in principle unknowable, e.g., the proposition expressed by the sentence,

(3.30) "On April 13, 1974, an extraterrestrial being, who has the ability, and who always exercises it, to thwart our attempts to detect him, stood on the tower of the Empire State Building."

47. Verificationist theories of meaning were formulated in different ways at different times (See Carl Hempel, "Problems and Changes in the Empiricist Criterion of Meaning", reprinted in A.J. Ayer, ed., *Logical Positivism*, Glencoe, Illinois, Free Press, 1959). Most sophisticated formulations allowed that noncontingent propositions did not need empirical backing and claimed only that this was essential only to contingent propositions if they were to be meaningful [*sic*].

For a contemporary exposition of Verificationism, see Michael Dummett's *Frege: Philosophy of Language*, London, Duckworth, 1973, esp. pp. 463–470. Dummett's account "dispenses altogether with the conception of objective truth-values, determined independently of our knowledge or means of knowledge, by a reality external to us" (p. 470). It is clear that this involves, among other things, a rejection of the correspondence theory of truth adopted in this book, and, as he recognizes, a rejection of classical two-valued logic. He concedes, however, that the consequences of his account "have never been systematically worked out" (p. 468).

In saying that the proposition expressed by (3.30) is *unknowable*, we are of course, saying that it is unknown; but we are also saying more. Remember that we are here examining the limits of *human* knowledge. And although there will be some possible worlds in which something or other (God, perhaps) knows the truth-value of the proposition expressed by (3.30), that fact is simply irrelevant to our claim.

To say that the proposition expressed by (3.30) is unknowable is merely to say that it is unknown by any sentient beings who have the same set of sensory modes and reasoning abilities as we human beings have and who find themselves in possible worlds which have the same physical laws as our actual world has. An extraterrestrial being who is undetectable by any human modes of experience and who is also undetectable by any physically constructible instrument is an extraterrestrial being whose existence must forever lie beyond human ken.

To be sure, the Positivists would claim that insofar as (3.30) does not assert anything which can be known to be true or known to be false, it does not express a proposition at all. But this seems to be little more than a piece of dogma rendered plausible, to some, by a faulty theory of meaning. It may be admitted, with the Positivists, that in the absence of evidence for the existence of an undetectable extraterrestrial being we have no good reason to believe that such a being really exists. We may even agree that (3.30) is a sentence the utterance of which need hardly be taken very seriously. But this is a far cry from saying that what (3.30) expresses cannot possibly be true, or false, let alone that (3.30) is literally meaningless.

There was a time when the verificationist theory of meaning seemed plausible not only to some philosophers but also to some scientists. It was often invoked in early formulations of the theories of relativity and quantum mechanics. But today it no longer seems plausible. Thus for example we find contemporary astrophysicists arguing with respect to black holes that “there are parts of the universe from which, in principle, we cannot get any information.”<sup>48</sup> To be sure, it seems on current theory to be physically impossible for us ever to obtain information about the internal states of black holes. Nonetheless, serious speculation abounds about what those internal states might in fact be. This speculation is not unbridled. It is seen by physicists to be subject to the constraint of consistency with currently well attested physical and cosmological theories. But of course the very question of consistency or inconsistency can arise only on the presumption that the speculative theory — even if unverifiable and unfalsifiable — nevertheless really *is* true or false.

6. *Are there any noncontingent propositions knowable empirically, that is, knowable experientially but not a priori?*

To begin, we must put to rest an argument which would attempt to prove that *experiential* knowledge of noncontingent truths is impossible. The flawed argument goes like this:

If a proposition is noncontingently true, i.e., is necessarily true, then it is true in all possible worlds. But our experience of how things are is of only one among this infinity of possible worlds; that is, it is restricted to the actual world. How, then, could we have *experience* which would show that a proposition is true in all possible worlds? Our experience is limited to the actual world and does not extend to non-actual ones. It could hardly suffice, therefore, to establish truths about non-actual but possible worlds. In short, it is not possible to have experiential knowledge of noncontingent truths.

