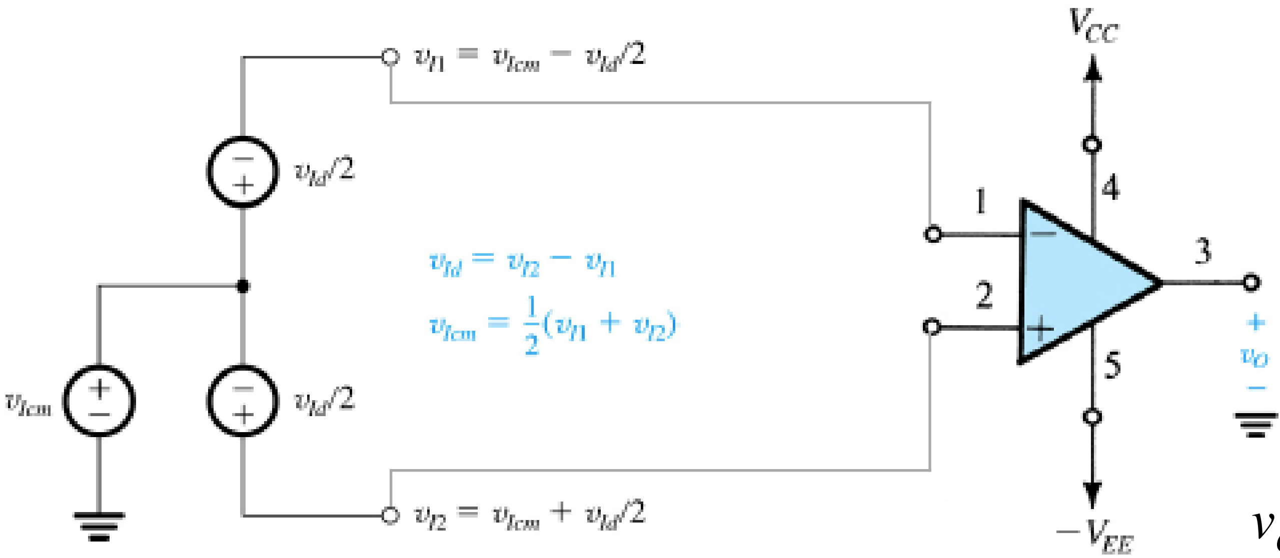
Applications of operational amplifiers

- ***Difference amplifier***
- ***Instrumentation amplifier***
- ***Opamp inverting integrator***
- ***Opamp differentiator***
- ***Charge-sensitive amplifier***

Textbook material for self-study:
Ch.2 - Operational Amplifiers,
Sections 2.4 to 2.5

Open-loop op amp as a difference amplifier

Goal: to amplify the difference of signals v_{I2}-v_{I1}, while supresssing the common mode gain



$$v_O = A_d v_{Id} + A_{cm} v_{Icm}$$

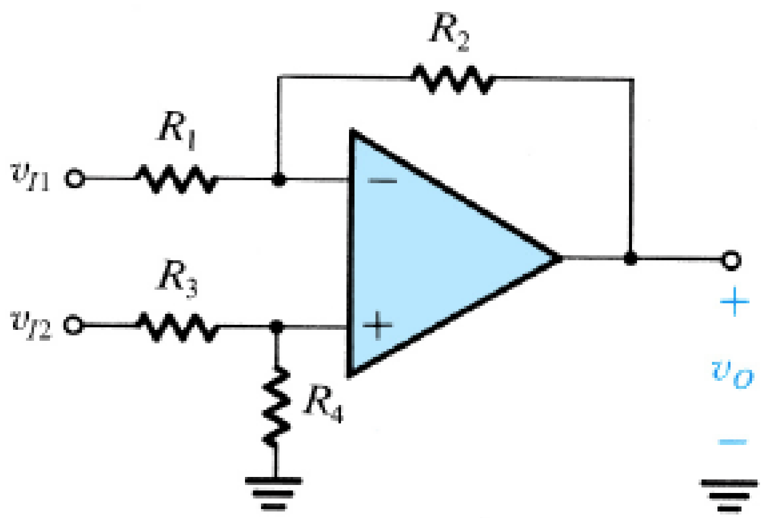
Common-Mode Rejection Ratio (CMRR) quantifies the op amp's ability to reject the common mode signals

$$CMRR = 20\log \frac{|A_d|}{|A_{cm}|}$$

Disadvantages of open-loop op amp as a difference amplifier:

- **very high gain but poor controllability**
- **closed-loop gain more stable and more predictable**

A single op amp difference amplifier



The condition:

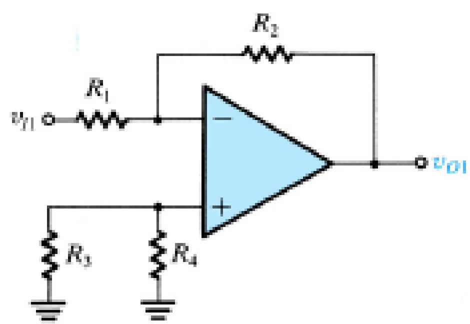
$$\frac{R_4}{R_3} = \frac{R_2}{R_1}$$

must be met

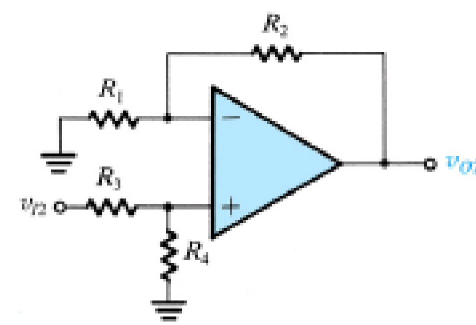
Goal: Find v_O **as a function of** v_{I1} **and** v_{I2}

