Applied Calculus Workshop (ACW) as a Complex Social System

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Project Presentation

Outline

ACW

- Introduction
- Defining Efficiency
- Questions

Models without & with Vital Dynamics

- Assumptions
- Conceptualization
- ODE
  - Classification
  - Steady State
  - Analysis Part I, II
- Data Collection
- Cellular Automaton

Closing Remarks

- Further Work
- History & Examples
- References

Introduction to the Workshop

Developed in the 1990s in the Department of Mathematics at SFU:

- drop-in based
- serves a variety of courses and student needs
- provides TA-ships for graduate students

Introduction to the Workshop

Similar centres are offered across the country:

- Mathematics Learning Centre at **Capilano** University
- Mathematics and Statistics Learning Centre at **Dalhousie** University
- Learning Centre at University of **Guelph**
- Math and Statistics Learning Centre at University of **Toronto** Scarborough

Introduction to the Workshop

Some facts:

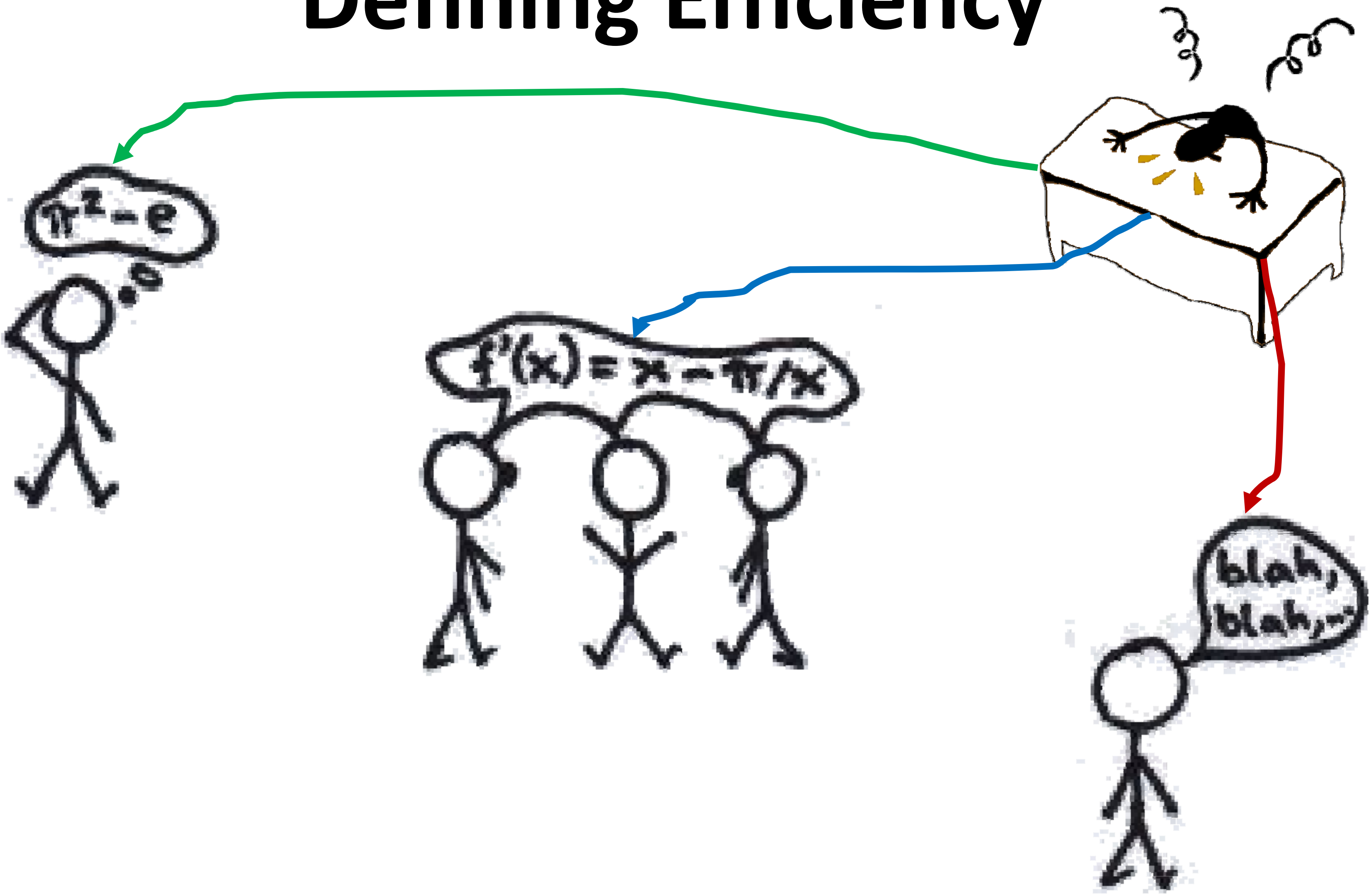
- 5 workshops servicing ~4000 students/fall
- ACW services ~1200 students/fall,spring
- accommodates 40-50 students
- 15-18 TAs
- open M-F with 6-10 h/day

Introduction to the Workshop

Primary goals of the workshop are twofold:

- provide economical help to the students
- students need to work well and receive good service

Defining Efficiency



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Defining Efficiency

Efficiency for a workshop is defined as the degree to which studious behaviour among students is observed under the influence of teaching assistants.

→complex social system

Defining Efficiency

The questions driving the project:

- Can we model the dynamics among students and teaching assistants and reproduce observed behaviours?
- Can this model be used to study the effect of the number of TAs present in terms of an increase/decrease in studiousness?
- Can this model be used to study how the type of TA present influences the learning environment?
- How efficient can a workshop be, i.e. under which conditions does the climate for learning flourish?

