Small-signal analysis of BJT amplifiers

- ***Amplifier design using BJTs***
  - ***voltage amplifier***
  - ***voltage transfer characteristics***
  - ***small-signal voltage gain***

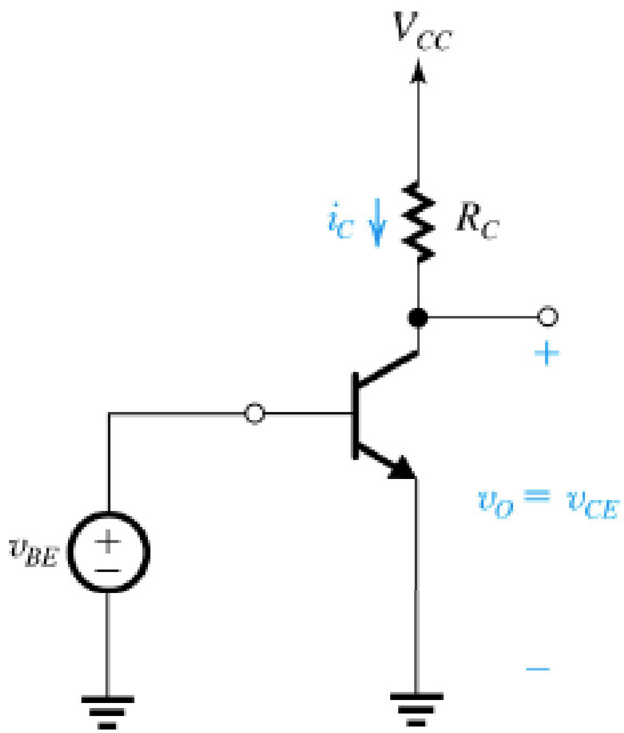
- ***Small-signal parameters of BJTs operating as amplifiers***

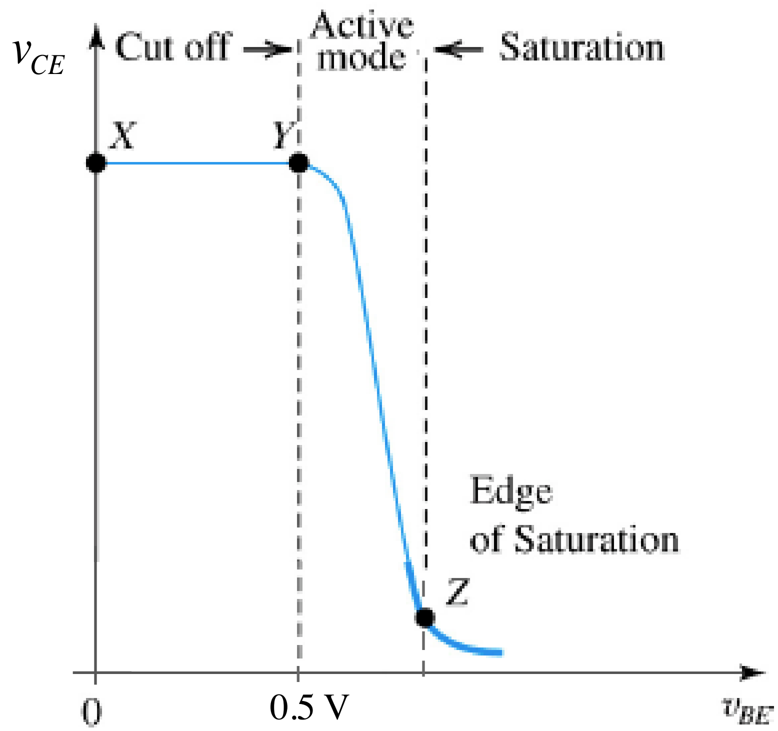
- ***Small-signal BJT models***
  - ***the hybrid-π model***
  - ***the T model***

- ***Small-signal analysis using small-signal models***

Textbook material for self-study: Sec.7.1-7.2 - BJT small-signal operation and models

Simple voltage amplifier using BJT





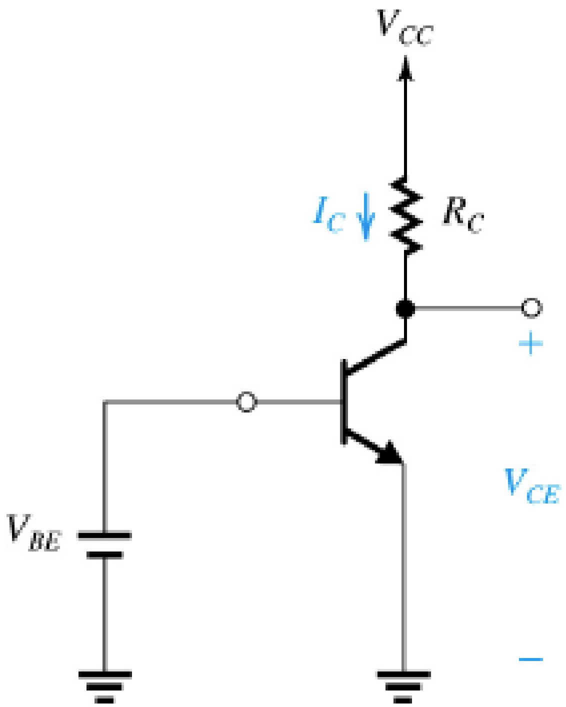
$$v_{CE} = V_{CC} - i_C R_C$$

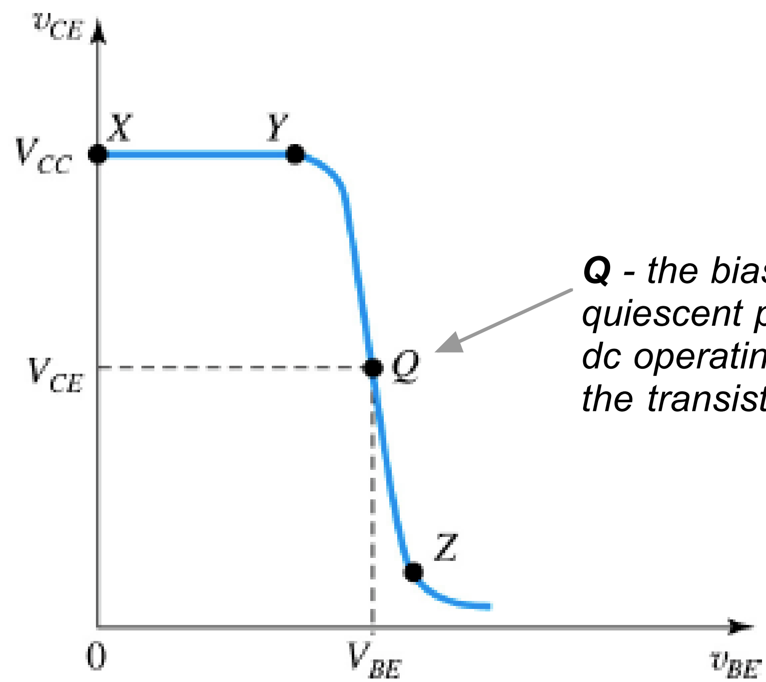
$$i_C = I_S \exp\left(v_{BE} / V_{th} \right)$$

$$v_{CE} = V_{CC} - R_C I_S \exp\left(v_{BE} / V_{th} \right)$$

$$V_{th} = \frac{kT}{q} = 25.9\,mV \quad at \quad T \approx 300K$$

Biasing the BJT in the voltage amplifier





***Q** - the bias point (the quiescent point, or the dc operating point) of the transistor*

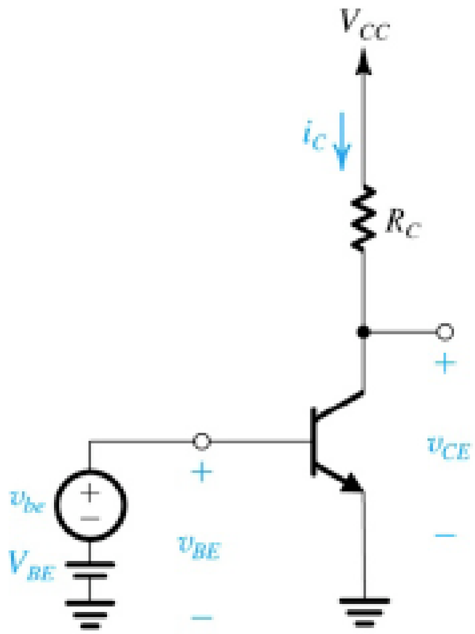
*The bias point **Q** must be located on the active-mode segment of the voltage transfer curve (VTC)*