48. Larry Smarr, quoted in “Those Baffling Black Holes”, in *Time*, vol. 112, no. 10 (Sept. 4, 1978), p. 58.



In spite of its initial plausibility, this argument is, in the final analysis, unsound.

The alleged difficulty which the argument purports to expose evaporates as soon as we distinguish between (1) knowing that a proposition is true, and (2) knowing that a proposition is necessarily true.

Since necessary truths are true in all possible worlds, and the actual world is a possible one, they are also true in the actual world. And hence our experience of how things are in the actual world may very well suffice to establish the truth (in the actual world) of a proposition which is true both in the actual world and in all other possible (but non-actual) worlds as well. That is to say, experience can sometimes suffice to show that a necessarily true proposition is true *simpliciter*. What experience cannot suffice to establish is that a necessarily true proposition is necessary; but it may show that it is true. As Kant put it: "Experience teaches us that a thing is so and so, but not that it cannot be otherwise."<sup>49</sup>

Having effectively rejected the argument which claims to show that experiential knowledge of noncontingent truths is impossible, we can now proceed to look for examples of such truths. They are easy to find. We need only cite the example of the necessarily true proposition

(3.31) It is raining or it is not raining.

The easiest way to come to know that (3.31) is true is by ratiocinative reasoning. But, nevertheless, one *could* determine its truth-value by looking out the window and seeing that it is raining, from which one could ascertain by valid inference that the proposition in question is true.

It cannot, then, be doubted that there is *experiential* knowledge of noncontingent truths.

But all this is just by way of scene-setting. Question 6, is not, after all, whether there is experiential knowledge of noncontingent propositions, but whether there are any noncontingent propositions which are knowable empirically, that is, knowable *only* experientially. And it is at this point that our answer must become somewhat lame.

We are quite unable to cite any examples of noncontingent propositions which we can confidently say are unknowable by any other than experiential means. We find ourselves in the situation in which the townspeople of Königsberg were in before Euler succeeded in producing his proof. Even before his proof, they already knew that there was no route of the stipulated sort which connected the seven bridges; but they did not know, nor is it easy to see how they could have known, that this conclusion could also be arrived at ratiocinatively. As it later happened, the problem did succumb to ratiocinative reasoning; but until it did, there was no way of knowing that what they knew experientially was also knowable ratiocinatively.<sup>50</sup> There do not seem to be any guarantees whatever in this matter. Although some noncontingent propositions may come to be known experientially, this does not imply that they may not, after all, be demonstrable ratiocinatively.

49. *Critique of Pure Reason*, Introduction, B3.

50. It is interesting to note that the case study of the Königsberg problem furnishes us with an example of a point we made in rebuttal to the specious argument which began this answer to Question 6. Before Euler's proof, the townspeople of Königsberg knew the truth-value of proposition (3.17). What they did not know was whether (3.17) was contingent or not; whether the fact that there was no route of the described kind was due to an idiosyncratic feature of the local landscape or whether it was more wide-ranging. The answer to the latter question came in Euler's proof. What he showed was that it was *impossible* that there should be such a route, effectively showing that the obstacle to such a route was not an idiosyncratic feature of the local landscape but a feature which would be found in any possible world whatever which contained seven bridges deployed as they were in Königsberg.

Summing up, our answer seems to be this: various noncontingent propositions are knowable experientially. Whether they are knowable *solely* experientially, that is, knowable empirically, we simply do not know. But, equally, we do not know of any good argument whatever which would suggest that there are no such propositions. There may be; there may not be. We have no reason here to prefer one answer to the other.

7. *Are there any noncontingent propositions which are knowable both experientially and ratiocinatively?*

Yes. We have just cited two examples: (3.31), the proposition that it is raining or it is not raining, and (3.17), the one dealing with the bridges of Königsberg.

