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Comparing the inference capabilities of binary, trivalent and sigmoid fuzzy cognitive maps

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ABSTRACT

In this paper, we compare the inference capabilities of three different types of fuzzy cognitive maps (FCMs). A fuzzy cognitive map is a recurrent artificial neural network that creates models as collections of concepts/neurons and the various causal relations that exist between these concepts/neurons. In the paper, a variety of industry/engineering FCM applications is presented. The three different types of FCMs that we study and compare are the binary, the trivalent and the sigmoid FCM, each of them using the corresponding transfer function for their neurons/concepts. Predictions are made by viewing dynamically the consequences of the various imposed scenarios. The prediction making capabilities are examined and presented. Conclusions are drawn concerning the use of the three types of FCMs for making predictions. Guidance is given, in order FCM users to choose the most suitable type of FCM, according to (a) the nature of the problem, (b) the required representation capabilities of the problem and (c) the level of inference required by the case.

1. Introduction

Fuzzy cognitive maps (FCMs) [13] is a well established technique for prediction and decision making, e.g., in industry, especially for situations where fuzziness and uncertainty exist. The main areas of FCM research concern the construction, the structure, the static/dynamic analysis and the inference capabilities of FCMs. Various types of FCMs have been proposed, each of them having different transfer function, leading to different representation and inference capabilities. Although the inference capabilities of FCMs have been studied separately for each type of FCM, no detailed comparison is contacted between them. In this paper:

(a) We compare the inference capabilities of the three most commonly used types of FCMs, which are the binary, the trivalent and the sigmoid FCM. The comparison is done by imposing a series of similar scenarios to the three types of FCMs and examining the predictions made by them.
(b) Based on the comparison above, we identify rules that propose the type of FCM that should be used, according to the problem in hand.

In the following section, a short introduction to FCMs is presented followed by an illustration of industrial/engineering FCM applications. The connection between artificial neural networks (ANNs) and FCMs is presented in Section 3, followed by a section discussing the transfer functions of FCMs. The various kinds of dynamic behavior that FCMs can exhibit are shown in Section 5. The inference capabilities of binary and trivalent FCMs are demonstrated and compared to Section 6, while in

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Section 7, the inference capabilities of sigmoid FCMs are presented and compared to those of binary and trivalent FCMs. In the final section, the conclusions drawn by the comparison of the three FCMs are presented, followed by rules concerning the right choice of FCM type, depending on the nature of the specific problem.

2. Fuzzy cognitive maps and their industrial/engineering applications

Fuzzy cognitive map is a type of recurrent artificial neural network that has been introduced by Kosko [13,14] based on Axelord's work on cognitive maps [2]. It combines elements of fuzzy logic and ANNs. FCMs create models as collections of concepts and the various causal relations that exist between these concepts. The concepts are represented by neurons and the causal relationships by directed arcs between the neurons. Each arc is accompanied by a weight that defines the type of causal relation between the two concepts/neurons. The sign of the weight determines the positive or negative causal relation between the two concepts/neurons. Many applications of FCMs have been developed concerning, e.g., economic predictions [4], political decisions [1], image processing [10], ecology [20], etc. In the following sections, we will present industrial and engineering applications of FCMs.

2.1. An FCM for a process control problem

An FCM can be created in order to handle the process control problem of Fig. 1 [23,27].

As it is shown in Fig. 1, the system consists of two tanks, each of them having an inlet valve and an outlet valve. The two tanks are connected sequentially, so the outlet valve of the first tank is the inlet valve of the second. The purpose of the control system is (a) to keep the height of liquid in both tanks, into a specific range $[H_{\min}, H_{\max}]$ and (b) to keep the temperature of the liquid in both tanks into a specific range $[T_{\min}, T_{\max}]$. A heating element can regulate the temperature of the liquid in tank 1 and a thermometer measures the temperature of the liquid in the tank 2. The control system, for example, should open valve 2 in order hot liquid from tank 1 to come into tank 2, when the temperature of the liquid 2 is decreased. The FCM created for the above control problem is shown in Fig. 2.

The FCM contains the following eight concepts that represent the variables of the system:

C1:	The height of the liquid in tank 1. This can change by opening/closing valve 1 and valve 2
C2:	The height of the liquid in tank 2. This can change by opening/closing valve 2 and valve 3
C3:	Valve 1 state (open, closed or partially open)
C4:	Valve 2 state (open, closed or partially open)
C5:	Valve 3 state (open, closed or partially open)
C6:	The temperature in tank 1
C7:	The temperature in tank 2
C8:	The operation of the heating element

Each of the concepts above can take a value from the interval [0,1] that corresponds to a similar physical state of the same degree. For example in Fig. 2, the fact that C3: Valve1 has value 0.55 means that Valve1 is opened to a degree of 55%.

Furthermore, the arcs between the concepts of the FCM of Fig. 2, represent the possible ways that these concepts can interact with each other. For example:

(a) The arc that connects C1 with C3, relates the amount of the liquid in tank 1 with the operation of the valve 1.