Superposition technique

case 1

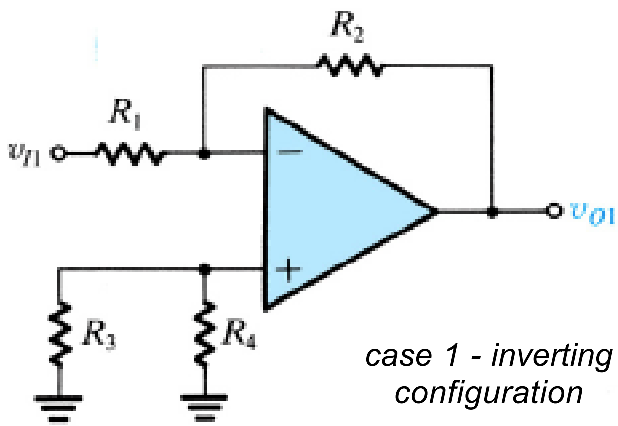


case 2



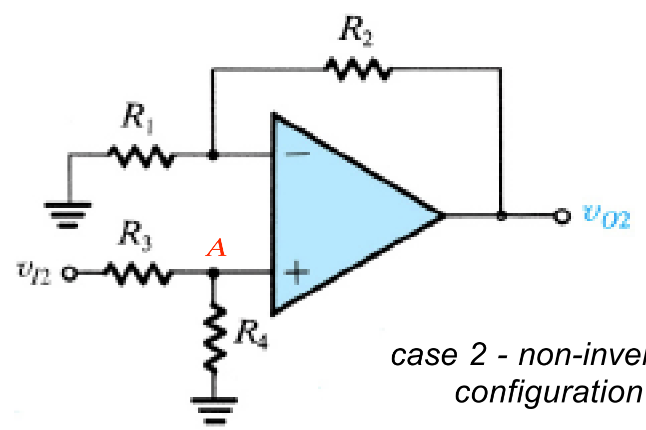
$$v_O = v_{O1(case1)} + v_{O2(case2)}$$

Differential gain of the difference amplifier



case 1 - inverting configuration

$$\frac{R_4}{R_3} = \frac{R_2}{R_1}$$



case 2 - non-inverting configuration

$$v_{O1} = -\frac{R_2}{R_1} v_{I1}$$

$$v_{O2} = v_A \left(1 + \frac{R_2}{R_1}\right) = v_{I2} \frac{R_4}{R_4 + R_3}\left(1 + \frac{R_2}{R_1}\right) = v_{I2} \frac{\frac{R_4}{R_4}}{\frac{R_4}{R_4} + \frac{R_3}{R_4}}\left(1 + \frac{R_2}{R_1}\right) =$$

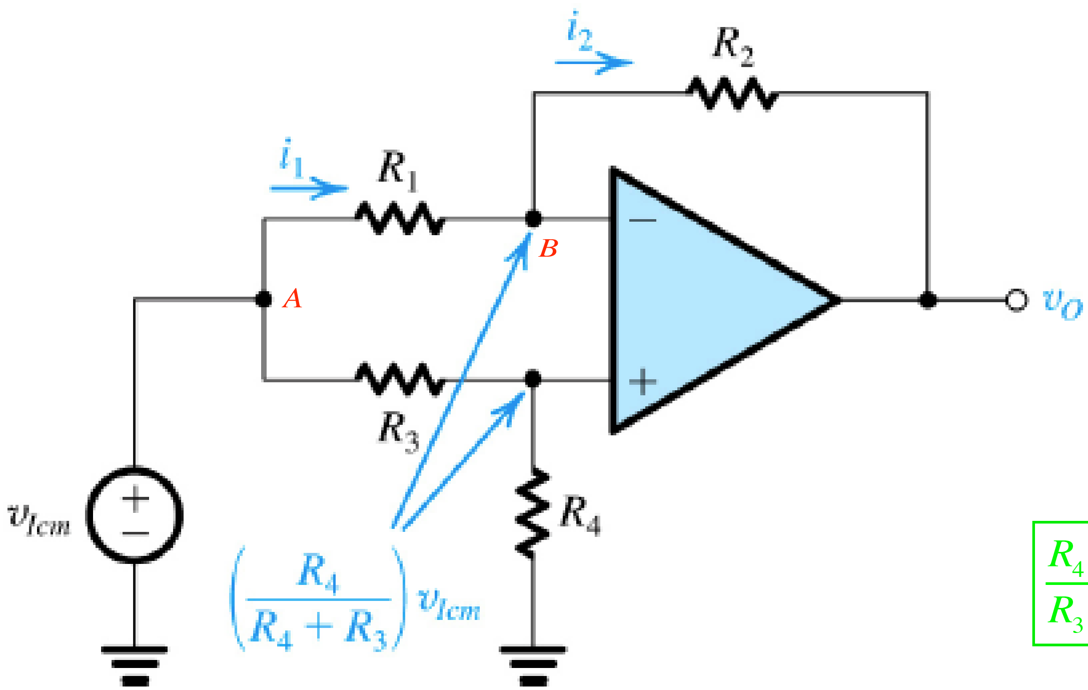
$$= v_{I2} \frac{1}{1 + \frac{R_1}{R_2}}\left(1 + \frac{R_2}{R_1}\right) = v_{I2} \frac{R_2}{R_2 + R_1} \frac{R_1 + R_2}{R_1} = v_{I2} \frac{R_2}{R_1}$$

$$v_O = v_{O1} + v_{O2} = -\frac{R_2}{R_1} v_{I1} + \frac{R_2}{R_1} v_{I2} = \frac{R_2}{R_1}\left(v_{I2} - v_{I1}\right) = \frac{R_2}{R_1} v_{Id}$$

Differential gain of the difference amplifier: $A_{diff} = \dfrac{R_2}{R_1}$ **when** $\dfrac{R_4}{R_3} = \dfrac{R_2}{R_1}$

To satisfy the $\frac{R_4}{R_3} = \frac{R_2}{R_1}$ *condition while minimizing a possible matching problem, we usually select* $R_3 = R_1$ *and* $R_4 = R_2$

Common-mode gain of the difference amplifier



CM - common-mode excitation only

$$\left(\frac{R_4}{R_4+R_3}\right)v_{Icm}$$

$$\frac{R_4}{R_3}=\frac{R_2}{R_1}$$

$$i_1=\frac{v_A-v_B}{R_1}=\frac{1}{R_1}\left(v_{Icm}-\frac{R_4}{R_4+R_3}v_{Icm}\right)=v_{Icm}\frac{R_3}{R_4+R_3}\frac{1}{R_1}$$

$$v_O=v_B-i_2R_2=\frac{R_4}{R_4+R_3}v_{Icm}-i_2R_2=\frac{R_4}{R_4+R_3}v_{Icm}-v_{Icm}\frac{R_3}{R_4+R_3}\frac{R_2}{R_1}=v_{Icm}\left(\frac{R_4}{R_4+R_3}-\frac{R_3}{R_4+R_3}\frac{R_4}{R_3}\right)=$$

$$=v_{Icm}\left(\frac{R_4}{R_4+R_3}-\frac{R_4}{R_4+R_3}\right)=0$$

Common-mode gain of the difference amplifier:

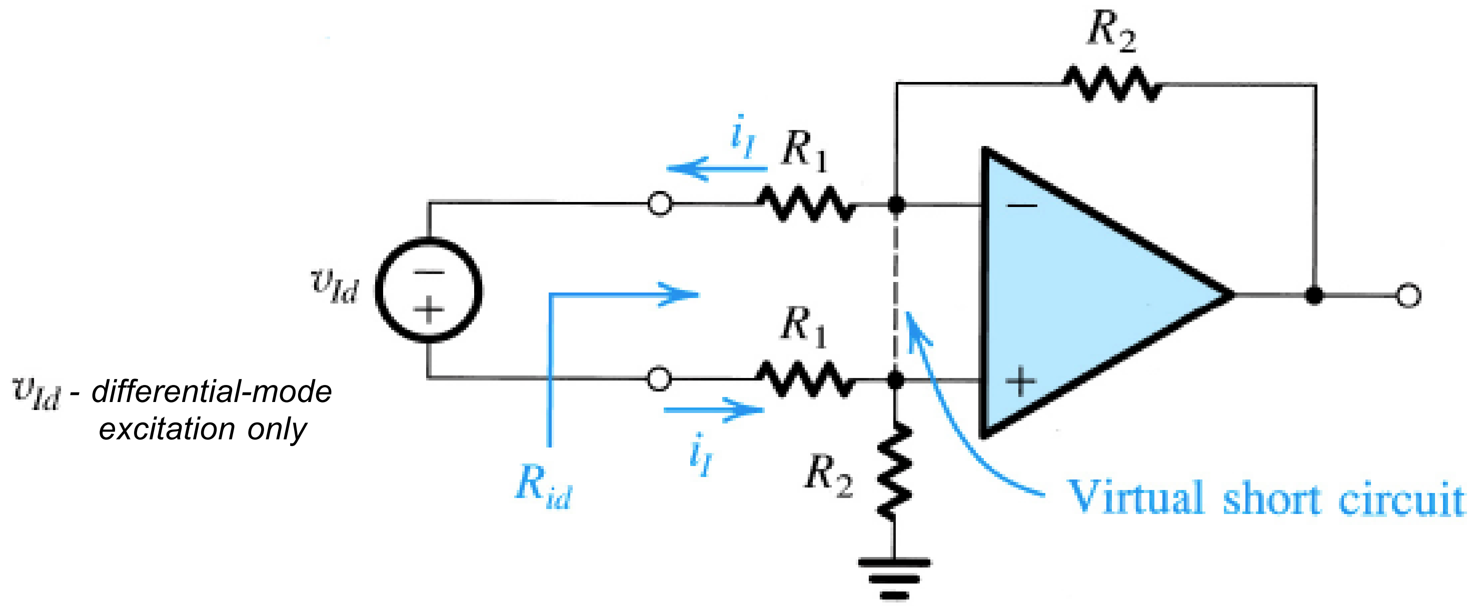
$$A_{cm}=\frac{v_O}{v_{Icm}}=\frac{R_4}{R_4+R_3}\left(1-\frac{R_2}{R_1}\frac{R_3}{R_4}\right)=0$$

when $\frac{R_4}{R_3}=\frac{R_2}{R_1}$

$R_3=R_1$

$R_4=R_2$

Differential input resistance of the difference amplifier



$$\frac{R_4}{R_3} = \frac{R_2}{R_1}$$

$$R_3 = R_1$$

$$R_4 = R_2$$

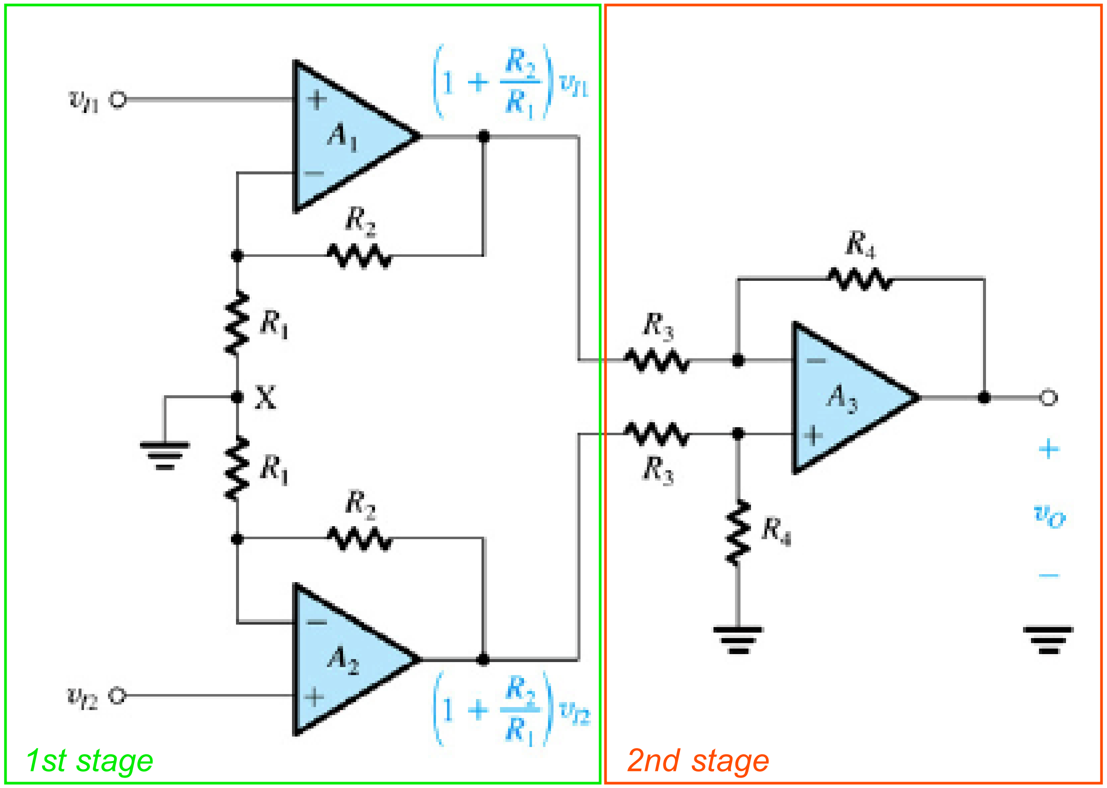
Differential input resistance of the difference amplifier:

$$R_{id} \equiv \frac{v_{Id}}{i_I} = \frac{R_1 i_I + 0 + R_1 i_I}{i_I} = 2R_1$$

Disadvantages of the single op amp difference amplifier:

- **small input resistance independent on op amp technology (gain R_2/R_1 must be large)**
- **to vary differential gain, resistors must be varied simultaneously (a difficult task)**

The instrumentation amplifier



1st stage - noninverting op amps A_1 and A_2 to increase the input resistance

$$A_{1d} = A_{2d} = 1 + \frac{R_2}{R_1}$$

$$R_{id} \approx \infty$$

2nd stage - a single op amp A_3 difference amplifier

$$A_{3d} = \frac{R_4}{R_3}$$

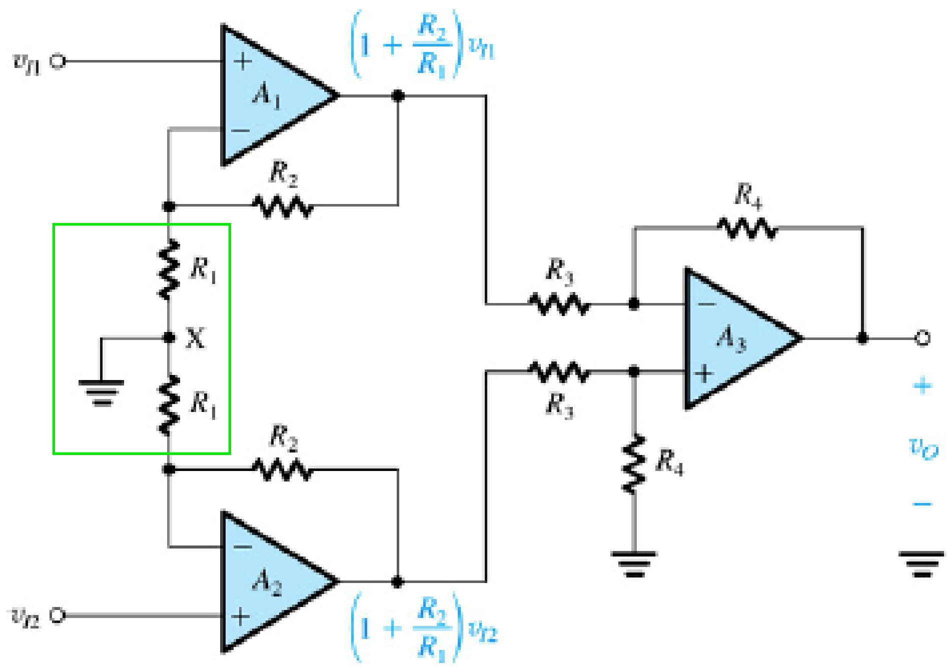
$$A_d = \frac{v_O}{v_{Id}} = \frac{v_O}{v_{I2} - v_{I1}} = A_{1,2\,d}A_{3d} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3}\right)$$

