Applied Calculus Workshop (ACW) as a Complex Social System

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Project Presentation
Outline

ACW
- Introduction
- Defining Efficiency
- Questions

Models without & with Vital Dynamics
- Assumptions
- Conceptualization
- ODE
  - Classification
  - Steady State
  - Analysis Part I, II
- Data Collection
- Cellular Automaton

Closing Remarks
- Further Work
- History & Examples
- References
Introduction to the Workshop

Developed in the 1990s in the Department of Mathematics at SFU:

• drop-in based
• serves a variety of courses and student needs
• provides TA-ships for graduate students
Introduction to the Workshop

Similar centres are offered across the country:

• Mathematics Learning Centre at **Capilano** University
• Mathematics and Statistics Learning Centre at **Dalhousie** University
• Learning Centre at University of **Guelph**
• Math and Statistics Learning Centre at University of **Toronto** Scarborough
Introduction to the Workshop

Some facts:
- 5 workshops servicing ~4000 students/fall
- ACW services ~1200 students/fall,spring
- accommodates 40-50 students
- 15-18 TAs
- open M-F with 6-10 h/day
Introduction to the Workshop

Primary goals of the workshop are twofold:
• provide economical help to the students
• students need to work well and receive good service
Defining Efficiency

\[ f'(x) = x - \pi/x \]

\[ \pi^2 - e \]
Defining Efficiency

Efficiency for a workshop is defined as the degree to which studious behaviour among students is observed under the influence of teaching assistants.

→complex social system
Defining Efficiency

The questions driving the project:

• Can we model the dynamics among students and teaching assistants and reproduce observed behaviours?

• Can this model be used to study the effect of the number of TAs present in terms of an increase/decrease in studiousness?

• Can this model be used to study how the type of TA present influences the learning environment?

• How efficient can a workshop be, i.e. under which conditions does the climate for learning flourish?
Model without Vital Dynamics

Assumptions:

• Student classification: working, attracted to work or socialize, socializing.

• Each student group is large enough so its size can be considered a continuous function of time.

• No student can leave or enter the system.
Model without Vital Dynamics

Assumptions cont’d:

• Attracted students can be influenced by working students to work.
• Attracted students can be influenced by teaching assistant(s) to work.
• Attracted students can be influenced by socializing students to socialize.
Model without Vital Dynamics

Conceptualization of Influences and Transitions:
Model without Vital Dynamics

Variables & Parameters: Let

- \( t \) be time
- \( W(t), A(t), S(t) \) be the # of working, attracted, socializing students
- \( \sigma \) be the rate constant for transfer from \( A \) to \( S \) due to the influence of \( S \)
- \( \omega \) be the rate constant for transfer from \( A \) to \( W \) due to the influence of \( W \)
- \( \tau \) be the rate constant for transfer from \( A \) to \( W \) due to the influence of teaching assistants
Model without Vital Dynamics

ODEs:

\[
\frac{dW}{dt} = \omega WA + \tau (N) A \\
\frac{dA}{dt} = -\omega WA - \tau (N) A - \sigma SA \\
\frac{dS}{dt} = \sigma SA
\]

with initial values \( W_0, A_0, \) and \( S_0. \)
Model without Vital Dynamics

Classification:
• non-linear
• aggregate
• deterministic
• dynamic
• continuous
• qualitative/quantitative
Model without Vital Dynamics

Steady State:
• The steady state is reached when $A=0$.
• This is not an interesting result at all. But ...
• What values need to be chosen for the parameters $\omega$ and $\sigma$?
• What information do the values of the parameter $\tau$ lead to?
Model with Vital Dynamics

Additional assumptions:

• If there is too much working, then socializing students leave.

• If there is too much socializing, then working students leave.

• Students that have left will be replaced with students entering in the same proportions as the initial values $W_0$, $A_0$, and $S_0$. 
Model with Vital Dynamics

Conceptualization of Influences and Transitions:
Additional Variables & Parameters: Let

- $\alpha$ be the rate constant for transfer out of $W$ due to the influence of $S$
- $\beta$ be the rate constant for transfer out of $S$ due to the influence of $W$
Model with Vital Dynamics

ODEs:

\[
\frac{dW}{dt} = \omega W A + \tau(N) A - \alpha SW + \alpha SWW_0 + \beta SSWW_0
\]

\[
\frac{dA}{dt} = -\omega W A - \tau(N) A - \sigma SA + \alpha SWA_0 + \beta SWA_0
\]

\[
\frac{dS}{dt} = \sigma SA + \alpha SWS_0 - \beta SW + \beta SWS_0
\]

with initial values \(W_0, A_0,\) and \(S_0\).
Model with Vital Dynamics

Classification:
• non-linear
• aggregate
• deterministic
• dynamic
• continuous
• qualitative/quantitative
Data Collection

Counting constraints:

• 3 separate days, 2 hour duration, 10 minute time intervals

• Working: individuals or small groups that were clearly working

• Socializing: individuals that were engaged non-mathematically with other students, or inappropriately using electronic devices

• Total number of students
Data Collection

Graphs of data normalized over total students:

- working
- attracted
- socializing
Data Collection

Graphs of data normalized over total students:

- working
- socializing
Data Collection

Data Analysis:

• # of working students is \( \approx 2-3 \) times that of the socializing students \( \rightarrow \) initial conditions
• high # of exits & entrances near the half hour
• changes in \( W \) and \( S \) occur \( \sim \) every 20 minutes
• changes in \( W, S \) are periodic
• changes in \( W \) and \( S \) are out-of-phase
Data Collection