8. *Are there any noncontingent propositions which are knowable ratiocinatively (and ipso facto a priori) but which are not knowable experientially?*

As a step towards finding the proper answer to this question, let us begin by considering the proposition,

$$(3.32) \quad 1 + 1 = 2$$

Let it be understood ("stipulated" if you prefer), that we are talking about the sum of two numbers 1 and 1, not about the consequences of putting one thing alongside another thing. So considered, the sentence " $1 + 1 = 2$ " expresses a noncontingently true proposition. Although the consequence of putting one thing alongside another may well be that there is nothing at all (in the case where sufficient quantities of uranium are put together to form a critical mass), or only one thing (in the case where one raindrop coalesces with another), or three or more (in the typical case where a male rabbit is put into a hutch with a female) —the *sum* of the numbers 1 and 1 cannot possibly be other than 2. Now this noncontingently true proposition (like (3.31)) can be known to be true both ratiocinatively and experientially. On the one hand, we can know the truth of (3.32) simply by virtue of understanding the *concepts* of oneness, addition, equality, and twoness. We can, therefore, know it to be true ratiocinatively. On the other hand, we can know (3.32) to be true simply by discovering, as a matter of general experience, that whenever there is one thing and another thing there are two things. A whole method of teaching children so-called mathematical "skills" is based on the presupposition that mathematical truths can be learned by generalization from experience.

But are *all* noncontingent truths which are knowable ratiocinatively, like the proposition that one plus one equals two, also knowable experientially? Or, are there some noncontingent truths which are knowable ratiocinatively, but which are *not* knowable experientially?

Although there seems to be no doubt that such simple truths of arithmetic as (3.32) can be learned experientially, it is hard to see how experience can suffice when it comes to dealing with large numbers or complicated calculations.<sup>51</sup> What sort of *experience* could it be which teaches us that proposition

$$(3.33) \quad 347091 \times 6038 = 2,095,735,458$$

51. It is interesting to note that John Stuart Mill argued in *A System of Logic, Ratiocinative and Inductive* [1843] that *all* mathematical truths have to be established by inductive generalization from experience rather than ratiocinatively. By way of reply, Gottlob Frege, in *The Foundations of Arithmetic* [1884] (translated by J.L. Austin, New York, Harper, 1960), pointed out that this would mean that we would not be entitled to assert that  $1,000,000 = 999,999 + 1$  unless we had observed a collection of a million things split up in exactly this way (pp. 10–11).

is true? Although proposition (3.32) can reasonably be claimed to be knowable both experientially and ratiocinatively (that is, to belong to Category 7), it is very implausible to make a similar claim for proposition (3.33). It seems that for all but the simplest mathematical propositions, experience could never suffice to give us knowledge of their truth-values and that we must instead have recourse to ratiocinative methods.

Or, consider the fact that we know, for example, that the mathematical constant  $\pi$  is a nonrepeating, nonterminating decimal. How could we possibly know this experientially? We could not, of course. The constant  $\pi$  is the ratio of the circumference of a (perfect) circle to its diameter. No matter how carefully we construct and measure a circle we can never get more than roughly ten decimal places of accuracy — that is, we can physically measure  $\pi$  only to about ten decimal places. But so doing would never show what we already know, namely, that the decimal expression for  $\pi$  runs on to an infinity of digits after the decimal point. It is clear in this case, again, that we have an instance of knowledge of a noncontingent truth which is quite unknowable experientially.

Thus the answer to this eighth question, whether there are any noncontingent propositions which are knowable ratiocinatively but not experientially, would seem to be: Yes.