$$A_{cm} = A_{1,2\,cm}A_{3cm} = A_{1,2\,cm} \cdot 0 = 0$$

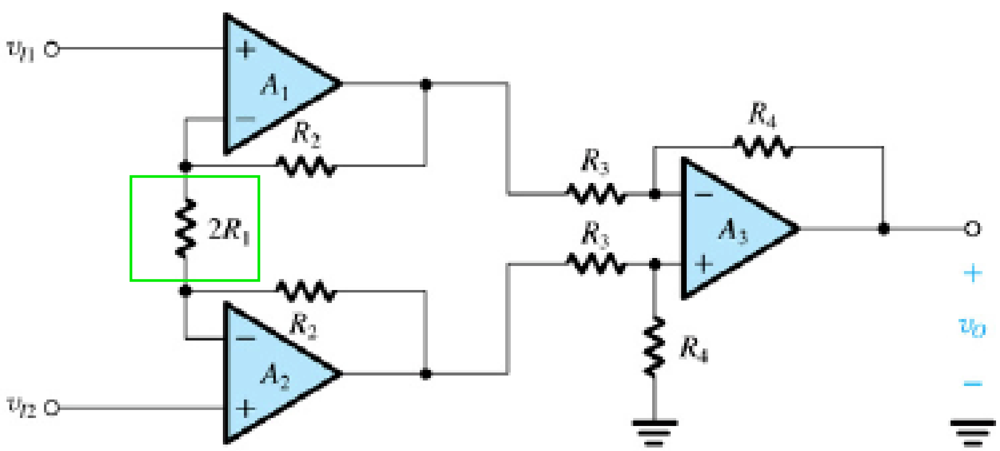
Disadvantages:

- *non-zero common-mode gain of the first stage enlarges CM signal for the second stage*
- *A_1 and A_2 must be perfectly matched, otherwise their mismatch willl be amplified by A_3*
- *to vary differential gain, two resistors (R_1, R_2, etc.)must be varied simultaneously (a difficult task)*

The improved instrumentation amplifier

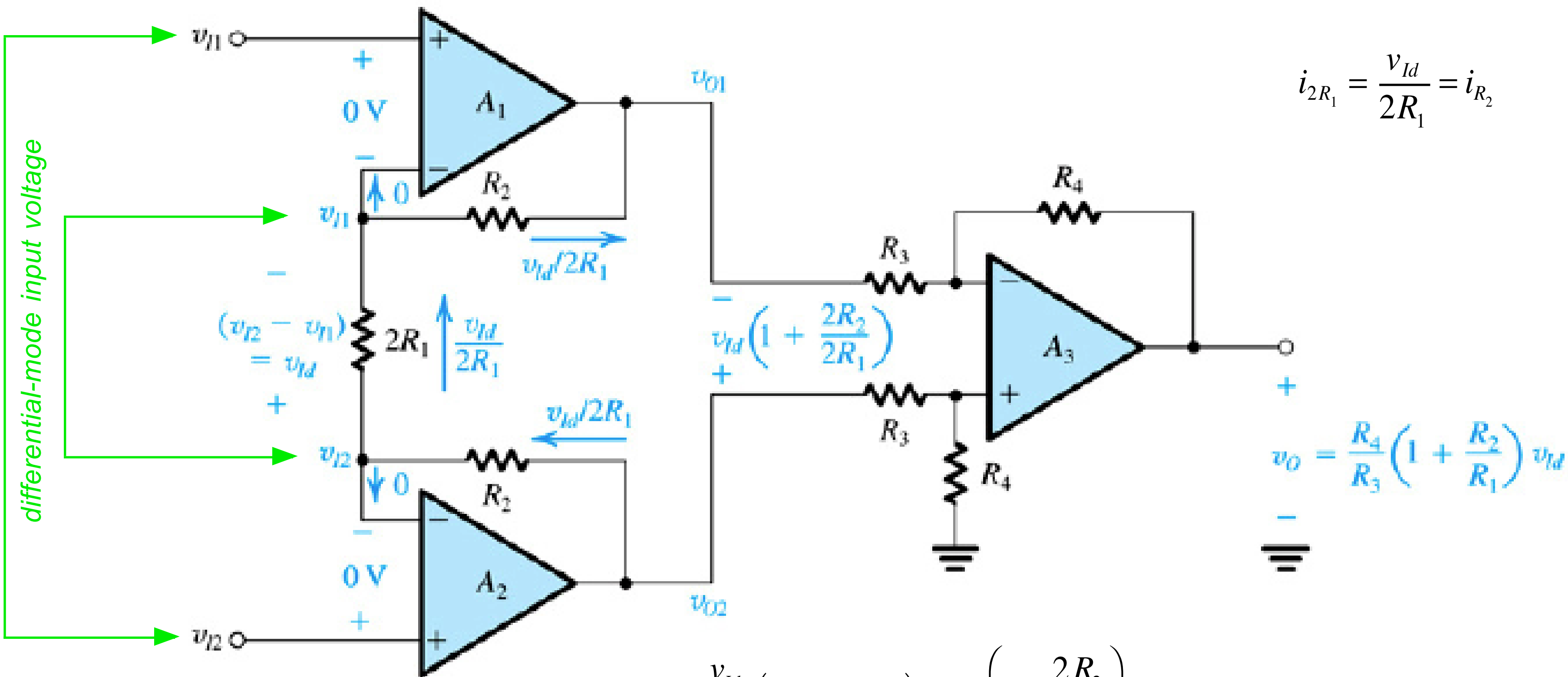


Original instrumentation amplifier



Improved instrumentation amplifier

Differential gain of the improved instrumentation amplifier



$$i_{2R_1} = \frac{v_{Id}}{2R_1} = i_{R_2}$$

$$v_{O2} - v_{O1} = \frac{v_{Id}}{2R_1}(2R_1 + 2R_2) = v_{Id}\left(1 + \frac{2R_2}{2R_1}\right)$$

$$v_O = \left(\frac{R_4}{R_3}\right)(v_{O2} - v_{O1}) = \frac{R_4}{R_3} v_{Id}\left(1 + \frac{R_2}{R_1}\right)$$