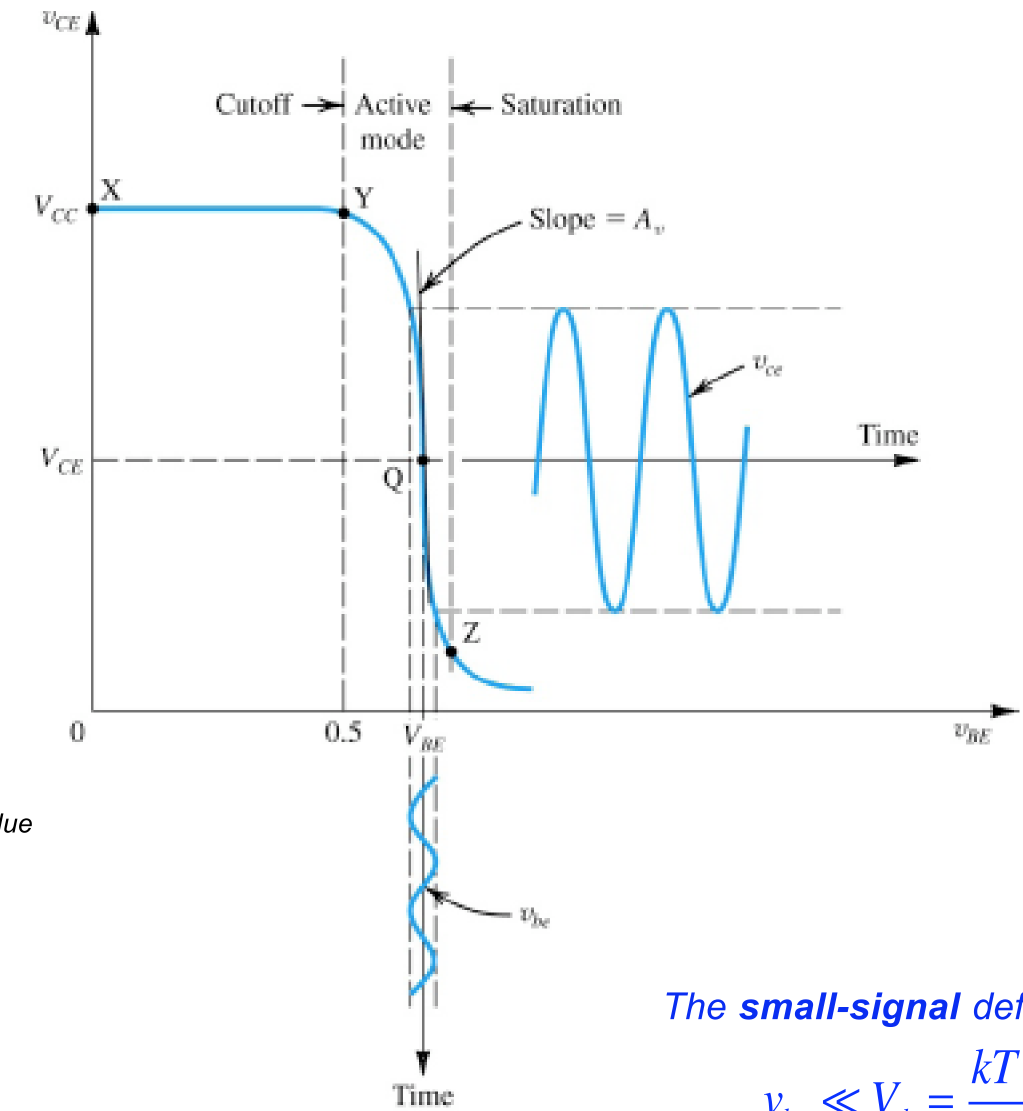
$$V_{CE} = V_{CC} - R_C I_S \exp\left(V_{BE} / V_{th} \right)$$

$$I_C = I_S \exp\left(V_{BE} / V_{th} \right)$$

$$V_{th} = \frac{kT}{q} = 25.9\,mV \quad at \quad T \approx 300K$$

Application of the small signal





the instantaneous value of Base-Emitter voltage

the ac value

$$v_{BE} = V_{BE} + v_{be}(t)$$

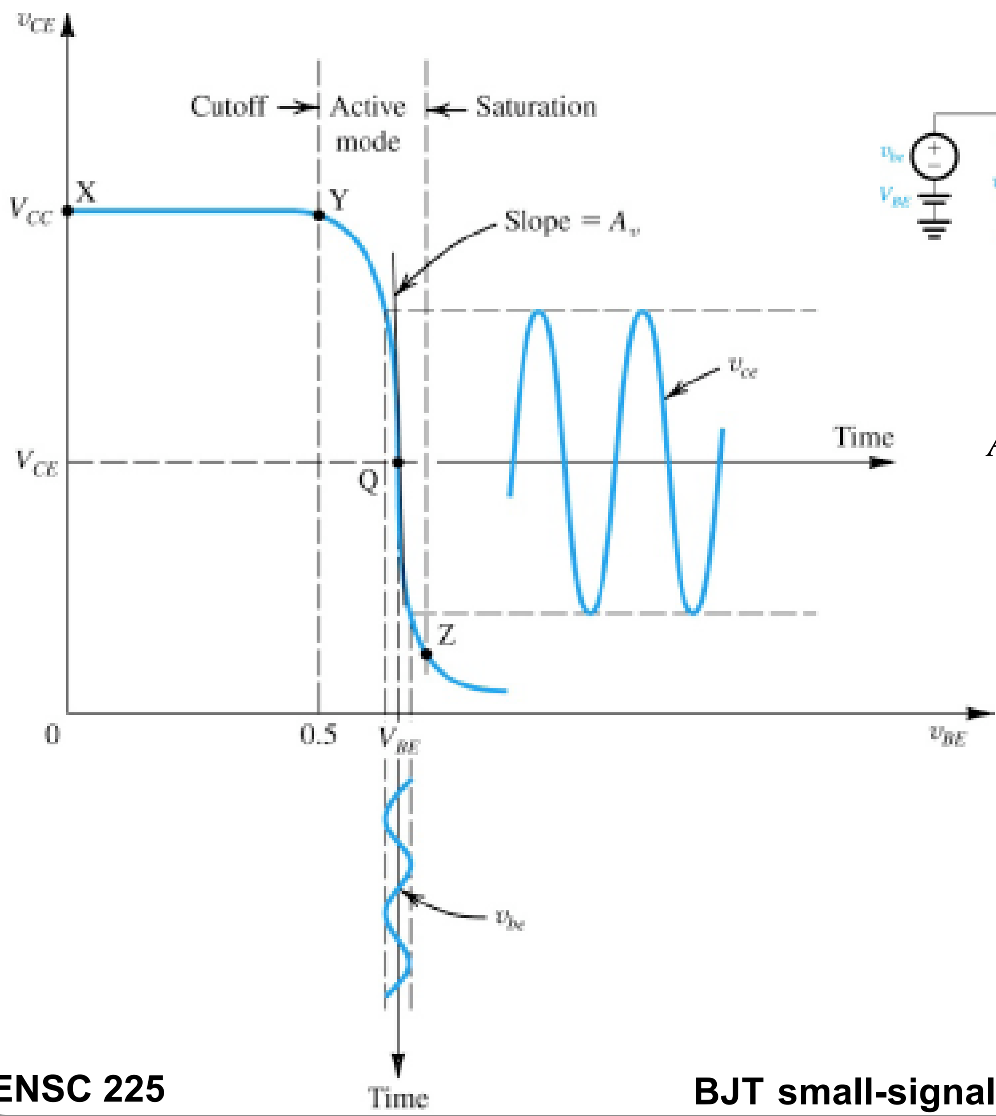
$$v_{CE} = V_{CE} + v_{ce}(t)$$

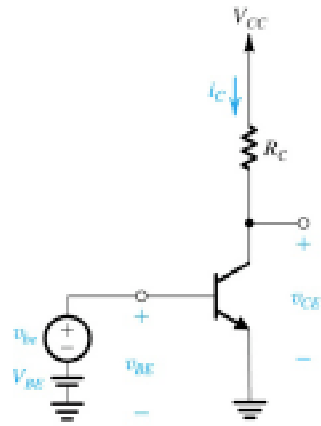
*the dc value (the **Q** point)*

*The **small-signal** definition*

$$v_{be} \ll V_{th} = \frac{kT}{q}$$

The small-signal voltage gain





$$A_v = \left.\frac{dv_{CE}}{dv_{BE}}\right|_{v_{BE}=V_{BE}}$$

$$v_{CE} = V_{CC} - R_C I_S \exp\left(v_{BE}/V_{th}\right)$$

$$A_v = \left.\frac{dv_{CE}}{dv_{BE}}\right|_{v_{BE}=V_{BE}} = \left.\frac{d\left[V_{CC} - R_C I_S \exp\left(v_{BE}/V_{th}\right)\right]}{dv_{BE}}\right|_{v_{BE}=V_{BE}} =$$