9. *Are there any noncontingent propositions which are knowable a priori but by means other than ratiocination?*

Earlier in this section we asked the comparable question about contingent propositions. Our answer, it may be remembered, was that neither of the serious candidates considered — those put forward by Kant and Kripke — could confidently be accepted. And that left only the forlorn possibility that more exotic examples of ESP, meditation, and so on, might fill the gap. The present case seems little different. No philosopher, so far as we know, has ever suggested that necessarily true propositions might be known a priori by transcendental argument. So even if transcendental arguments *were* to prove their worth as ways of providing wholly a priori knowledge, it seems unlikely that they would ever be invoked in support of a positive answer to Question 9. Again, although Kripke's claim that a priori knowledge of contingent propositions is made possible by reference-fixing definitions is paralleled by the standard conventionalist claim that a priori knowledge of noncontingent propositions is made possible by meaning-assigning definitions, the conventionalists would hasten to add that any knowledge so acquired arises through *analysis* of the meaning so assigned or by inference therefrom. In other words, the comparable conventionalist view about a priori knowledge of noncontingent propositions attributes this knowledge to a paradigm form of ratiocination. Finally, so far as the exotic cases are concerned, it seems safe to say that no one at all — philosopher or otherwise — has ever supposed that ESP or the like could yield a *a priori* knowledge of *noncontingent* truths.

The only case that seems worth discussing is that of intuition. Mathematicians, in particular, and sometimes logicians as well, are prone to ascribe their discoveries of new truths in mathematics and logic to "sheer intuition". We have already — in section 4 — discussed the near cousin of intuition, viz., self-evidence, and have found it wanting. But whereas those who talked of self-evidence would almost certainly have counted it as a form of ratiocinative knowledge, those who talk of intuition tend not to. The appeal to intuition is often made in such a way as to suggest that intuited propositions are neither known by experience nor known by ratiocination.

But are such propositions really known at all? Once more we need to remind ourselves of the difference between coming to believe a proposition which is true and knowing that that proposition is true. We might well allow that a person can come to believe — by intuition or whatnot — that a proposition is true when it is in fact true and yet deny, on the grounds that the justificatory condition of knowledge is not satisfied, that that person has *knowledge* of the truth of that proposition. Intuition, though it may be a means of discovery, can hardly count as a genuine mode of knowledge and a fortiori can hardly count as a mode of a priori knowledge.

Frege saw that very clearly in *The Foundations of Arithmetic* when he bemoaned the fact that

We are all too ready to invoke inner intuition, whenever we cannot produce any other ground of knowledge.<sup>52</sup>

He claimed, correctly, that when we judge a proposition to be a priori or empirical,

this is not a judgement about the conditions, psychological, physiological and physical, which have made it possible to form the content of the proposition in our consciousness; nor is it a judgement about the way in which some other man has come, perhaps erroneously, to believe it true; rather, it is a judgement about the ultimate ground upon which rests the justification for holding it to be true.<sup>53</sup>

And for this reason he sought, in his own investigations of mathematics and logic, never to invoke an appeal to intuition but always to give rigorous proofs of the propositions which he took to be true. Then, and only then, was he satisfied that he *knew* the proposition to be true.

Frege seems clearly to be correct in all this. But if so then intuition can hardly be counted as a nonratiocinative means of acquiring genuine knowledge of noncontingent propositions.

True, there *may* be still other ways of knowing such propositions than those we have considered. And it must be admitted that we know of no good argument which demonstrates the impossibility of nonratiocinative knowledge of noncontingent propositions. But it is surely safe to conclude that the prospects for finding any cases of such knowledge look very bleak indeed.

10. *Are there any noncontingent propositions which are unknowable?*

We begin by asking ourselves, "What conditions would have to be satisfied in order that *all* noncontingent propositions could be known?" For, if it turns out that no such set of conditions is satisfiable, then there will exist some noncontingent propositions which are unknowable. What might such a set of necessary conditions be?

The number of noncontingent propositions is infinite. To see this, one need only recall that every proposition of arithmetic (whether true or false) is noncontingent, and that clearly there must be an infinity of these since there is an infinity of numbers and there is an infinity of true propositions about *each* number and an infinity of false propositions about *each* number. Thus there are as many false noncontingent propositions (viz., an infinite number) as there are true noncontingent propositions (viz., an infinite number).