Model without Vital Dynamics

Assumptions:

- Student classification: working, attracted to work or socialize, socializing.
- Each student group is large enough so its size can be considered a continuous function of time.
- No student can leave or enter the system.

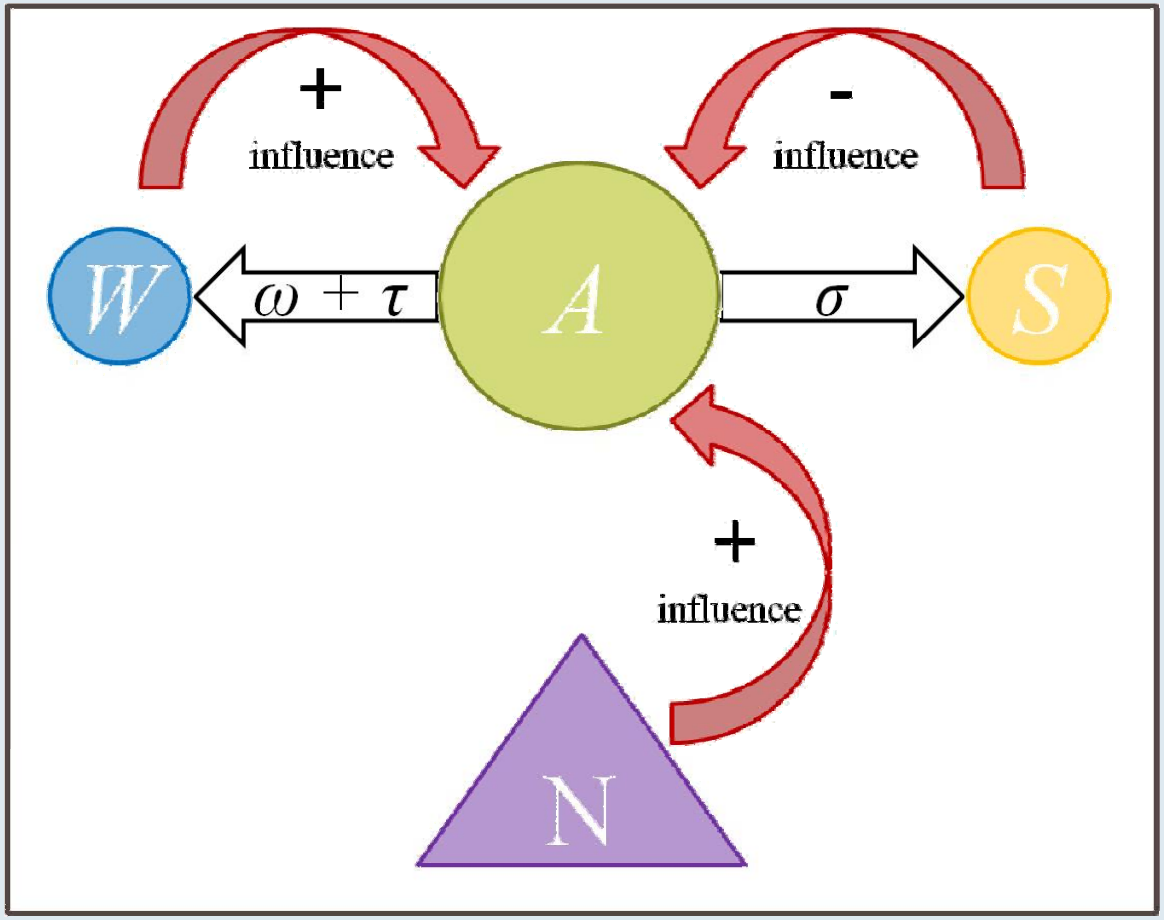
Model without Vital Dynamics

Assumptions cont'd:

- Attracted students can be influenced by working students to work.
- Attracted students can be influenced by teaching assistant(s) to work.
- Attracted students can be influenced by socializing students to socialize.

Model without Vital Dynamics

Conceptualization of Influences and Transitions:



Model without Vital Dynamics

Variables & Parameters: Let

- ***t*** be time
- ***W(t), A(t), S(t)*** be the # of working, attracted, socializing students
- ***σ*** be the rate constant for transfer from *A* to *S* due to the influence of *S*
- ***ω*** be the rate constant for transfer from *A* to *W* due to the influence of *W*
- ***τ*** be the rate constant for transfer from *A* to *W* due to the influence of teaching assistants

Model without Vital Dynamics

ODEs:

$$\frac{dW}{dt} = \omega WA + \tau(N)A$$

$$\frac{dA}{dt} = -\omega WA - \tau(N)A - \sigma SA$$

$$\frac{dS}{dt} = \sigma SA$$

with initial values W_0, A_0, and S_0.

Model without Vital Dynamics

Classification:

- non-linear
- aggregate
- deterministic
- dynamic
- continuous
- qualitative/quantitative

Model without Vital Dynamics

Steady State:

- The steady state is reached when $A=0$.
- This is not an interesting result at all. But …
- What values need to be chosen for the parameters ω and σ?
- What information do the values of the parameter τ lead to?