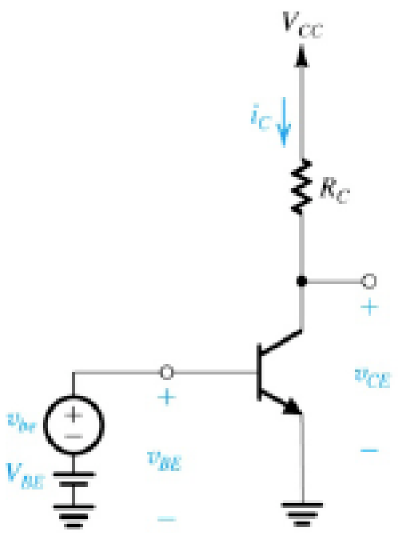
$$= -\left(I_S \exp\left(V_{BE}/V_{th}\right)\right)\frac{R_C}{V_{th}} = -\left(\frac{I_C}{V_{th}}\right)R_C$$

- The amplifier is inverting - there is a 180° phase shift between the output and the input

$$A_v = -\left(\frac{I_C}{V_{th}}\right)R_C = -\frac{V_{R_C}}{V_{th}} = -\frac{V_{CC} - V_{CE}}{V_{th}}$$

- The gain of the amplifier is proportional to the collector bias current and the load resistance