Given the immensity of the class of noncontingent true propositions, in what ways can we expect to know them? Experiential means will do for some of them. We have already seen this in the case of proposition (3.32), the proposition that one plus one is two. Experience, we said, could teach us that this proposition is true. But the numbers referred to by our hypotheses need not be too large before experiential methods fail us. This was illustrated in the case of proposition (3.33). So although some noncontingent truths are knowable experientially, others, indeed an infinite number of others, would seem to be knowable, if at all, only ratiocinatively.

Now what conditions would have to be satisfied for these remaining propositions to be knowable ratiocinatively? Some of them, of course, can be known analytically, i.e., can be known directly by conceptual analysis. But again, the vast bulk of them cannot be known by these methods. The

52. G. Frege, *The Foundations of Arithmetic*, Oxford, Blackwell (2nd revised ed.), 1959, p. 19.

53. *Ibid.*, p. 3.

Goldbach Conjecture would seem to be a case in point. Of the infinitely many noncontingent propositions which cannot be known experientially or analytically, there would appear to be only one means remaining for knowing their truth, viz., inferring them from analytical truths already established. This is the way most mathematicians have come to view the problem. The limits of the knowable among noncontingent propositions have come to be regarded as virtually coinciding with the limits of the validly inferable.

Is the method of inference up to the task? By taking recourse to inferential methods can we hope to be able to establish (at least in principle) the truth-value of every noncontingent proposition?

In the first half of this century the question was seriously pursued for a special class of noncontingent propositions, viz., arithmetical propositions. Although this class hardly exhausts the class of noncontingent propositions, a negative answer about the class of arithmetical propositions would obviously require a negative answer concerning the larger class of which it is a part. That is, it is a necessary condition for all noncontingent propositions being knowable, that all arithmetical propositions be knowable, and the latter class is knowable only if inferential methods can suffice to establish all those among them which are not knowable by other means.

A serious limit on the power of inferential methods was established in a famous paper published in 1931 by the mathematician-logician, Kurt Gödel.<sup>54</sup> In this paper Gödel showed that not *all* true propositions of arithmetic can be inferred from a finite set of consistent axioms.

But to say that not *all* true propositions may be derived from a finite set of true axioms is *not* to say that there exists *any one* true proposition which could not so be deduced. For every finite consistent set<sup>55</sup> of axioms there will always exist some propositions which cannot be inferred from that set, but this still allows that any of those propositions could be inferred from another or from an enlarged set of axioms. Any proposition one chooses could always be inferred from one set or other; the only point is that not *all* propositions could be inferred from any *one* finite set no matter how large.

But what exactly does this imply in regard to the question we are pursuing? Does Gödel's Proof show that there are some noncontingent propositions which are unknowable?

Gödel's Proof shows us that in order to know greater and greater numbers of noncontingent propositions, we should have to use larger and still larger sets of axioms: a small finite set of axioms is demonstrably inadequate to the task. With this said, our answer would seem to be near at hand.

Ever-larger sets of axioms can be used to *establish* the truth of their implications only if those axioms are themselves *known* to be true. (Although any axiom can, of course, be derived from itself, this does nothing to show that that axiom is true. To show that an axiom is true, it is necessary to demonstrate the truth of the axiom extrasystematically, i.e., by some means other than by derivation from itself or from other axioms in the axiom-set.) Can we establish the truth of the members of ever-increasing axiom-sets without falling into a vicious infinite regress? Experiential means are obviously inadequate to the task; similarly, conceptual analysis would also seem to be inadequate. This leaves inference. But if we are to rely on inference, from which propositions are we to infer the problematic axioms? Presumably more powerful axioms. But if this is so, then the epistemic problem — far from being solved — is compounded, and our hope for a solution skirting the problem of a vicious regress is confounded.