Observation:

• excellent TAs → 5-6 students
• good TAs → 3-4 students
• average TAs → 1 student
Model without Vital Dynamics

Analysis of parameter values $\omega$ and $\sigma$:

<table>
<thead>
<tr>
<th>$W_0$</th>
<th>$S_0$</th>
<th>$\omega$</th>
<th>$\sigma$</th>
<th>$\tau$</th>
<th>$W_\infty = kS_\infty$</th>
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<td>0.30</td>
<td>0.15</td>
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<td>0.50</td>
<td>0.50</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>$\omega=2\sigma$</td>
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<td>0.25</td>
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<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>$\omega=3\sigma$</td>
<td>0.60</td>
<td>0.20</td>
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<tr>
<td></td>
<td></td>
<td>$\omega=3\sigma$</td>
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<td>0.05</td>
<td>0.00</td>
</tr>
<tr>
<td>0.36</td>
<td>0.12</td>
<td>$\omega=\sigma$</td>
<td>0.50</td>
<td>0.50</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\omega=\sigma$</td>
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<td>0.05</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Model without Vital Dynamics

Analysis of parameter values $\omega$ and $\sigma$ and graphs:

$W_0 = 0.30, S_0 = 0.15$ with $\tau = 0$
$\omega = \sigma = 0.12$

$W_0 = 0.36, S_0 = 0.12$ with $\tau = 0$
$\omega = 0.50, \sigma = 0.25$

Figure 15

Figure 22
Model without Vital Dynamics

Analysis of parameter value $\tau$:

<table>
<thead>
<tr>
<th>$W_0$</th>
<th>$S_0$</th>
<th>$\omega$</th>
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<th>$\tau$</th>
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<td>5</td>
<td>0.07</td>
<td>5.5</td>
</tr>
</tbody>
</table>
Model without Vital Dynamics

Analysis of parameter value $\tau$ and graphs:

$W_0 = 0.30, S_0 = 0.15$ with $\tau = 0.06$

$\omega = \sigma = 0.12$

$W_0 = 0.36, S_0 = 0.12$ with $\tau = 0.15$

$\omega = 0.50, \sigma = 0.25$

Figure 27

Figure 36
Analysis of parameter values $\alpha$ and $\beta$ and graphs:

For every value of $\alpha$ there seems to be a value for $\beta$, call this value $\beta_c$, such that for $\beta \leq \beta_c$ the socializing group wins, and for $\beta \geq \beta_c$ the working group wins. $A$ is always depleted!

$W_0 = 0.30, S_0 = 0.15, \omega = \sigma = 0.12, \tau = 0.06$. Then $\beta_c \approx 0.0310344$.

$\beta = 0.10, \beta = 0.01$

Figure 5a  Figure 6a
Public Perception of Science

Hmm I wonder if...

Start →

Do science

Read science

Science in Reality

Start

Hmm I wonder if

CHALLENGE ACCEPTED

Do science

Find out someone already did this

React science

Hm not quite going as expected...

UNBELIEVABLE

AMAZING RESULTS

Results turn out to be bullsh**

Wait, no it doesn't

Thinking

Hm, that's funny

Oh hey, this makes sense!

Sweet, maybe I can publish this

50 years ago

Go back to

Start
Cellular Automaton

Conceptualization of Influences and Transitions:
Cellular Automaton

Individuals:

• **W**: working student, who exudes a positive influence on A to become W

• **A**: student attracted to either work or socialize

• **S**: socializing student, who exudes a negative influence on A to become S

• **TA**: teaching assistant, who exudes a positive influence on A to become W, but cannot influence W or S
Cellular Automaton

Neighbourhood, Variables, and Parameters:

- **I**: the individual in this system
- **D**: the neighbourhood of I
- **n=5**: the number of neighbours of I
- **Ni**: the number of neighbours of I of type $i = TA, W, A, S$ in the neighbourhood $D$
Cellular Automaton

Neighbourhood, Variables, and Parameters:

- \( \tau: \) TA influence on \( I \)
- \( \omega, \sigma: \) social influence on \( I \)
- \( P_W: \) the probability that an entering student is of type \( W \)
- \( P_A: \) the probability that an entering student is of type \( A \)
- \( P_S: \) the probability that an entering student is of type \( S \)
Cellular Automaton

Social Influence Counters:

• Initial values: \( C_i(0) = 0 \), with \( i = W, A, \) or \( S \)

• \( C_W(t) = C_W(t-1) - N_S \sigma \)

• \( C_A(t) = C_A(t-1) + N_{TA} \tau + N_W \omega - N_S \sigma \)

• \( C_S(t) = C_S(t-1) - N_W \sigma \)
Cellular Automaton

Transition Rules:
Let $0 \leq \tau, \sigma, \omega \leq 1$ and $P_W = 0.3$, $P_A = 0.6$, $P_S = 0.1$.

1. $W$: probabilistic transition:
   If $C_W(t) \leq -1$ then leaves and another student enters with probability $P_i$.

2. $A$: deterministic transition:
   a) If $C_A(t) \geq 1$ then this student becomes $W$.
   b) If $C_A(t) \leq -1$ then this student becomes $S$.

3. $S$: probabilistic transition:
   If $C_S(t) \leq -1$ then leaves and another student enters with probability $P_i$. 
Cellular Automaton

Classification:

- linear
- individual
- stochastic
- dynamic
- discrete
- qualitative/quantitative
Thank You!

\[
\pi^2 = e
\]

\[
f'(x) = x - \frac{\pi}{x}
\]

...blah, blah, blah...