Graphical method of constructing VTC

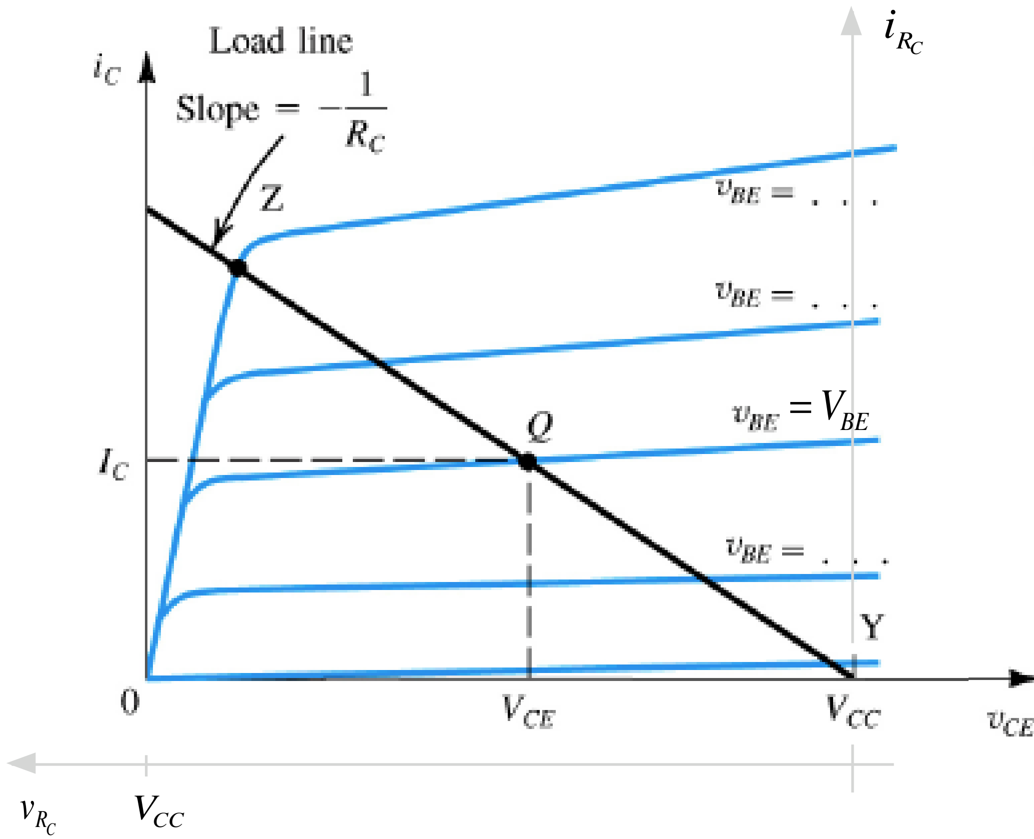


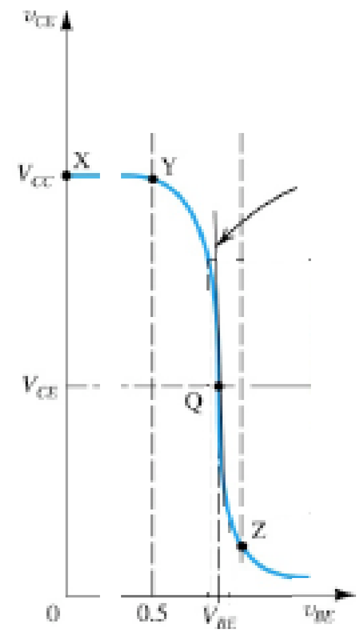
$$v_{CE} = V_{CC} - i_C R_C$$

$$i_C = \frac{1}{R_C}(V_{CC} - v_{CE})$$

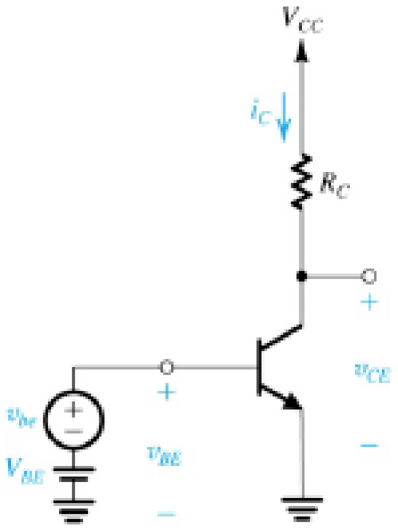
$$i_C = i_{R_C}$$

$$i_C = i_{R_C}$$



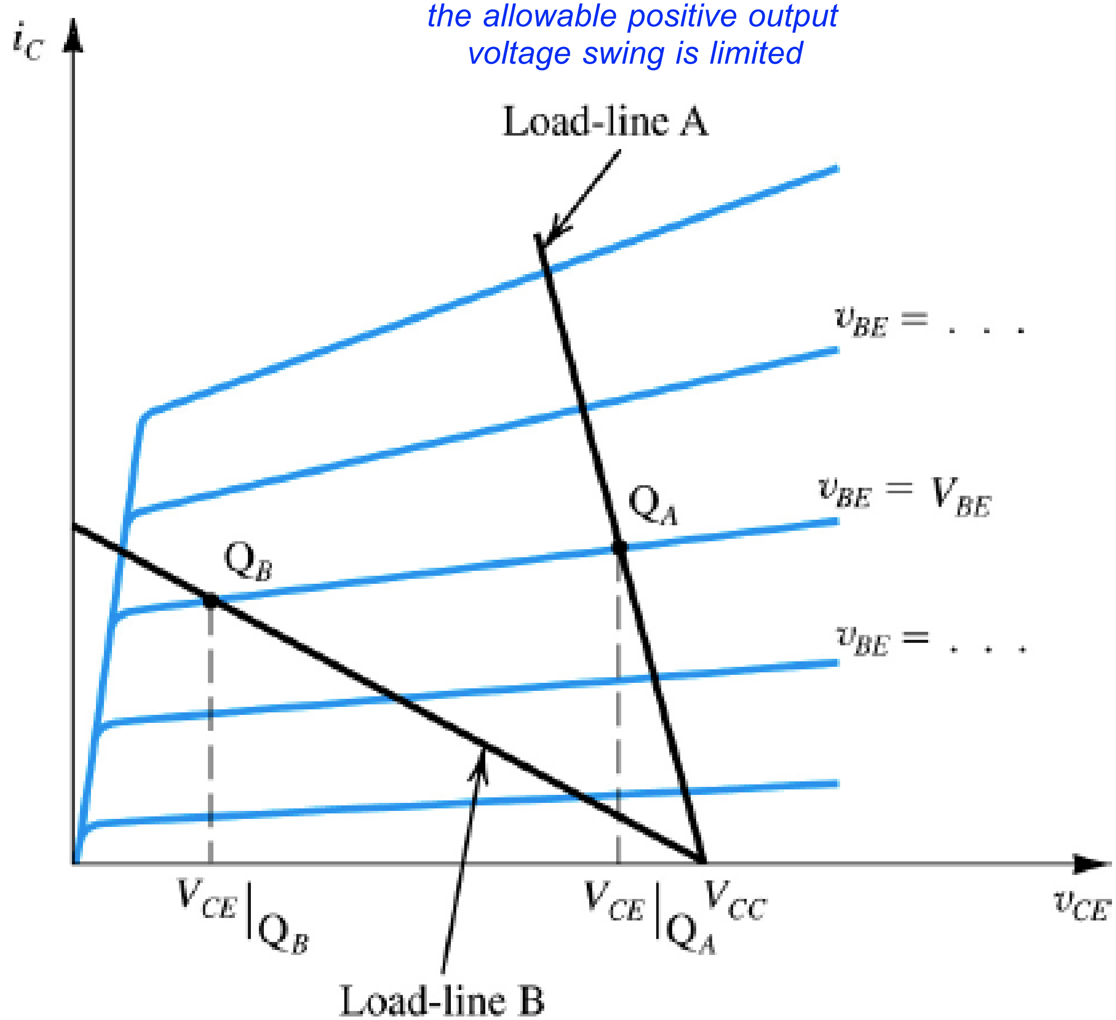


Effect of bias-point location on allowable voltage swing



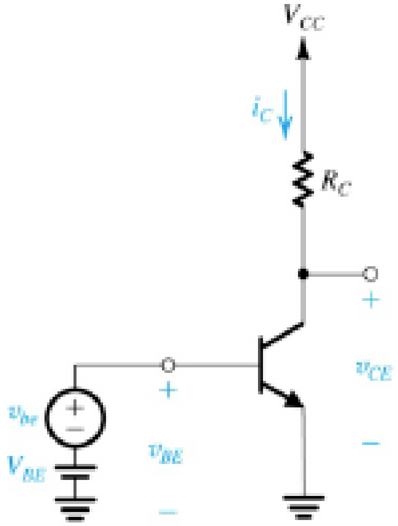
$$i_C = \frac{1}{R_C}\left(V_{CC} - v_{CE}\right)$$

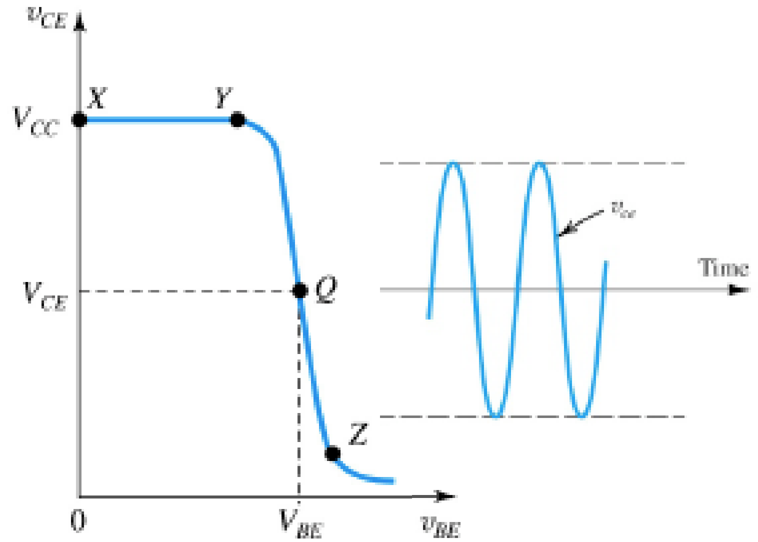
***Q_A** too close to V_{CC} - the allowable positive output voltage swing is limited*

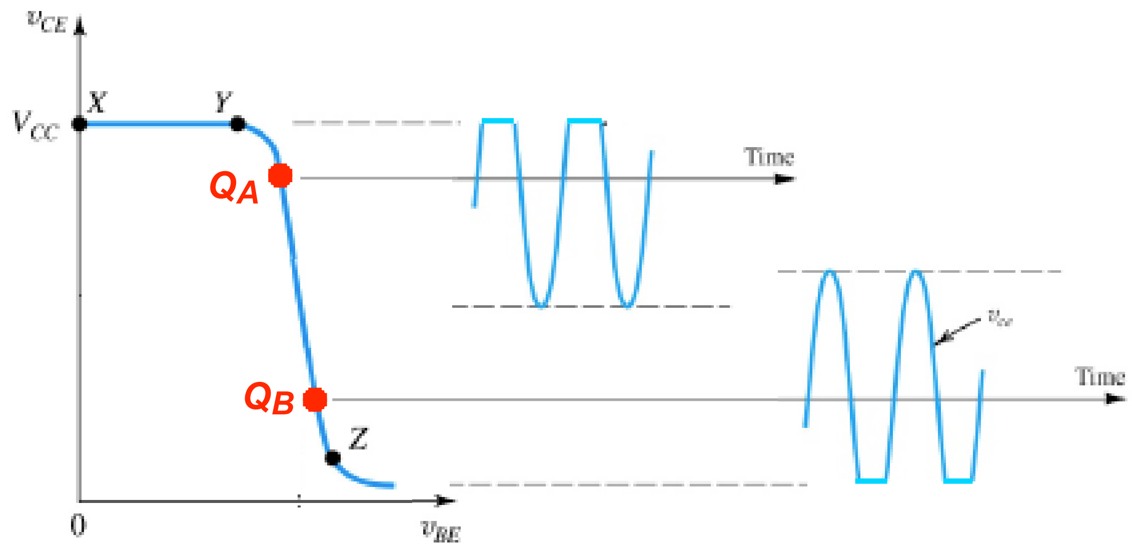


***Q_B** too close to saturation region - BJT saturation limits the allowable negative output voltage swing*

Amplifier's linearity limits





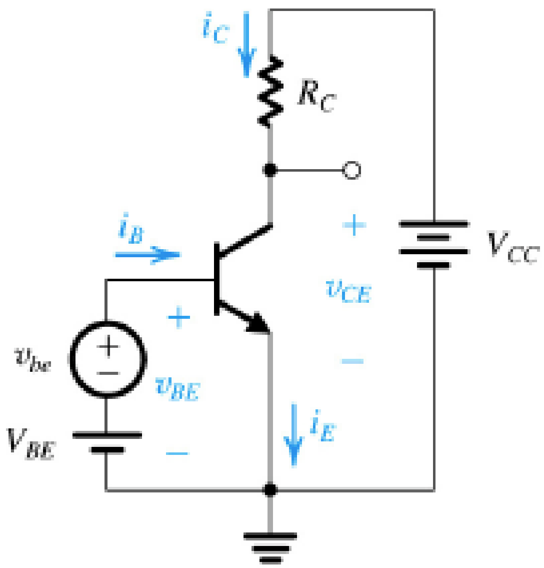


Q_A* too close to V_{CC} - the allowable positive output voltage swing is limited

Q_B* too close to saturation region - BJT saturation limits the allowable negative output voltage swing