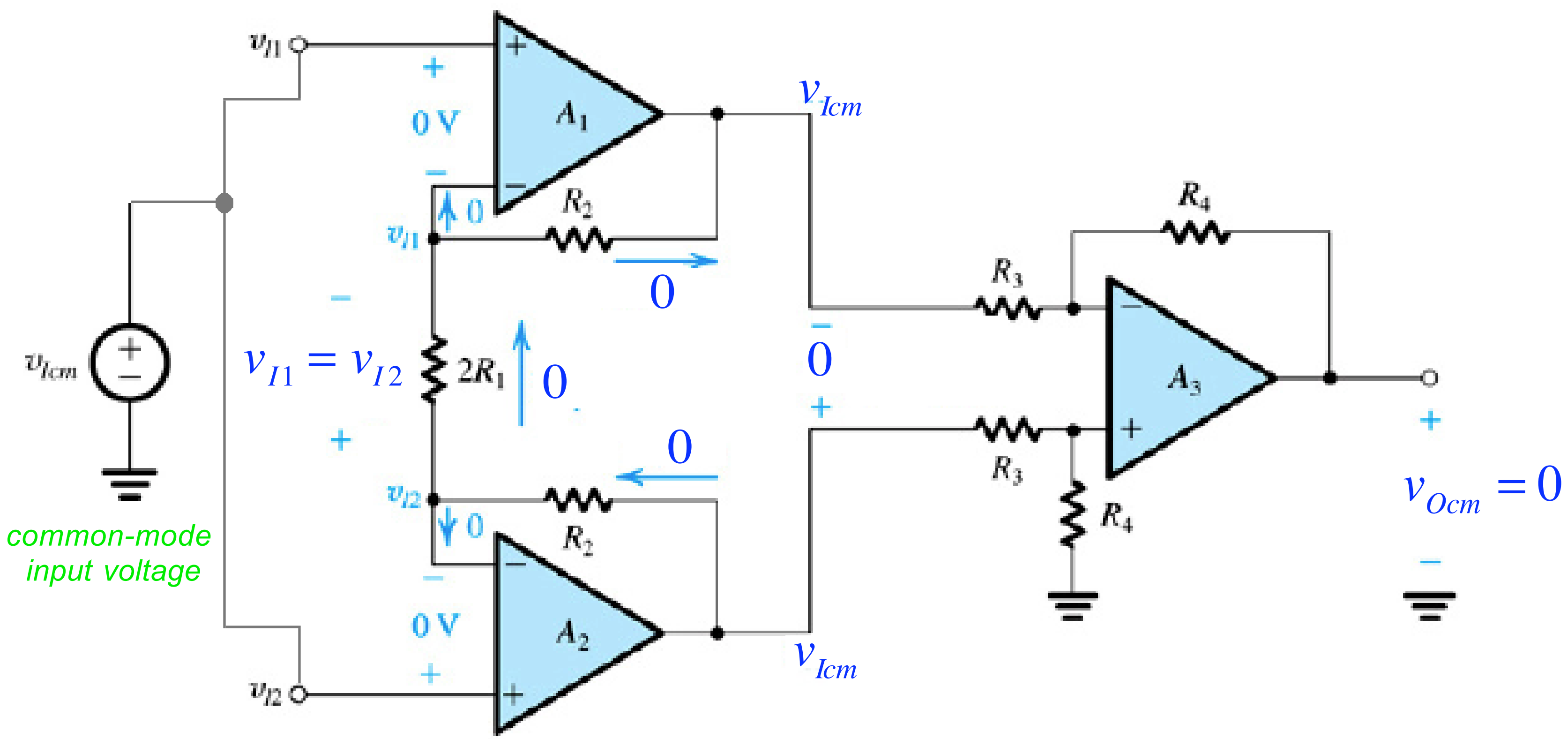
- *Differential gain*

$$A_d = \frac{v_O}{v_{Id}} = \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right)$$

$$A_d = \frac{v_O}{v_{Id}} = \frac{R_4}{R_3}\left(1 + \frac{R_2' + R_2''}{2R_1}\right)$$

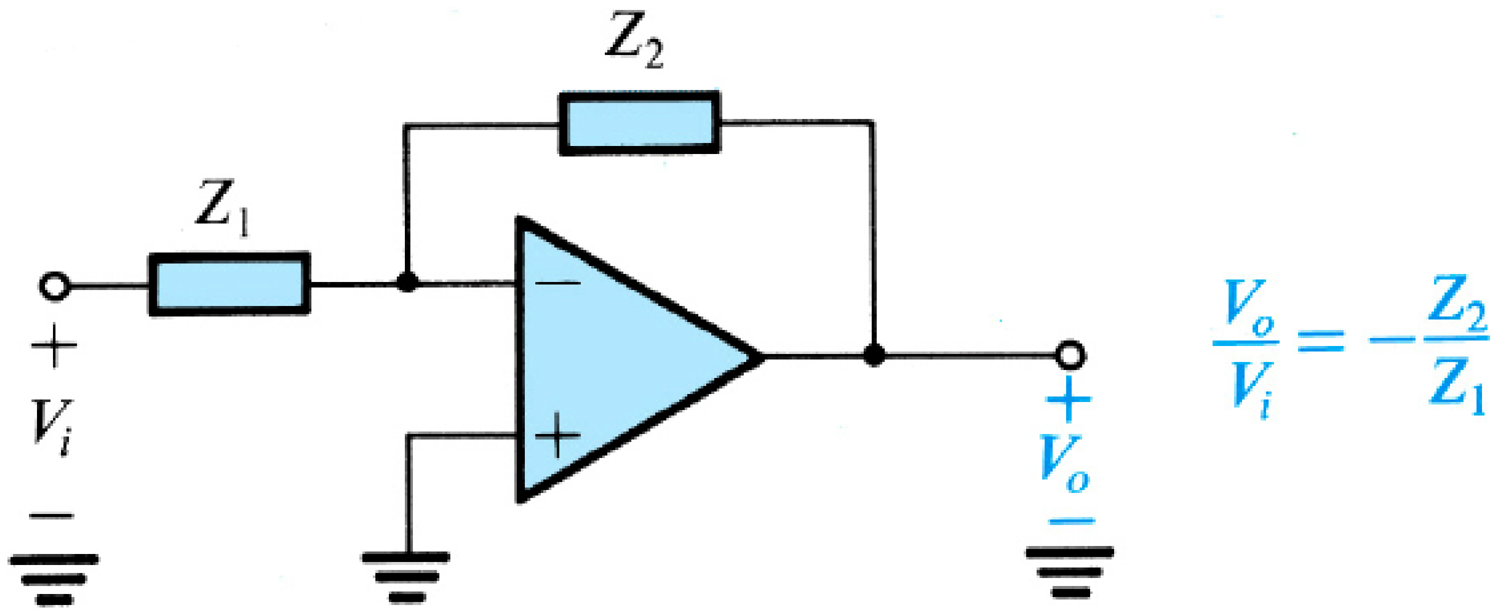
- *Differential gain no longer affected by R_2 mismatch*
- *Gain can be varied by changing only one resistor, $2R_1$*