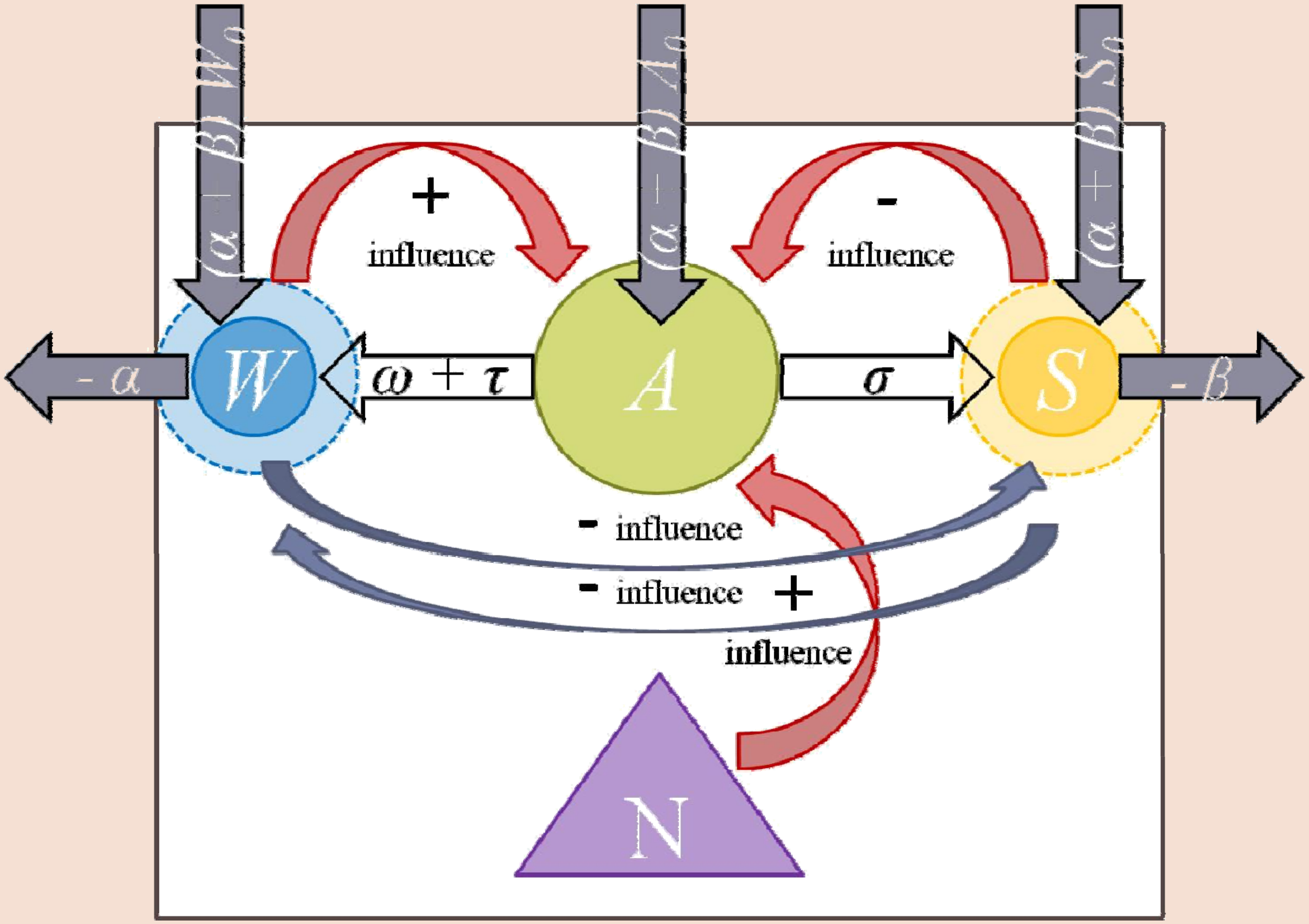
Model with Vital Dynamics

Additional assumptions:

- If there is too much working, then socializing students leave.
- If there is too much socializing, then working students leave.
- Students that have left will be replaced with students entering in the same proportions as the initial values W_0, A_0, and S_0.

Model with Vital Dynamics

Conceptualization of Influences and Transitions:



Model with Vital Dynamics

Additional Variables & Parameters: Let

- $\boldsymbol{\alpha}$ be the rate constant for transfer out of W due to the influence of S
- $\boldsymbol{\beta}$ be the rate constant for transfer out of S due to the influence of W

Model with Vital Dynamics

ODEs:

$$\frac{dW}{dt} = \omega WA + \tau(N)A - \alpha SW + \alpha SWW_0 + \beta SWW_0$$

$$\frac{dA}{dt} = -\omega WA - \tau(N)A - \sigma SA + \alpha SWA_0 + \beta SWA_0$$

$$\frac{dS}{dt} = \sigma SA + \alpha SWS_0 - \beta SW + \beta SWS_0$$

with initial values W_0, A_0, and S_0.

Model with Vital Dynamics

Classification:

- non-linear
- aggregate
- deterministic
- dynamic
- continuous
- qualitative/quantitative

Data Collection

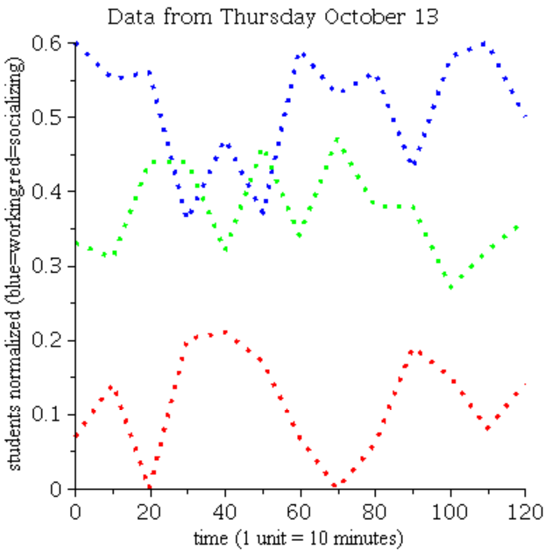
Counting constraints:

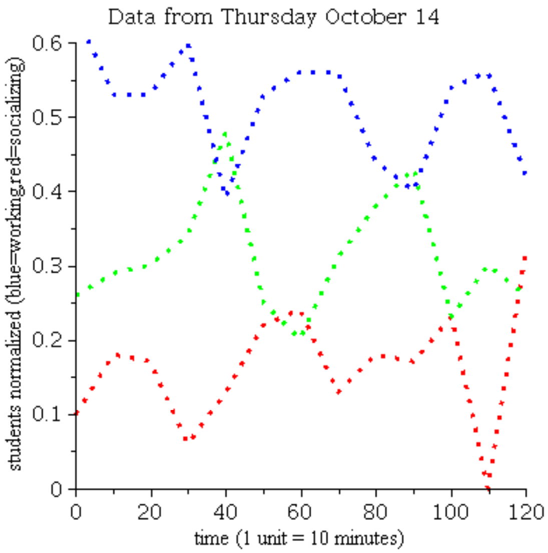
- 3 separate days, 2 hour duration, 10 minute time intervals
- Working: individuals or small groups that were clearly working
- Socializing: individuals that were engaged non-mathematically with other students, or inappropriately using electronic devices
- Total number of students

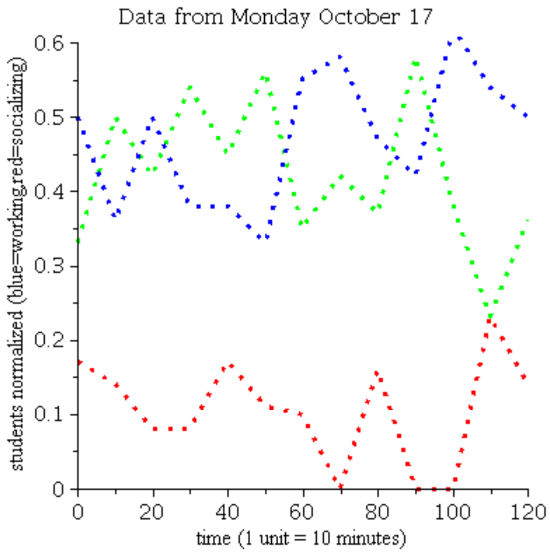
Data Collection

Graphs of data normalized over total students:

working attracted socializing





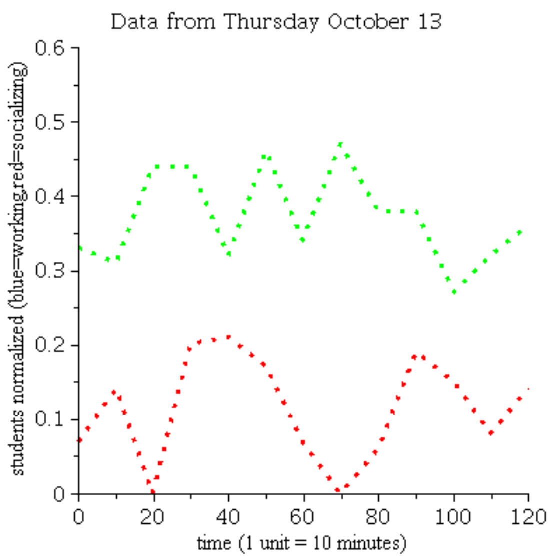


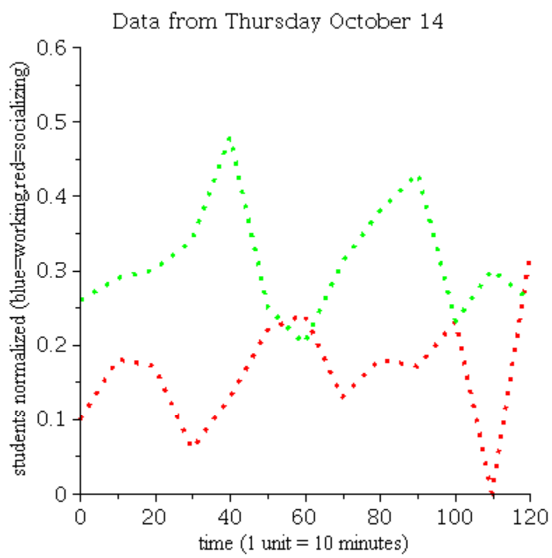
Data Collection

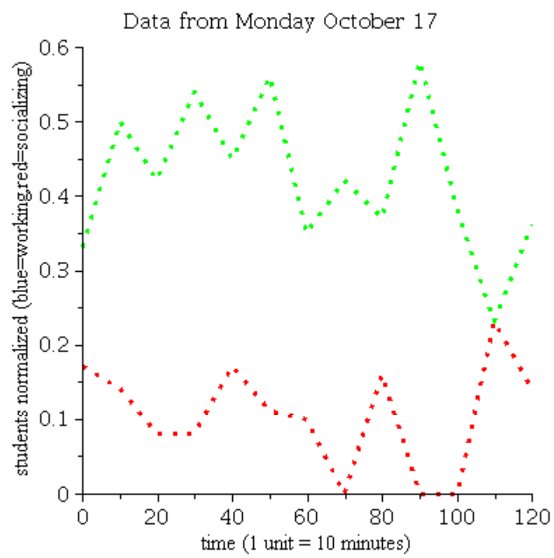
Graphs of data normalized over total students:

working

socializing







Data Collection

<u>Data Analysis:</u>

- # of working students is ≈2-3 times that of the socializing students → initial conditions
- high # of exits & entrances near the half hour
- changes in *W* and *S* occur ~ every 20 minutes
- changes in *W*, *S* are periodic
- changes in *W* and *S* are out-of-phase

Data Collection

Observation:

- excellent TAs → 5-6 students
- good TAs → 3-4 students
- average TAs → 1 student

$W_\infty = kS_\infty$ SFU

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Model without Vital Dynamics

Analysis of parameter values ω and σ:

W_0	S_0		ω	σ	τ	$W_\infty = kS_\infty$
0.30	0.15	$\omega=\sigma$	0.50	0.50	0	2
			0.12	0.12	0	2
		$\omega=2\sigma$	0.50	0.25	0	3
			0.14	0.07	0	3
		$\omega=3\sigma$	0.60	0.20	0	4
			0.15	0.05	0	4
0.36	0.12	$\omega=\sigma$	0.50	0.50	0	3
			0.12	0.12	0	3
		$\omega=2\sigma$	0.50	0.25	0	4.5
			0.14	0.07	0	4.5
		$\omega=3\sigma$	0.60	0.20	0	5.25
			0.15	0.05	0	5.25

Model without Vital Dynamics

Analysis of parameter values ω and σ and graphs:

$W_0 = 0.30$, $S_0 = 0.15$ with $\tau = 0$
$\omega = \sigma = 0.12$

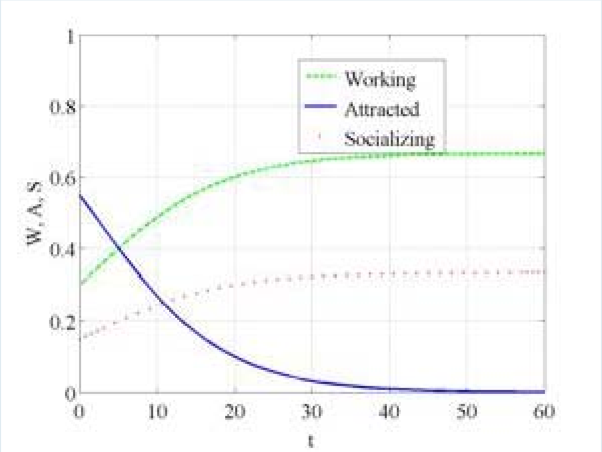

Figure 15

$W_0 = 0.36$, $S_0 = 0.12$ with $\tau = 0$
$\omega = 0.50$, $\sigma = 0.25$

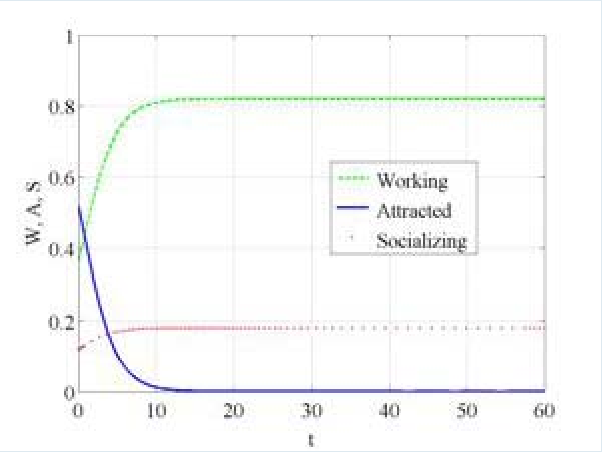

Figure 22

Model without Vital Dynamics

Analysis of parameter value τ:

W_0	S_0		ω	σ	τ	$W_\infty = kS_\infty$	τ	$W_\infty = kS_\infty$	τ	$W_\infty = kS_\infty$
0.30	0.15	$\omega=\sigma$	0.50	0.50	0	2	0.25	3	0.5	4
			0.12	0.12	0	2	0.06	3	0.12	4
		$\omega=2\sigma$	0.50	0.25	0	3	0.2	4	0.6	5
			0.14	0.07	0	3	0.05	4	0.2	5
0.36	0.12	$\omega=\sigma$	0.50	0.50	0	3	0.15	4	0.58	5
			0.12	0.12	0	3	0.04	4	0.13	5
		$\omega=2\sigma$	0.50	0.25	0	4.5	0.15	5	0.22	5.5
			0.14	0.07	0	4.5	0.04	5	0.07	5.5

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Model without Vital Dynamics

<u>Analysis of parameter value τ and graphs:</u>

$W_0 = 0.30$, $S_0 = 0.15$ with $\tau = 0.06$
$\omega = \sigma = 0.12$

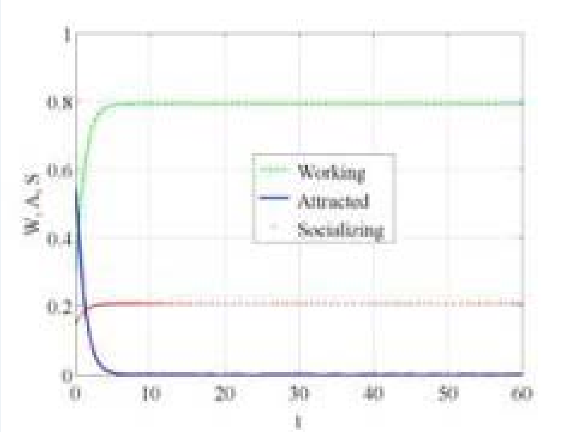


Figure 27

$W_0 = 0.36$, $S_0 = 0.12$ with $\tau = 0.15$
$\omega = 0.50$, $\sigma = 0.25$

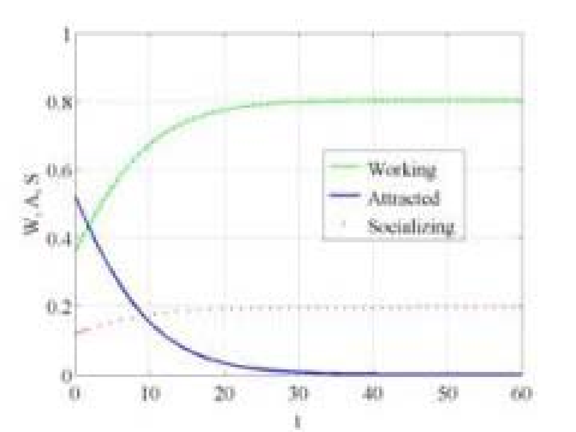


Figure 36

Model with Vital Dynamics

Analysis of param[illegible]ter values [illegible] and β and graphs:

For every value of α there see[illegible]s to be [illegible]e for β, [illegible]is value[illegible], such that for $\beta \leq \beta_c$ the socializing group wins, and fo[illegible]$\beta \geq \beta_c$ th[illegible]king gro[illegible]ns. A is a[illegible]ays depleted!