DC and small-signal analysis

Amplifier circuit with both dc bias and signal source

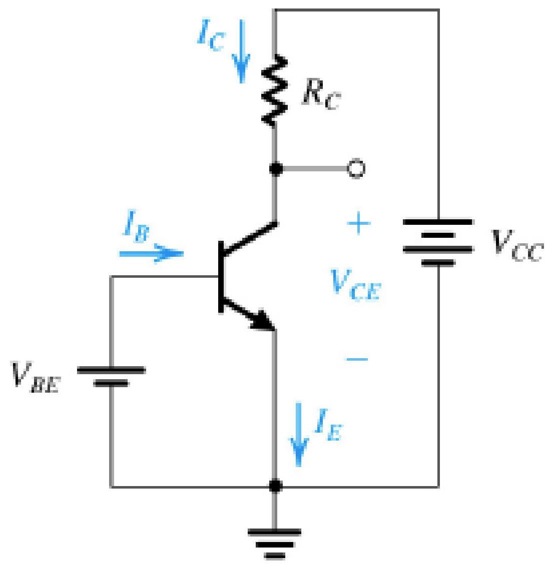


$$v_{BE} = V_{BE} + v_{be}$$

$$v_{CE} = V_{CE} + v_{ce}$$

$$i_C = I_C + i_c$$

Circuit for dc (bias) analysis



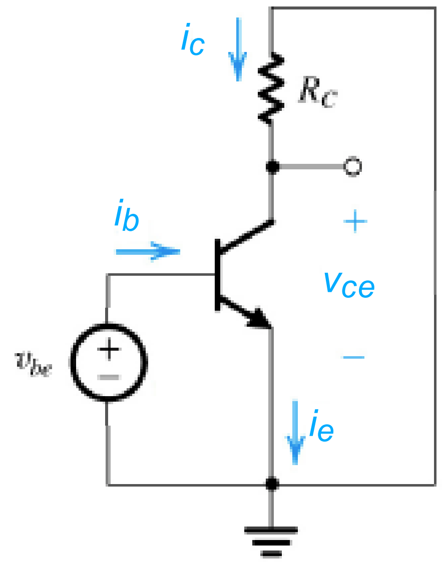
$$I_C = I_S \exp\left(V_{BE}/V_{th}\right)$$

$$I_E = \frac{I_C}{\alpha}$$

$$I_B = \frac{I_C}{\beta}$$

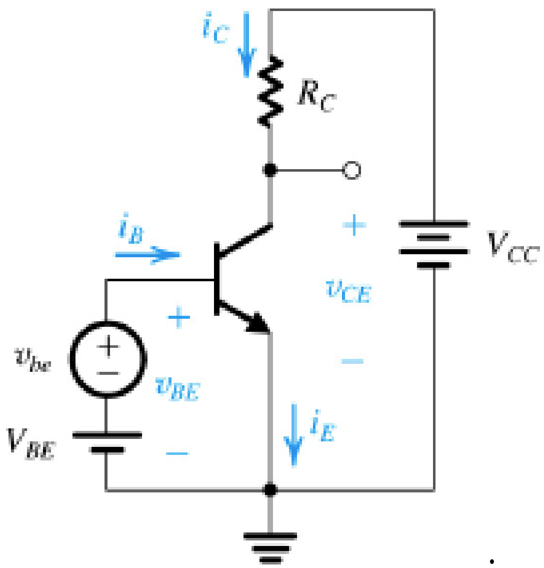
$$V_{CE} = V_{CC} - R_C I_C$$

Circuit for small-signal analysis

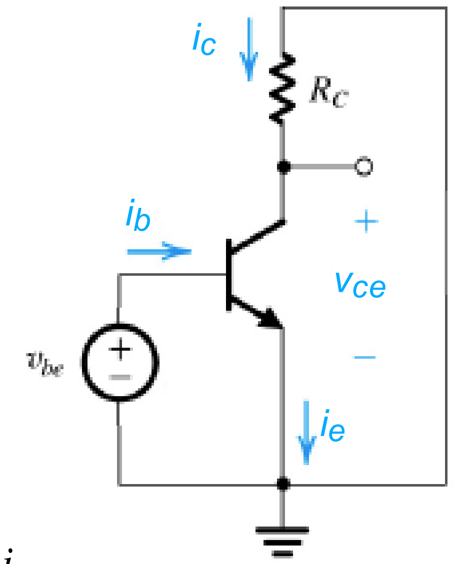


Small-signal transconductance of the BJT

Amplifier circuit with both dc bias and signal source



Circuit for small-signal analysis



$$v_{BE} = V_{BE} + v_{be}$$

$$i_C = I_S \exp\left(\frac{v_{BE}}{V_{th}}\right) = I_S \exp\left(\frac{V_{BE} + v_{be}}{V_{th}}\right) =$$

$$= I_S \exp\frac{V_{BE}}{V_{th}} \exp\frac{v_{be}}{V_{th}} = I_C \exp\frac{v_{be}}{V_{th}}$$

$$i_C = I_C \exp\frac{v_{be}}{V_{th}} \simeq I_C\left(1 + \frac{v_{be}}{V_{th}}\right) = I_C + \frac{I_C}{V_{th}} v_{be} = I_C + i_c$$

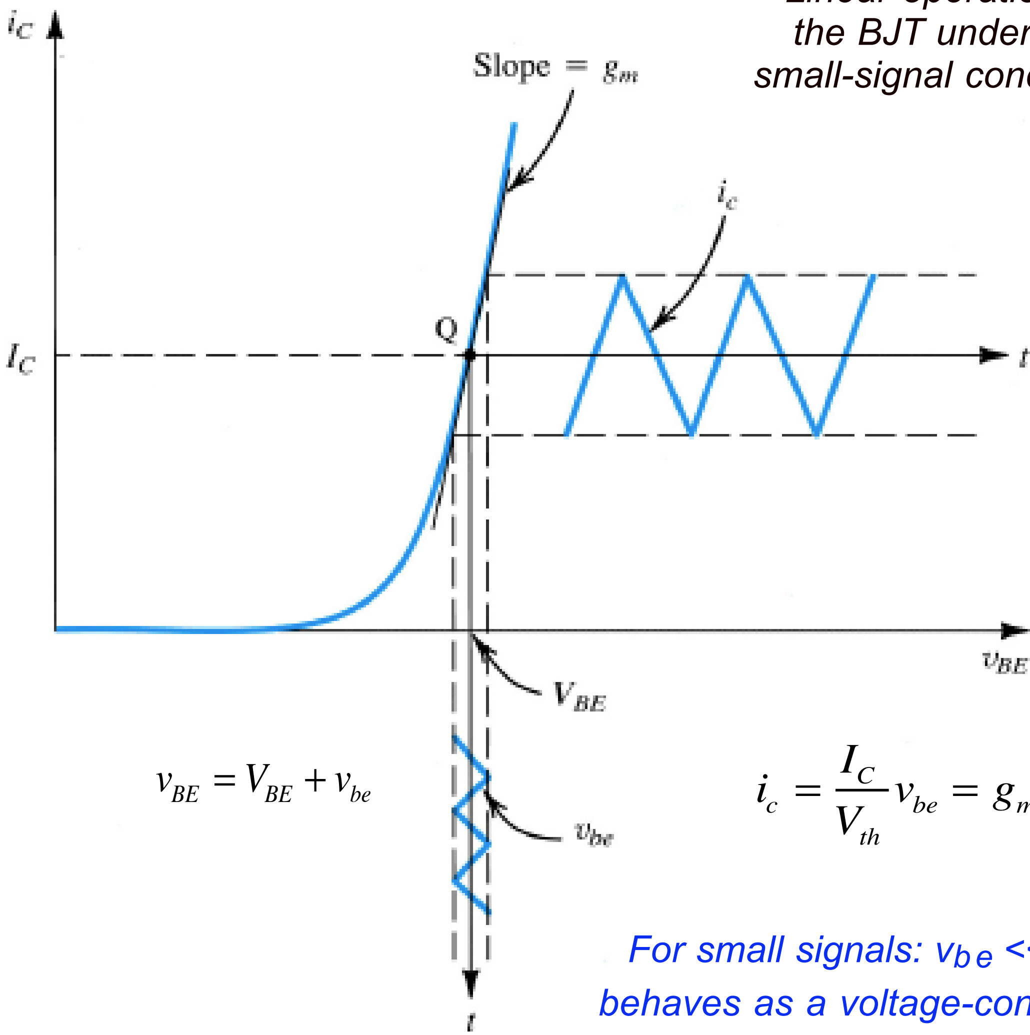
$$i_c = \frac{I_C}{V_{th}} v_{be} = g_m v_{be} \qquad g_m = \frac{I_C}{V_{th}}$$