The two means we have available to gain knowledge — experience and ratiocination — taken singly or in concert with inference appear to be inadequate to the magnitude of the task which Gödel has

54. Kurt Gödel, *On Formally Undecidable Propositions*, translated by J. van Heijenoort, in *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931*, ed. by J. van Heijenoort, Cambridge, Mass., Harvard University Press, 1967, pp. 596–616. For a popularized exposition of this important but difficult paper, see *Gödel's Proof* by Ernest Nagel and James R. Newman, New York, New York University Press, 1958.

55. Or more exactly, for every recursively generable consistent set of axioms, finite or infinite.

shown confronts us. Faced as we are with an infinitely large set of arithmetical propositions which are not derivable within any arithmetic having a finite number of axioms, we seem driven to the conclusion that some (indeed an infinite number) of these unknown propositions must remain unknowable.

For all we know, Goldbach's Conjecture might be just such a proposition. We do not have experiential knowledge of its truth in advance of a mathematical demonstration of its truth, as we had in dealing with the Königsberg Problem. Perhaps Goldbach's Conjecture is unprovable. We will come to know that it is *not*, only if someone should happen (by inferential means) to prove it true or prove it false. Until then, it remains unknown, and for all we know, unknowable.

*Appendix to section 6: a complete classificatory scheme for the epistemic and modal distinctions*

At the expense of complicating figure (3.1) somewhat, we can drop the simplifying assumption we made earlier concerning that part of the figure labeled with the question mark.

It is clear that the three categories, (1) *knowable experientially*, (2) *knowable ratiocinatively*, and (3) *knowable by some other means*, can combine in *seven* ways, not merely four as depicted in figure (3.1). The three additional ways, not represented, are:

- (a) knowable experientially and by some other, nonratiocinative means;
- (b) knowable experientially, ratiocinatively, and by some other means as well; and
- (c) knowable ratiocinatively and by some other, nonexperiential means.

These three additional categories can all be represented by subdividing that section on figure (3.1) labeled with the question mark. Doing so will give us the following figure:

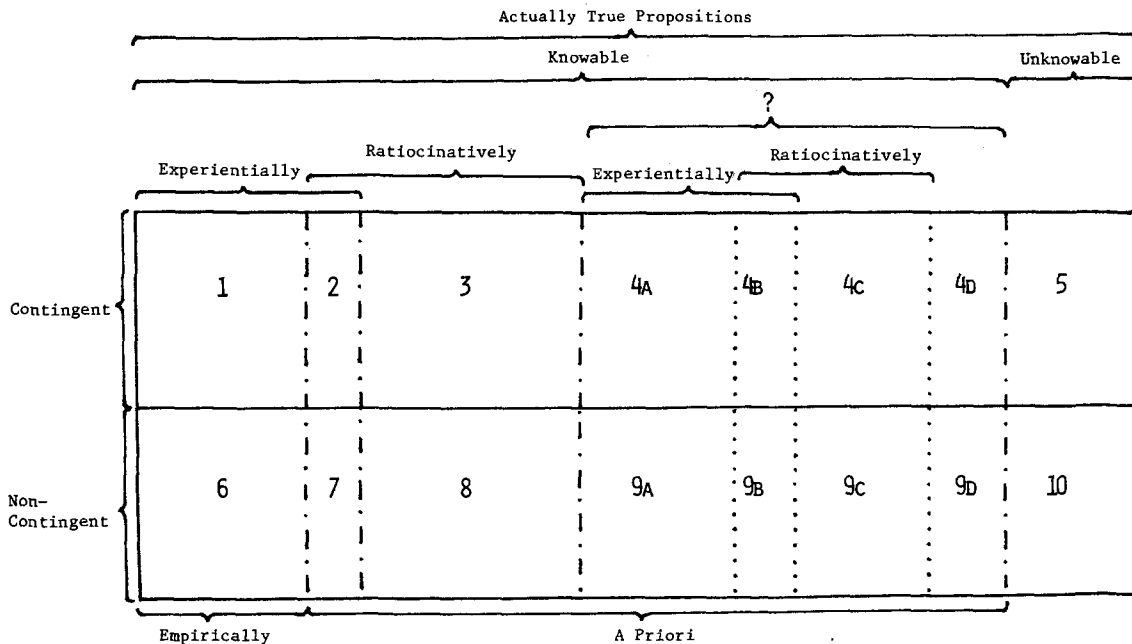


FIGURE (3.m)