$W_0 = 0.30$, $S_0 = 0.15$, $\omega = \sigma =$ [illegible]12, $\tau = 0.06$ [illegible] $\beta_c \approx 0.0310$[illegible]4

$\beta = 0.01$

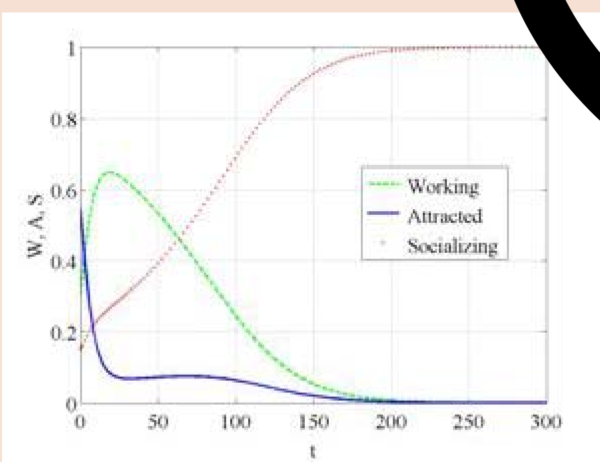


Figure 5a

$\beta = 0.$[illegible]

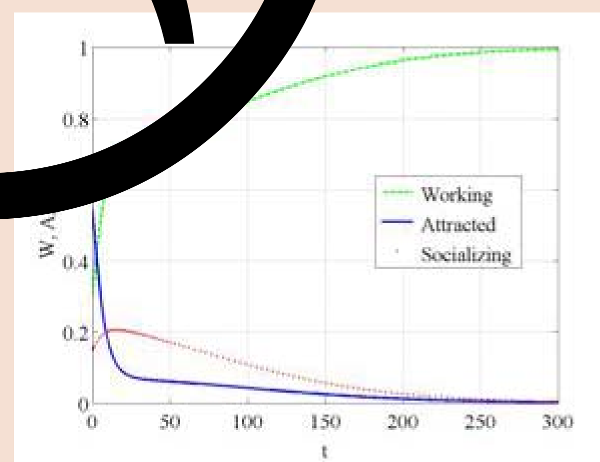
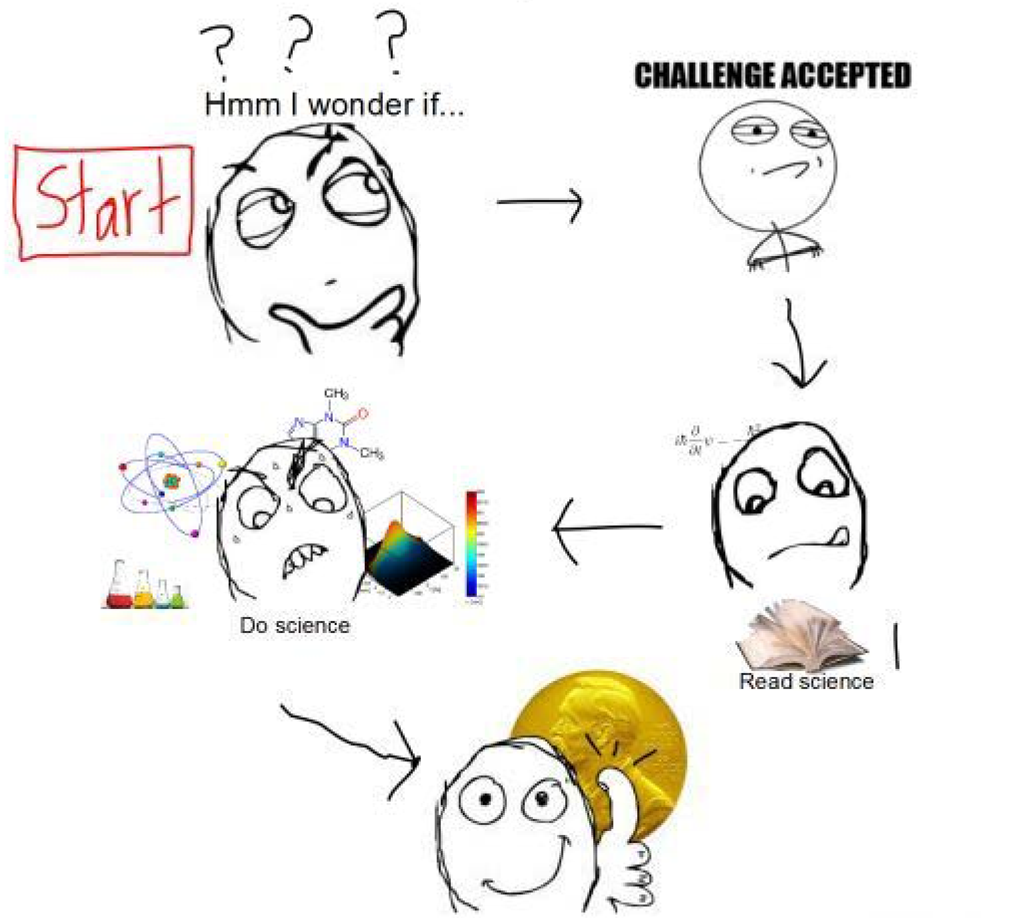
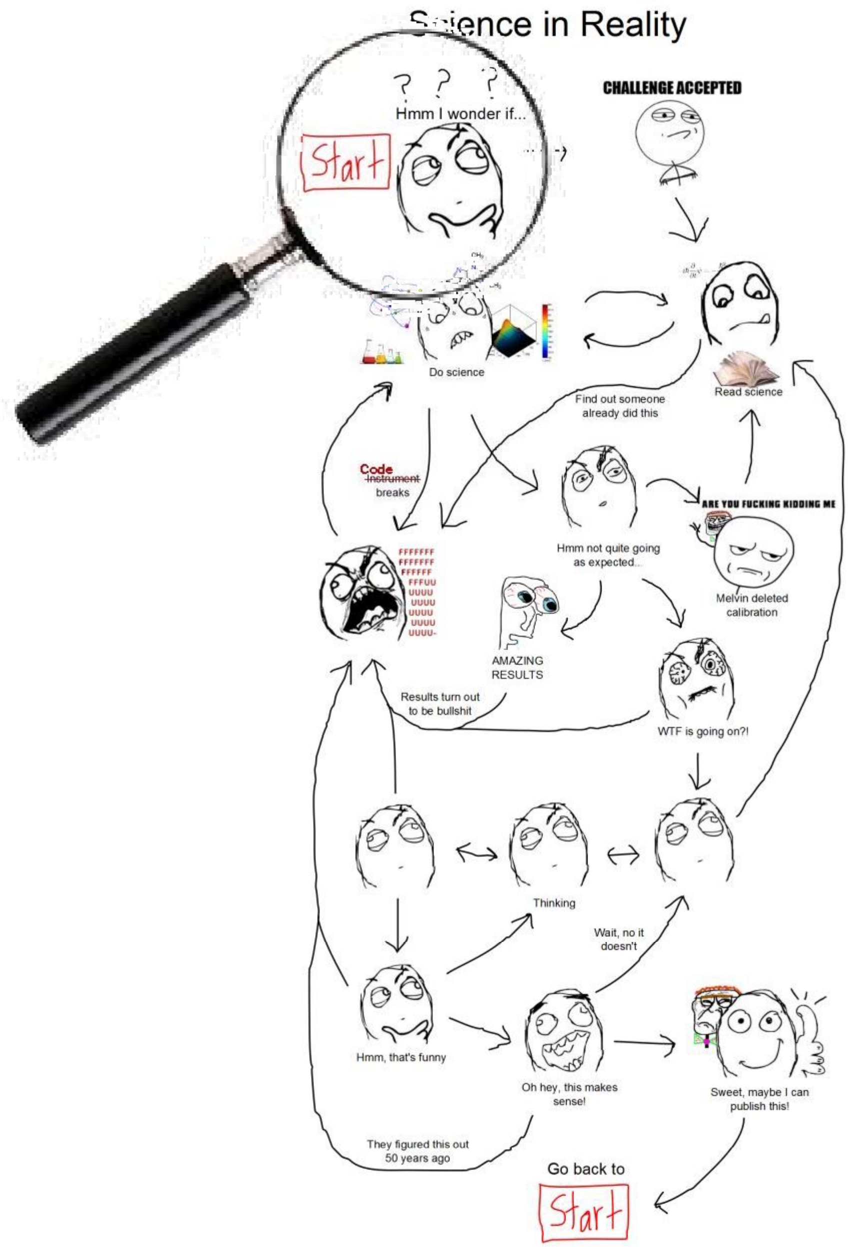