Transconductance of the bipolar transistor

$$g_m = \frac{\delta i_C}{\delta v_{BE}}\bigg|_{i_C = I_C}$$

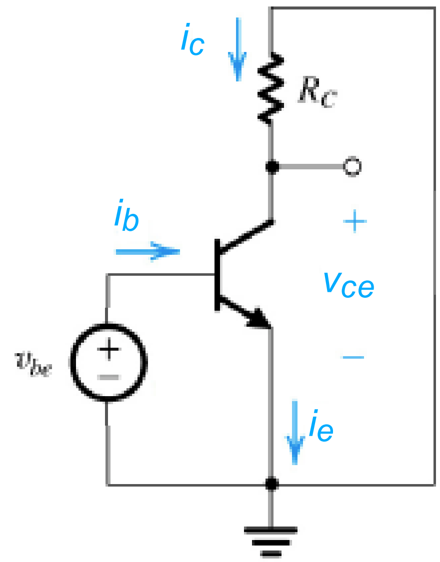
Small-signal transconductance of the BJT

Linear operation of the BJT under the small-signal conditions



$$v_{BE} = V_{BE} + v_{be}$$

Circuit for small-signal analysis



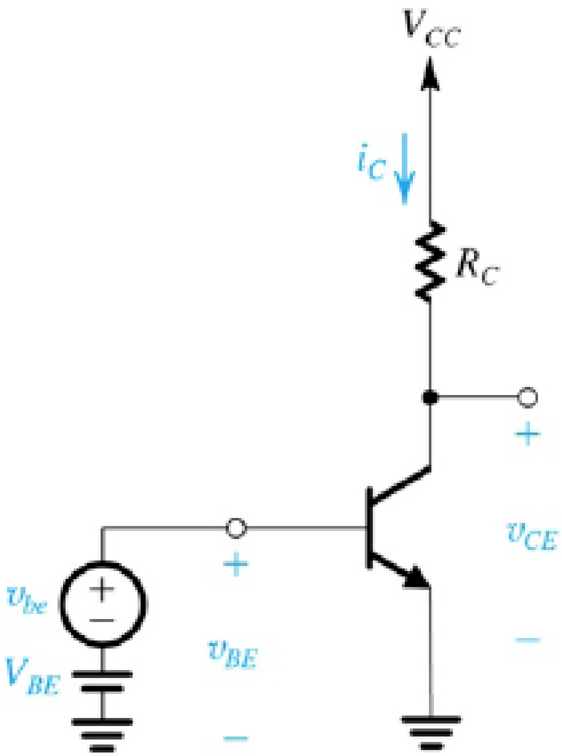
$$i_c = \frac{I_C}{V_{th}} v_{be} = g_m v_{be}$$

$$g_m = \frac{I_C}{V_{th}}$$

For small signals: $v_{be} << V_{th}$, the transistor behaves as a voltage-controlled current source

BJT transconductance : $g_m \approx 40$ mA/V at I_C = 1mA

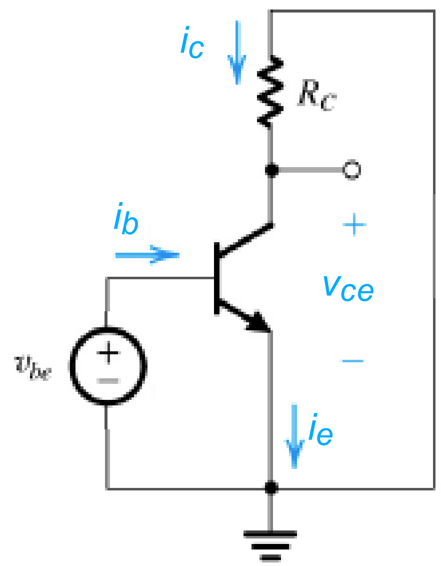
Voltage gain of the BJT amplifier



$$A_v = -\left(\frac{I_C}{V_{th}}\right)R_C = -g_m R_C$$

$$g_m = \frac{I_C}{V_{th}}$$

Circuit for small-signal analysis



$$v_{ce} = -i_c R_C = -g_m v_{be} R_C$$

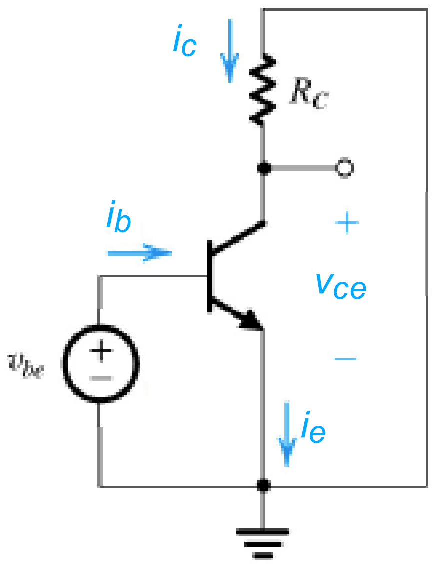
$$A_v = \frac{v_{ce}}{v_{be}} = -g_m R_C$$

$$A_v = -g_m R_C$$

Voltage gain of the BJT amplifier shown above is directly proportional to BJT transconductance g_m and the load resistance R_C .

Small-signal input resistance at the Base

Circuit for small-signal analysis



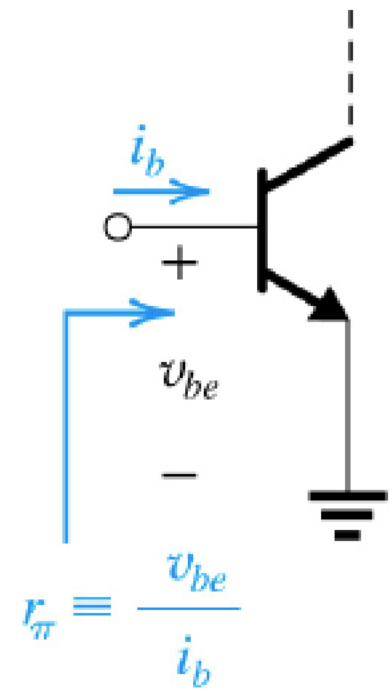
$$i_B = \frac{i_C}{\beta} = \frac{1}{\beta}\left(I_C + \frac{I_C}{V_{th}}v_{be}\right) = \frac{I_C}{\beta} + \frac{I_C}{\beta V_{th}}v_{be} = I_B + i_b$$

$$i_b = \frac{I_C}{\beta V_{th}}v_{be} = \frac{g_m}{\beta}v_{be}$$

r_π - *small-signal input resistance between Base and Emitter, looking into the Base.*

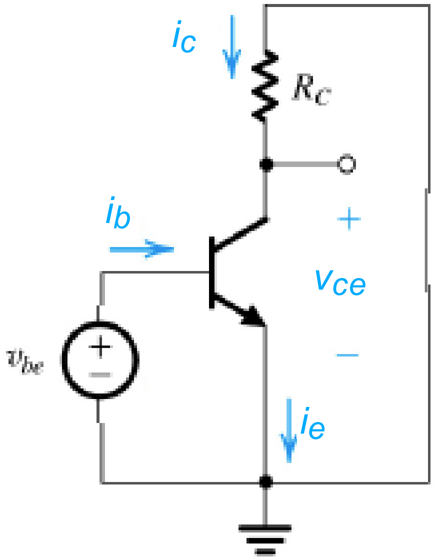
$$r_\pi \equiv \frac{v_{be}}{i_b} = \frac{\beta}{g_m}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{\beta}{I_C}V_{th} = \frac{V_{th}}{I_B}$$