Common-mode gain of the improved instrumentation amplifier



- *The input stage A_1, A_2 no longer amplifies the common-mode signal; it simply propagates it to the A_3 input, where it becomes supressed by A_3.*

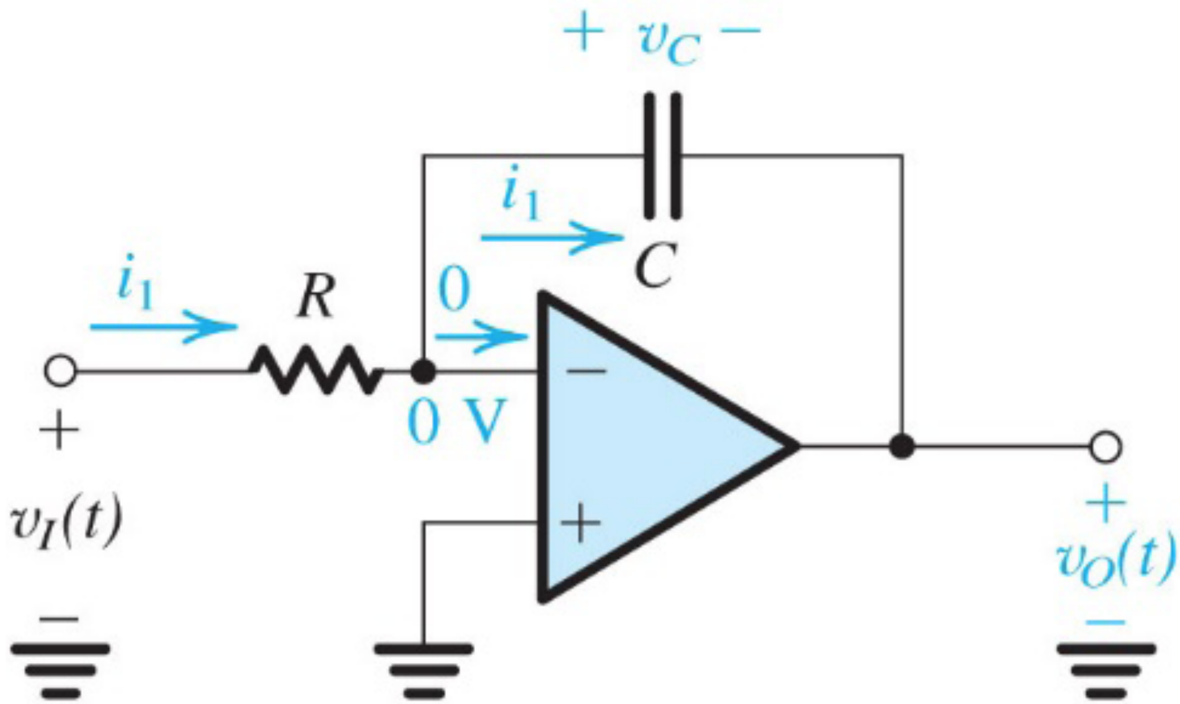
Inverting configuration with RC circuits



$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

- *Using combinations of RC feedback circuits instead of resistor-only feedback results in new circuits featuring the unique transfer functions that depend on signal frequency ω.*

The inverting integrator



Analysis in the time domain

current through R $i_1(t) = \frac{v_I(t)}{R}$

charge build across C $\int_0^t i_1(t)dt$

voltage across C $\frac{1}{C}\int_0^t i_1(t)dt$

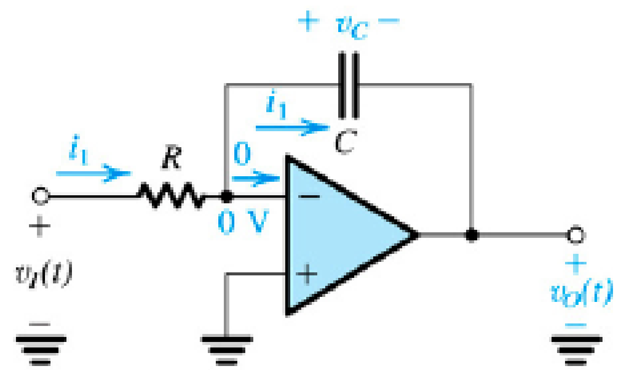
$$v_C(t) = V_C + \frac{1}{C}\int_0^t i_1(t)dt$$

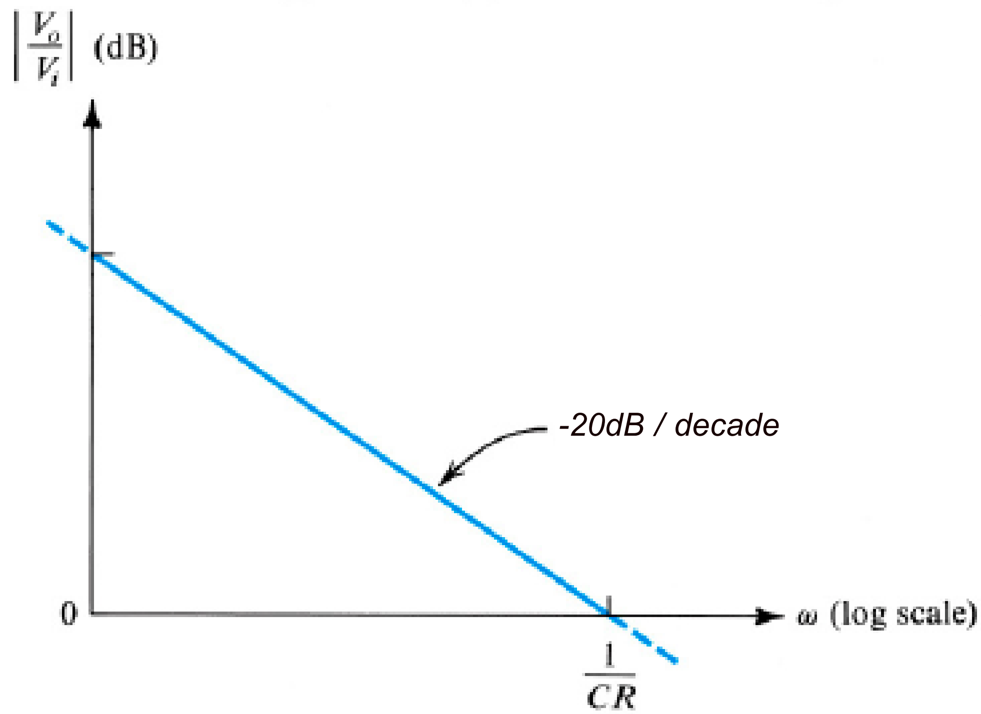
The output voltage $$v_O(t) = -\frac{1}{RC}\int_0^t v_I(t)dt - V_C$$

where V_C is the initial voltage across C

- *The circuit provides an output voltage $v_O(t)$ that is proportional to the time integral of the input voltage $v_I(t)$. The RC product is the integrator time constant.*

The inverting integrator





Frequency response of the integrator.