Figure 6a

Public Perception of Science

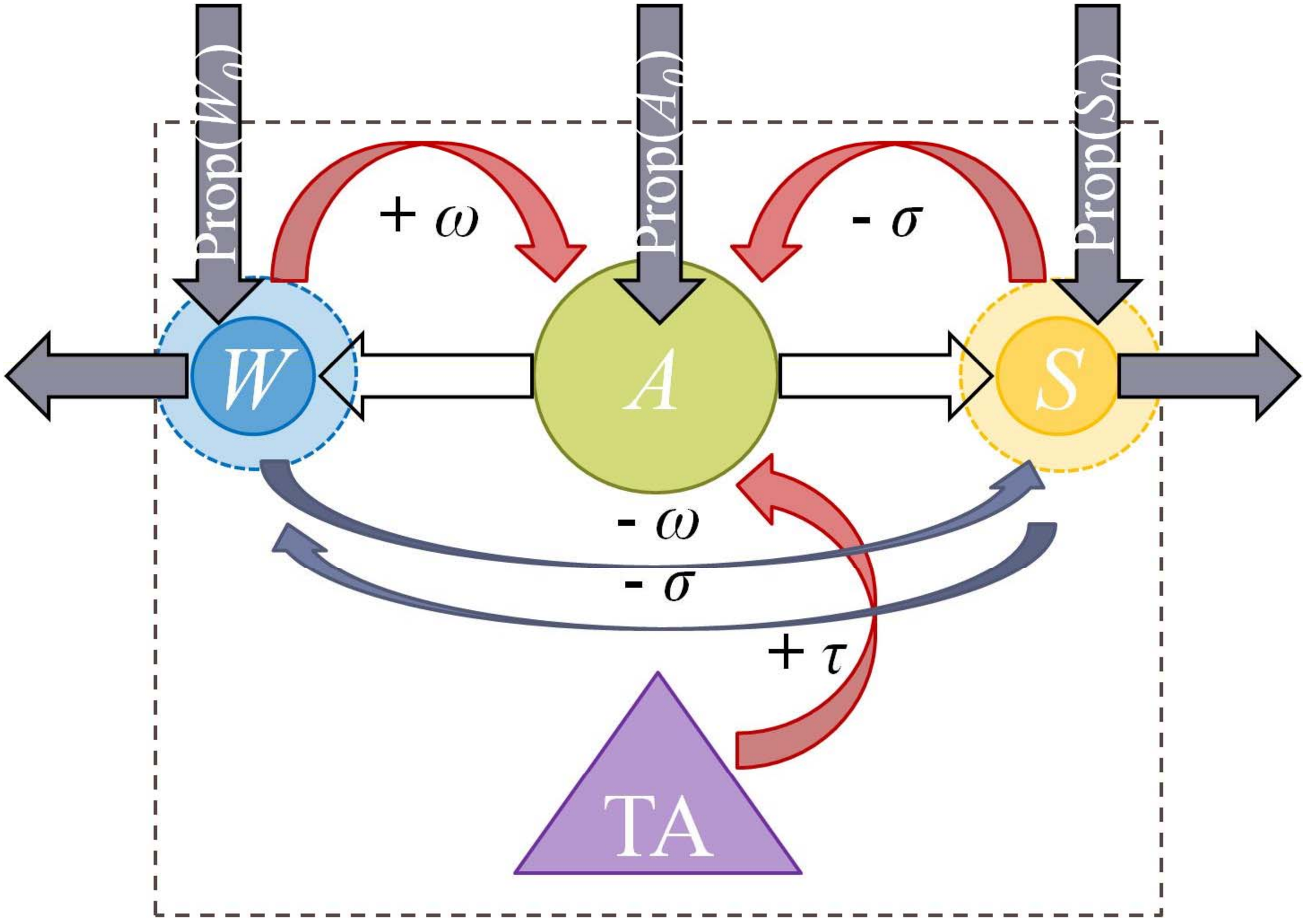


http://boredinpostconflict.blogspot.com/2011/06/public-perceptions-of-science-vs.html


Science in Reality
Hmm I wonder if...
Start
CHALLENGE ACCEPTED
Do science
Read science
Find out someone already did this
Code ~~Instrument~~ breaks
Hmm not quite going as expected...
ARE YOU FUCKING KIDDING ME
Melvin deleted calibration
FFFFFF FFFFFF FFFFF FFFUU UUUU UUUU UUUU UUUU UUUU-
AMAZING RESULTS
Results turn out to be bullshit
WTF is going on?!
Thinking
Wait, no it doesn't
Hmm, that's funny
Oh hey, this makes sense!
Sweet, maybe I can publish this!
They figured this out 50 years ago
Go back to
Start

Cellular Automaton

Conceptualization of Influences and Transitions:



Cellular Automaton

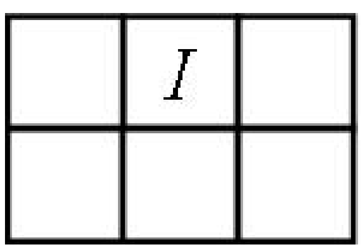
Individuals:

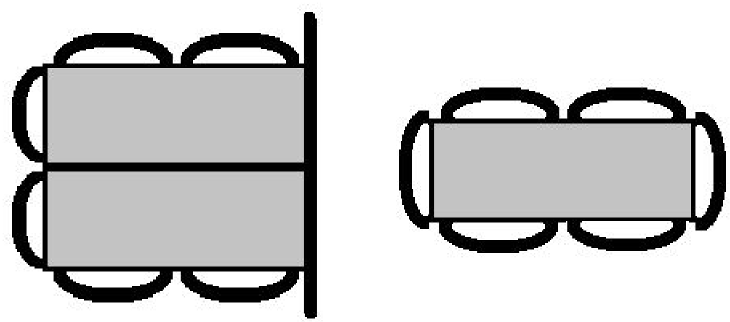
- ***W:*** working student, who exudes a positive influence on *A* to become *W*
- ***A:*** student attracted to either work or socialize
- ***S:*** socializing student, who exudes a negative influence on *A* to become *S*
- ***TA:*** teaching assistant, who exudes a positive influence on *A* to become *W*, but cannot influence *W* or *S*

Cellular Automaton

Neighbourhood, Variables, and Parameters:

- ***I*:** the individual in this system
- ***D*:** the neighbourhood of *I*
- ***n*=5:** the number of neighbours of *I*
- **N_i :** the number of neighbours of *I* of type i = *TA, W, A, S* in the neighbourhood *D*





Cellular Automaton

Neighbourhood, Variables, and Parameters:

- ***τ:*** *TA* influence on *I*
- ***ω, σ:*** social influence on *I*
- P_W**:** the probability that an entering student is of type *W*
- P_A**:** the probability that an entering student is of type *A*
- P_S**:** the probability that an entering student is of type *S*

Cellular Automaton

Social Influence Counters:

- Initial values: $C_i\ (0)=0$, with i = W, A, or S
- $C_W(t) = C_W(t-1) - N_S\sigma$
- $C_A(t) = C_A(t-1) + N_{TA}\tau + N_W\omega - N_S\sigma$
- $C_S(t) = C_S(t-1) - N_W\sigma$

Cellular Automaton

Transition Rules:

Let $0 \le \tau, \sigma, \omega \le 1$ and $P_W = 0.3, P_A = 0.6, P_S = 0.1$.

1. ***W:*** probabilistic transition:
   If $C_W(t) \le -1$ then leaves and another student enters with probability P_i.
2. ***A:*** deterministic transition:
   a) If $C_A(t) \ge 1$ then this student becomes ***W***.
   b) If $C_A(t) \le -1$ then this student becomes ***S***.
3. ***S:*** probabilistic transition:
   If $C_S(t) \le -1$ then leaves and another student enters with probability P_i.

Cellular Automaton

Classification:

- linear
- individual
- stochastic
- dynamic
- discrete
- qualitative/quantitative

Thank You!