Small-signal input resistance at the Emitter

Circuit for small-signal analysis



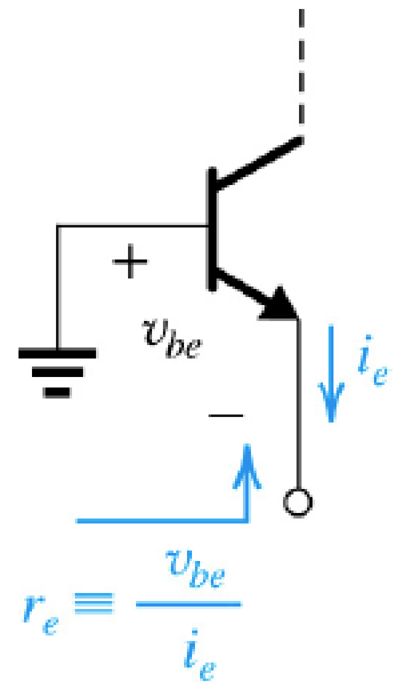
$$i_E = \frac{i_C}{\alpha} = \frac{I_C}{\alpha} + \frac{i_c}{\alpha} = I_E + i_e$$

$$i_e = \frac{i_c}{\alpha} = \frac{g_m v_{be}}{\alpha} = \frac{I_C}{\alpha V_{th}} v_{be} = \frac{I_E}{V_{th}} v_{be}$$

r_e - small-signal input resistance between Base and Emitter, looking into the Emitter.

$$r_e \equiv \frac{v_{be}}{i_e} = \frac{V_{th}}{I_E}$$

$$r_e = \frac{V_{th}}{I_E} = \frac{V_{th}\alpha}{I_C} = \frac{\alpha}{g_m} \simeq \frac{1}{g_m}$$



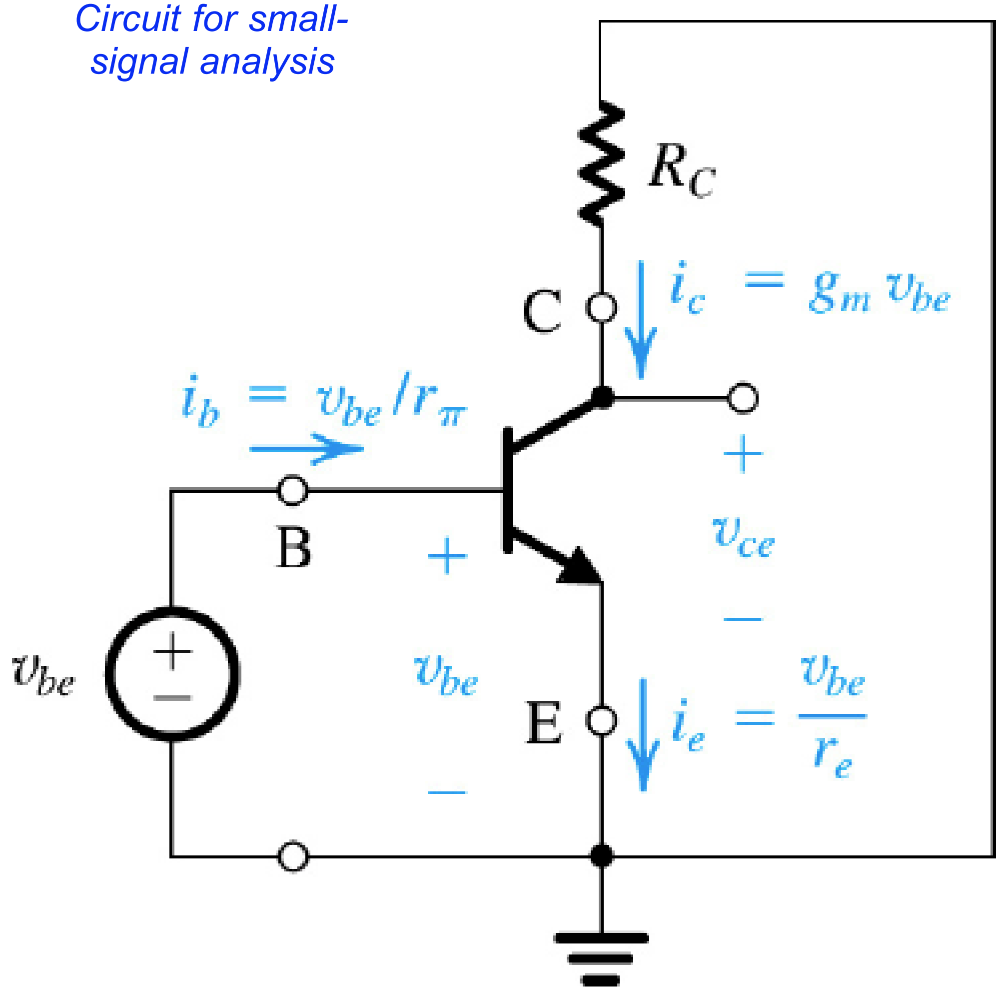
Relation between **r_π** and **r_e**

$$v_{be} = i_b r_\pi = i_e r_e$$

$$r_\pi = \frac{i_e}{i_b} r_e = \frac{i_c + i_b}{i_b} r_e = \frac{(\beta+1) i_b}{i_b} r_e = (\beta+1) r_e$$

Small-signals in the BJT amplifier

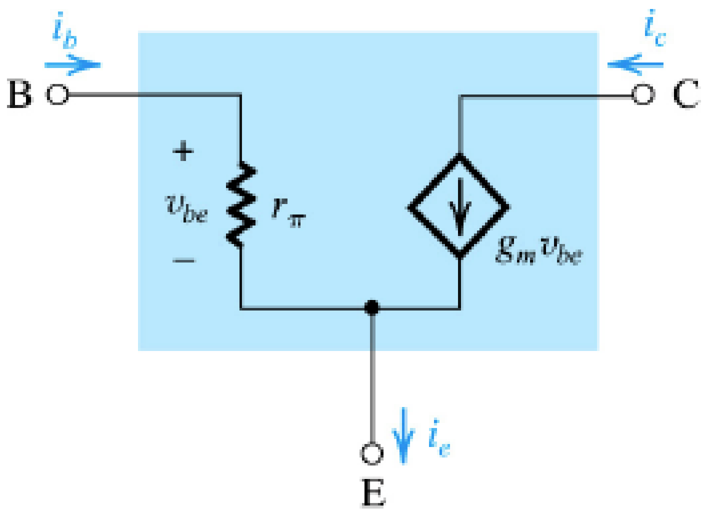
Circuit for small-signal analysis



- *In the circuit for small-signal analysis, all dc voltage sources must be replaced with short circuits, and all dc current sources must be replaced with open circuits*
- *In the small-signal analysis, the bipolar transistor must be replaced with its small-signal model*

Small-signal hybrid-π model of the BJT

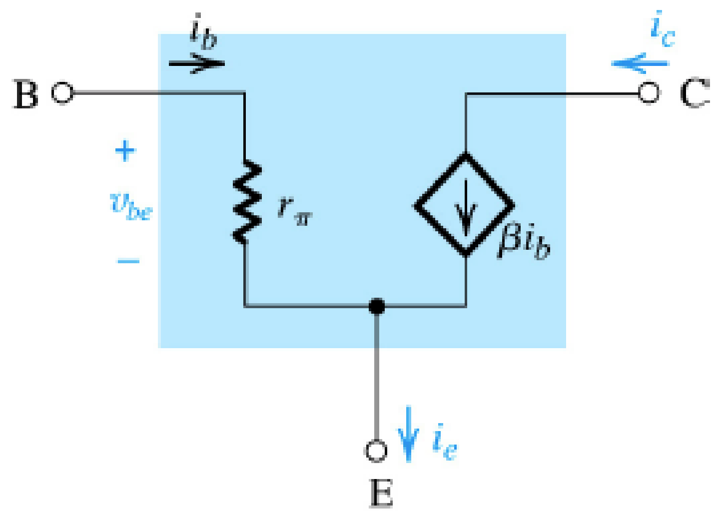
Voltage-Controlled Current Source BJT representation



$$g_m = \frac{I_C}{V_{th}}$$

$$r_\pi = \frac{\beta}{g_m}$$

Current-Controlled Current Source BJT representation

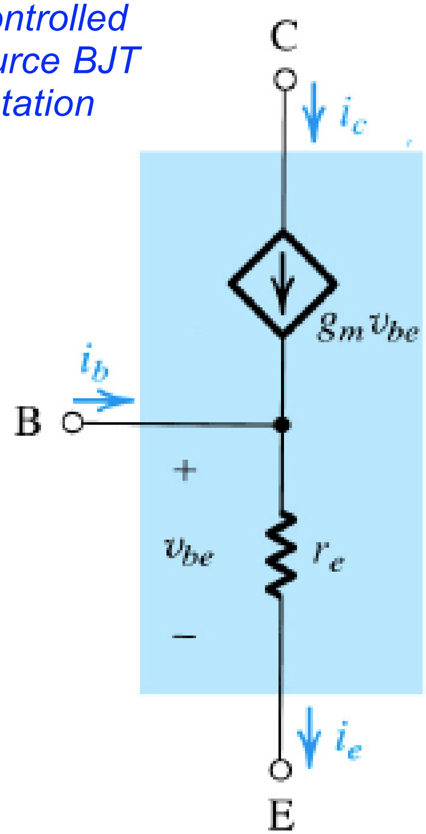


$$g_m v_{be} = g_m\left(i_b r_\pi\right) = \left(g_m r_\pi\right) i_b = \beta i_b$$

$$i_e = i_b + i_c = \frac{v_{be}}{r_\pi} + g_m v_{be} = \frac{v_{be}}{r_\pi}\left(1 + g_m r_\pi\right) = \frac{v_{be}}{r_\pi}\left(1+\beta\right) = \frac{v_{be}}{\dfrac{r_\pi}{\left(1+\beta\right)}} = \frac{v_{be}}{r_e}$$