Analysis in the frequency domain

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{\frac{1}{sC}}{R} = -\frac{1}{sCR}$$

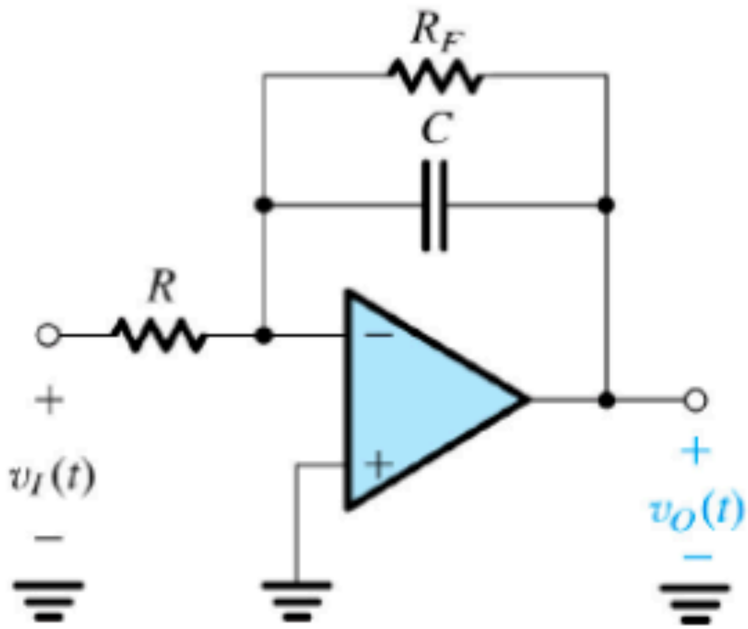
For physical frequencies $s = j\omega$

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{1}{j\omega CR}$$

Magnitude of the transfer function: $\left|\frac{V_o}{V_i}\right| = \frac{1}{j\omega CR}$

Disadvantage: There is no negative feedback at $\omega = 0$ (-3dB corner frequency), and the gain magnitude is infinite (in practice, the amp does not behave like a linear amplifier and its output voltage is saturated.

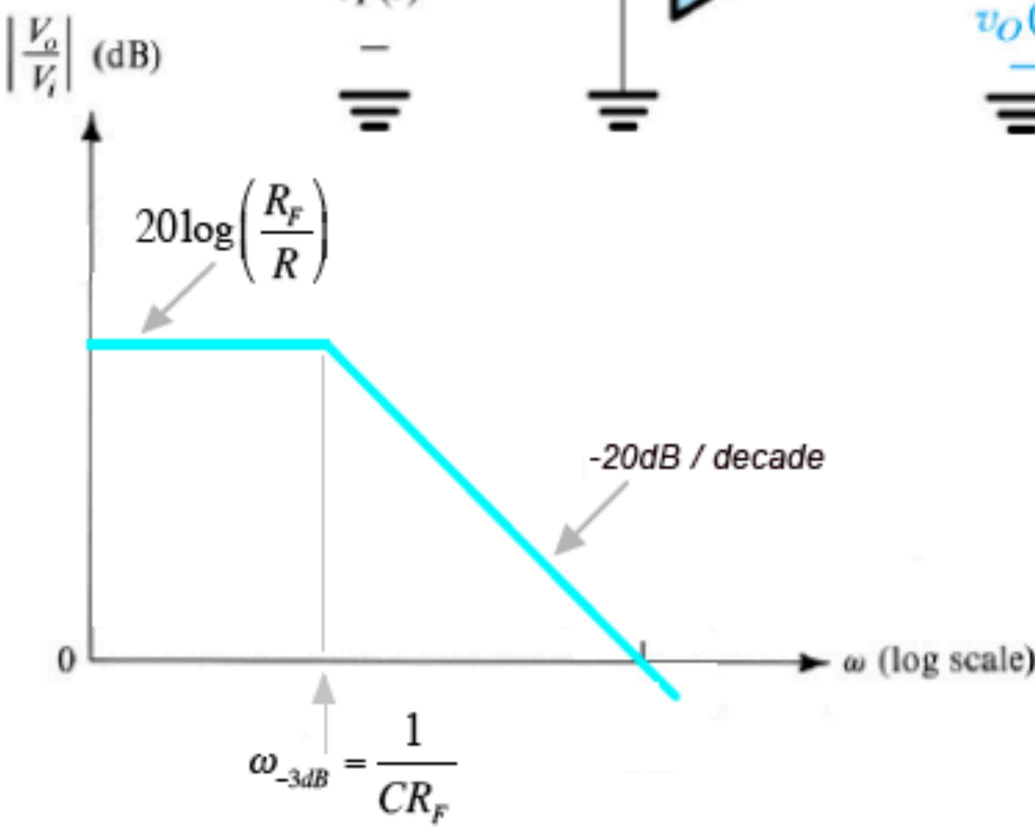
The inverting integrator with a finite dc gain



Analysis in the frequency domain

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{1}{Z_1(s)Y_2(s)} =$$

$$= -\frac{1}{R\left(\frac{1}{R_F} + sC\right)} = -\frac{1}{\frac{R}{R_F} + sCR} = \frac{-\frac{R_F}{R}}{1 + sCR_F}$$

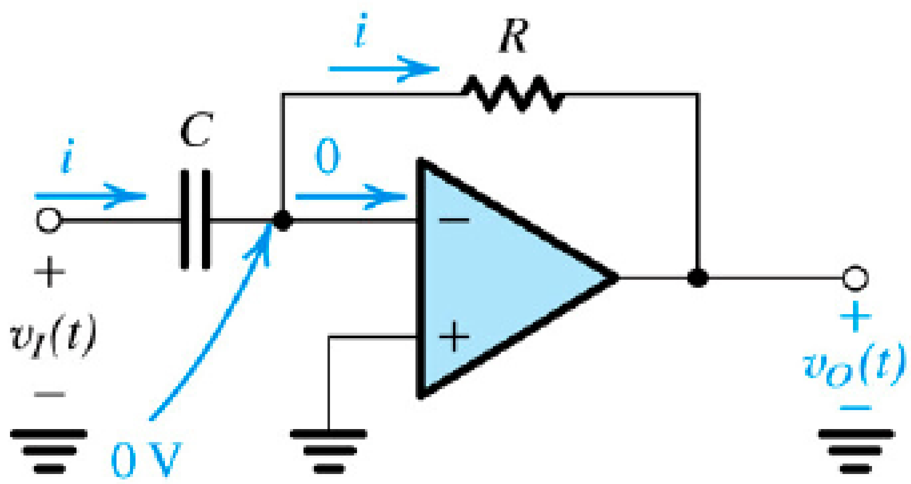


For physical frequencies $s = j\omega$

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{\frac{R_F}{R}}{1 + j\omega CR_F}$$