Now, then, we must ask, “Which of these eight categories, viz., 4a through 4d and 9a through 9d, are empty and which have members?” The answers we give to each of these eight questions will depend on the answers we have given to the original questions, 1 through 4 and 6 through 9. Let us take one example. Consider the first of this new set of questions, viz., “Are there any contingent propositions which are knowable experientially and by some other, nonratiocinative means?” That is, “Is category 4a empty or not?” The answer to this question will depend on the answers we have given for the original questions 1 and 4, viz., [in (1)] whether there are any contingent propositions knowable experientially and not ratiocinatively, [in (4)] whether there are any contingent propositions knowable by other than experiential or ratiocinative means. We replied that undoubtedly there *are* contingent propositions knowable experientially which are not knowable ratiocinatively, and that there *may* be contingent propositions which are knowable by other than experiential or ratiocinative means. In *combining* these two answers, the answer to the question whether there are any propositions in category 4a becomes: We simply do not know; there may be.

If one goes through all the categories, 4a through 4d and 9a through 9d, the strongest claim that can be made in any case, is that the category *may* have some members. In none of these cases can we say with any good reason that there *is* a proposition in that category.

### EXERCISE

*Try to argue for each of the categories 4b through 4d and 9a through 9d whether that category is empty or not.*

## 7. THE EPISTEMOLOGY OF LOGIC

We began this chapter by asking how knowledge of the subject matter of logic is possible and whether that knowledge is obtained in a way different from knowledge in physics and chemistry. To help answer these questions we have spent much time discussing some fundamental epistemological distinctions and have just completed a classificatory scheme containing no fewer than ten major categories of propositions.

The scene is now set for us to ask where exactly in this elaborate scheme the subject matter of logic is to be placed.

Let us begin with the exhaustive and exclusive classification comprising the horizontal division on figure (3.m): the contingent/noncontingent classification. Are the propositions which make up the subject matter of logic contingent or noncontingent? Let us remind ourselves just which propositions these are. They are those propositions which ascribe to other propositions modal attributes. They are not propositions which ascribe attributes to physical things in the actual world or in any other world; and they are not propositions which ascribe truth-values (simpliciter) to other propositions. They are a very specific set of propositions. They are propositions *about* propositions, and more specifically about a certain set of attributes (viz., modal attributes) of propositions.

Now what about these propositions? Are they contingent or noncontingent? Or, perhaps, are some of them contingent and some noncontingent? At this point we will merely assert an answer: they are all of them noncontingent. Although a proof for this claim could be given now, we have chosen to reserve it for later in this book. A nonformal *proof* that all the propositions of logic are noncontingent is given in chapter 6.

This means that the subject matter of logic is to be found distributed somewhere across categories 6

through 10, and that the *science* of logic will be restricted to at most these five categories. As a matter of fact, however, it turns out the science of logic is concerned with no more than *two* of these categories, viz., 7 and 8. Let us see why.

Immediately we can see that the science of logic can have no commerce with propositions in category 10. It is logically impossible to have a science of unknowable propositions. For all we know, there may be some logical propositions which are humanly unknowable. But if there are any, they lie beyond the reach of human science.

This leaves four categories, viz., categories 6 through 9, as possible categories of logical propositions which are subject to investigation by the science of logic. But two of these, viz., categories 6 and 9, are, as we have already seen, problematic. We have no way of knowing whether either of them has *any* members. And if it is difficult to say, in the first instance, whether there are *any* noncontingent truths knowable empirically (category 6) or knowable other than experientially or ratiocinatively (category 9), then it is doubly difficult to say whether there are any truths of logic in these categories, for truths of logic form a proper subset of the class of noncontingent truths. Even if we could be assured that categories 6 and 9 do have members, it would require further argumentation to establish that among these members are some propositions of logic.