Small-signal T model of the BJT

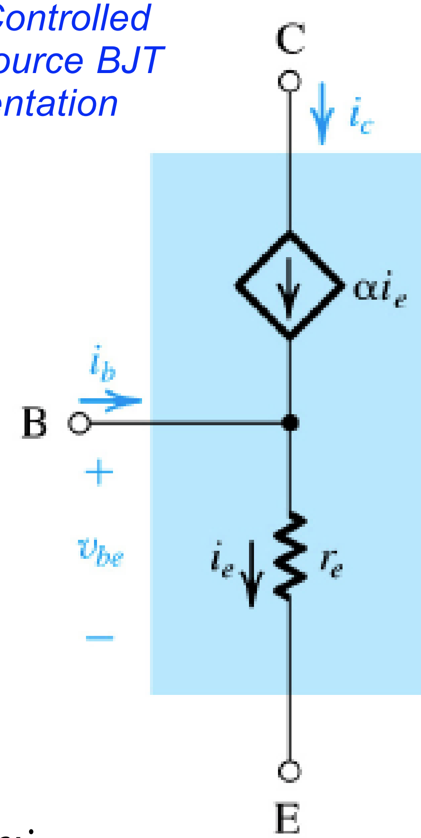
Voltage-Controlled Current Source BJT representation



$$g_m = \frac{I_C}{V_{th}}$$

$$r_e = \frac{V_{th}}{I_E} = \frac{\alpha}{g_m}$$

Current-Controlled Current Source BJT representation



$$g_m v_{be} = g_m \left(i_e r_e \right) = \left(g_m r_e \right) i_e = \alpha i_e$$

$$i_b = i_e - i_c = \frac{v_{be}}{r_e} - g_m v_{be} = \frac{v_{be}}{r_e}\left(1 - g_m r_e\right) = \frac{v_{be}}{r_e}\left(1-\alpha\right) = \frac{v_{be}}{r_e}\left(1 - \frac{\beta}{\beta+1}\right) = \frac{v_{be}}{r_e}\left(\frac{\beta+1-\beta}{\beta+1}\right) = \frac{v_{be}}{\left(1+\beta\right) r_e} = \frac{v_{be}}{r_\pi}$$

Small-signal analysis procedure

1. Eliminate the signal sources and determine the dc operating point of the BJT (I_C)

2. Eliminate the dc sources by replacing each dc voltage source with a short circuit and each dc current source with an open circuit

3. Replace the BJT with one of its small-signal models

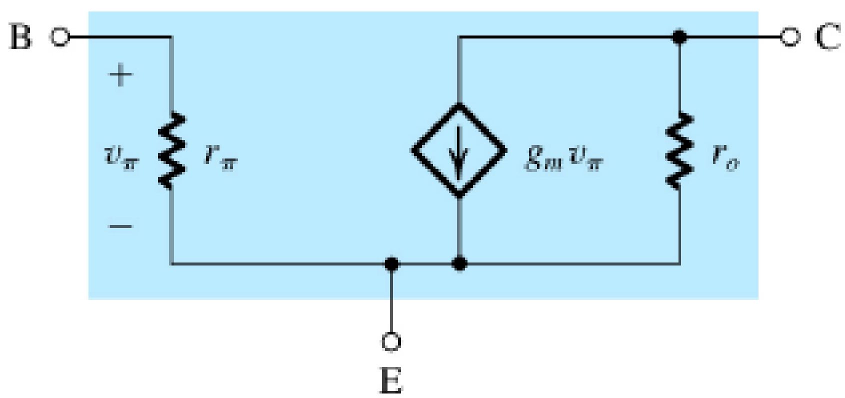
4. Calculate the values of small-signal model parameters

$$g_m = \frac{I_C}{V_{th}} \qquad r_\pi = \frac{\beta}{g_m} \qquad r_e = \frac{V_{th}}{I_E} = \frac{\alpha}{g_m}$$

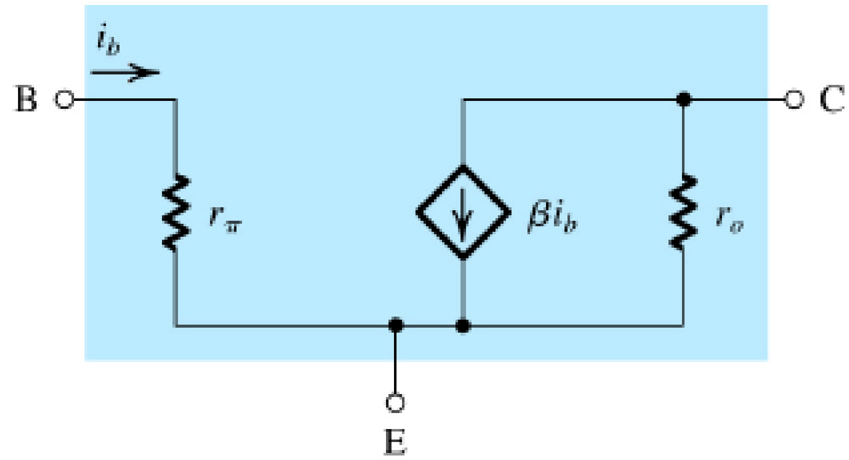
5. Analyze the resulting small-signal circuit to determine the required quantities (voltage gain, input and output resistances, etc.)

Small-signal hybrid-π model of the BJT including the output resistance

Voltage-Controlled Current Source BJT representation



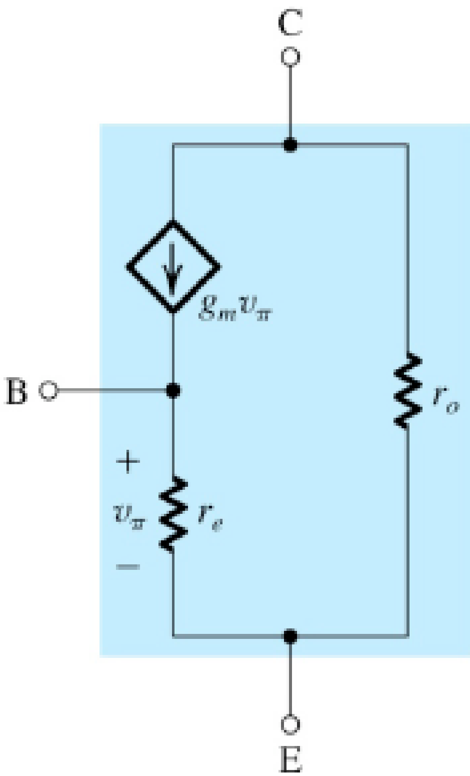
Current-Controlled Current Source BJT representation



$$g_m = \frac{I_C}{V_{th}} \qquad r_\pi = \frac{\beta}{g_m} \qquad r_o = \frac{V_A}{I_C}$$

Small-signal T model of the BJT including the output resistance

Voltage-Controlled Current Source BJT representation



$$r_o = \frac{V_A}{I_C}$$

$$g_m = \frac{I_C}{V_{th}}$$

$$r_e = \frac{V_{th}}{I_E} = \frac{\alpha}{g_m}$$

Current-Controlled Current Source BJT representation