Magnitude of the DC gain: $\left|\frac{V_o}{V_i}\right| = \frac{R_F}{R}$

The op amp differentiator

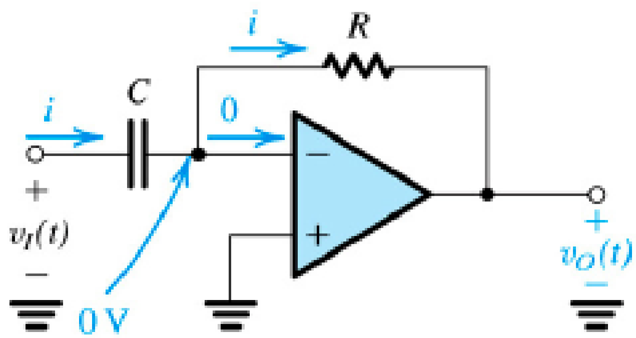


Analysis in the time domain

current through C $$i(t) = C\frac{dv_I(t)}{dt}$$

the output voltage $$v_O(t) = -CR\frac{dv_I(t)}{dt}$$

The op amp differentiator



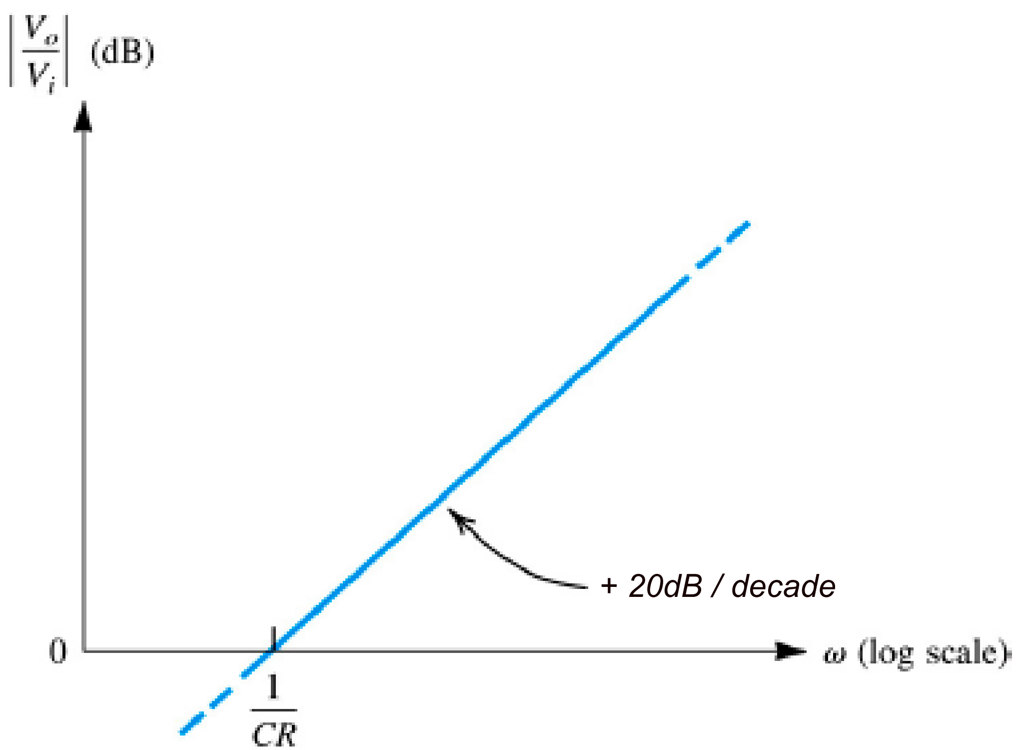
Analysis in the frequency domain

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{R}{\frac{1}{sC}} = -sCR$$

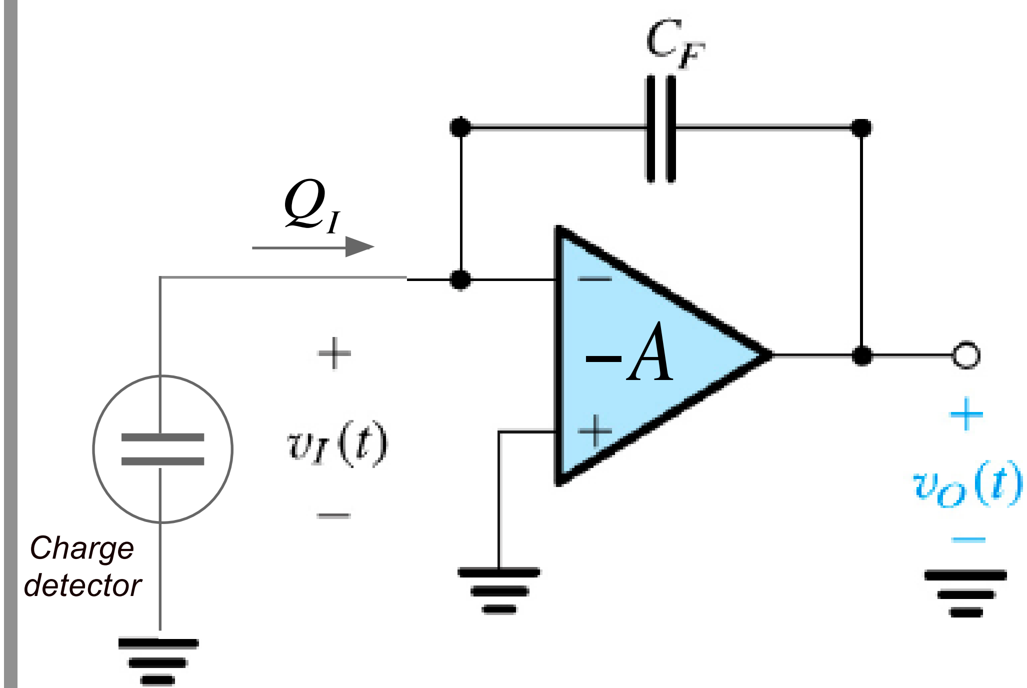
For physical frequencies $s = j\omega$

$$\frac{V_o(j\omega)}{V_i(j\omega)} = -j\omega CR$$

Magnitude of the transfer function: $\left|\frac{V_o}{V_i}\right| = \omega CR$



Charge Sensitive Amplifier (CSA)



The output voltage $v_O = -Av_I$

The input impedance $Z_I = \infty$

Voltage across C_F

$$v_{C_F} = v_I - v_O = v_I - Av_I = v_I(1+A)$$

Charge deposited on C_F

$$Q_{C_F} = C_F v_{C_F} = C_F v_I (1+A)$$

$Q_I = Q_{C_F}$ *since* $Z_I = \infty$

Effective input capacitance $C_I = \dfrac{Q_I}{v_I} = C_F(1+A)$

"Charge" gain $A_Q = \dfrac{dV_O}{dQ_I} = \dfrac{Av_I}{C_I v_I} = \dfrac{A}{C_I} = \dfrac{A}{C_F(1+A)} \approx \dfrac{1}{C_F}$

$$[A_Q] = \frac{Volt}{Coulomb} = \frac{1}{Farad}$$