Having utterly no guarantee, then, that any of its subject matter lies outside of categories 7 and 8, it is hardly any wonder that the science of logic is concerned exclusively with propositions knowable ratiocinatively.

Whether it is a case of the subject matter dictating the methodology (i.e., ratiocination), or the methodology dictating the subject matter, is a question which is idle and profitless to pursue. For the fact of the matter is that by a long series of historical adjustments the subject matter of logic and the methodology of logic have been adapted to fit one another.

The natural sciences (e.g., physics, chemistry, astronomy), the life sciences (e.g., botany, zoology, medicine) and the social sciences (e.g., history and economics) are all *empirical* sciences. Their subject matter is concerned with but one possible world, the actual one. The science of logic (and the science of mathematics, too, for that matter) is an *a priori* science. Not only is its subject matter different from the other sciences in that it is concerned with all possible worlds; its methodology differs too in that it adopts exclusively the ratiocinative method of gaining knowledge.

It has not always been this way. The natural sciences and the science of logic have not always been characterized by such different methodologies. If we look into ancient texts in mathematics and physics we discover that their methodologies were often confused amalgams of both ratiocinative and empirical methods. In Euclid's geometry, for instance, we find his attempt to prove the congruence of a certain pair of triangles by the expedient of moving one of them (in imagination) in space on top of the other. But this is now commonly seen as an unwitting intrusion of an empirical method into what we now regard as an *a priori* science. What happens to the shape of a thing when it moves about in space cannot be known *a priori* but can be known only by appeal to experience. Similarly, on the other side, we can find the inappropriate intrusion of *a priori* methodologies in the physical sciences. An example is the ancient physical theory that objects of unequal weight fall at speeds proportional to their weights. The justification of this theory was that '*it stood to reason* that heavy things fall faster than light things.' It was only in the seventeenth century that a man of extraordinary genius, Galileo, realized that this 'obvious truth' was a contingent matter and in need of experimental verification. And in actually performing the test he discovered that this seeming 'truth of reason' was in fact false. The *a priori* method in physics was thereby struck a blow from which it never recovered. And over the course of subsequent centuries, scientists and philosophers alike have tried to remove all vestiges of *apriorism* in physics.

The conscious attempt to purge unwitting appeals in *a priori* sciences to natural phenomena knowable empirically came later. It is only in the nineteenth century that such illicit methods as the



one of Euclid's just mentioned were purged from geometry and other mathematical and logical sciences. Here the credit, in part, goes to such persons as Riemann, Gauss, and especially David Hilbert.

Universal agreement that logic is, or should be, a ratiocinative science has, however, never been achieved. For a variety of reasons, some philosophers have asserted the contrary. Mill, we have already seen, argued against this view of logic on the basis of his belief that the only knowledge possible was empirical knowledge, that there was no such thing, ultimately, as a priori knowledge. Logic, as conceived by him, is just a high-order set of generalizations knowable empirically.

That there should be some controversy regarding the epistemology of logic is not surprising. Indeed it would be surprising if there were not. For the controversy surrounding its epistemology is a ramification of the controversy surrounding its ontology. To each of the ontologies discussed in chapter 2, there corresponds a distinctive epistemology. We have argued for an ontology which makes the subject matter of logic noncontingent *propositions*, and we have opted for the corresponding epistemology which, in our view, is that of ratiocination. In our view, logic is an a priori science.

Our answer, then, to the question "How is knowledge in logic possible?" is: by a priori reasoning, or more specifically, by ratiocinative reasoning. It remains now, in our next chapter, to examine how ratiocinative reasoning might assume different degrees of sophistication — how, in effect, the propositions of logic might be subjected to minimal degrees of analysis or to maximal degrees.