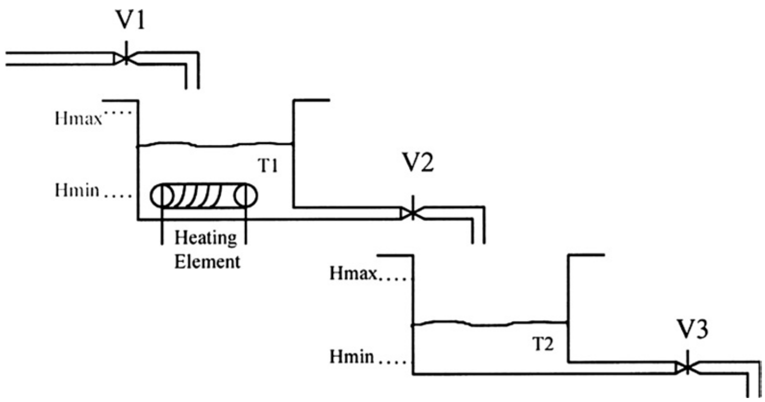

Fig. 1. A process control problem of two tanks and three valves [22,26].

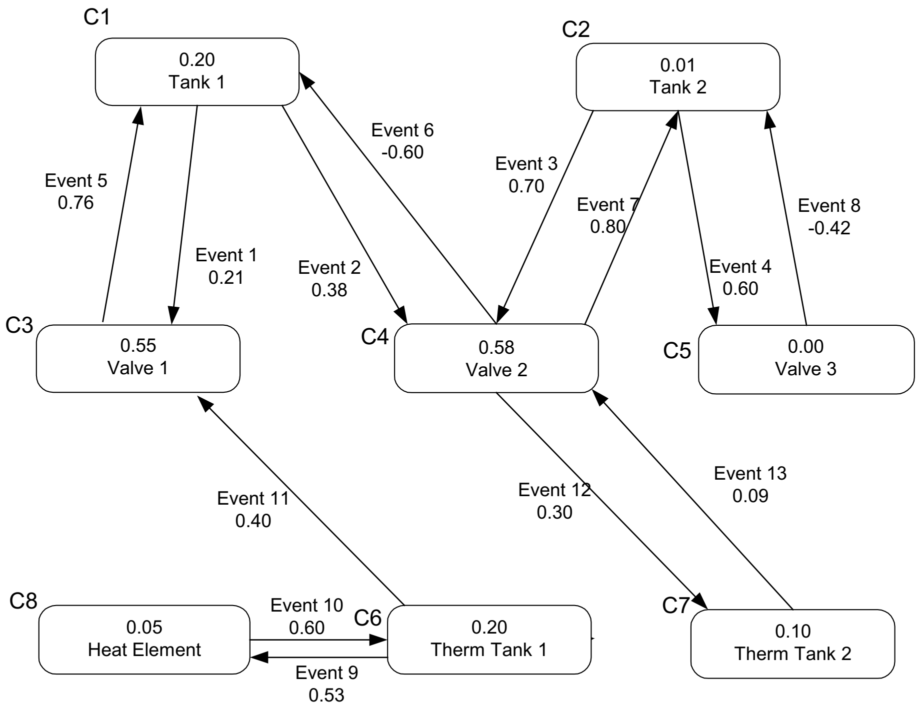

Fig. 2. FCM of a process control problem [14,18].

(b) The arc the connect C4 with C1, represents that the opening or not of valve 2 will cause the decrease or not of the height of the liquid in tank1.

The sign and the weight of each such connection are determined by a human that is expert in the above process. A study of this FCM can be found in [23,27].

2.2. An FCM concerning a car industry

Another FCM concerning a car industry [7] is shown in Fig. 3. In this more qualitative than quantitative model, the concepts that are involved are the following:

C1:	High Profits
C2:	Customer Satisfaction
C3:	High Sales
C4:	Union Raises
C5:	Safer Cars
C6:	Foreign Competition
C7:	Lower Prices

The weights of the arcs that connect the concepts are in the interval $[-1,1]$. They are determined by car industry experts and represent the causal relationships that, according to the experts, exist between the concepts. So for example a decision of the car industry to create C5: "Safer Cars", will cause an increase in C2: "Customer Satisfaction" to a degree $w_{52} = 0.8$ (that is, in linguistic terms, "very high") and also cause a decrease in C7: "Lower Prices" to a degree $w_{57} = -0.5$ (that is, in linguistic terms, "moderate"). Scenarios can be introduced to this FCM, examining for example what will be the consequences of producing Safer Cars to Sales, Profits, Customer Satisfaction, Union Raises and Competition from other car industries. A study of this FCM for various scenarios can be found in [29].

2.3. A civil engineering FCM

A Civil Engineering FCM that studies the consequences of the increase of a city's population and modernization to the city's public health [18] is shown in Fig. 4. In this qualitative model, the concepts that are used are the following:

C1:	Number of People in city
C2:	Migration into city
C3:	Modernization
C4:	Garbage per area
C5:	Sanitation Facilities
C6:	Number of Diseases per 1000 residents
C7:	Bacteria per Area

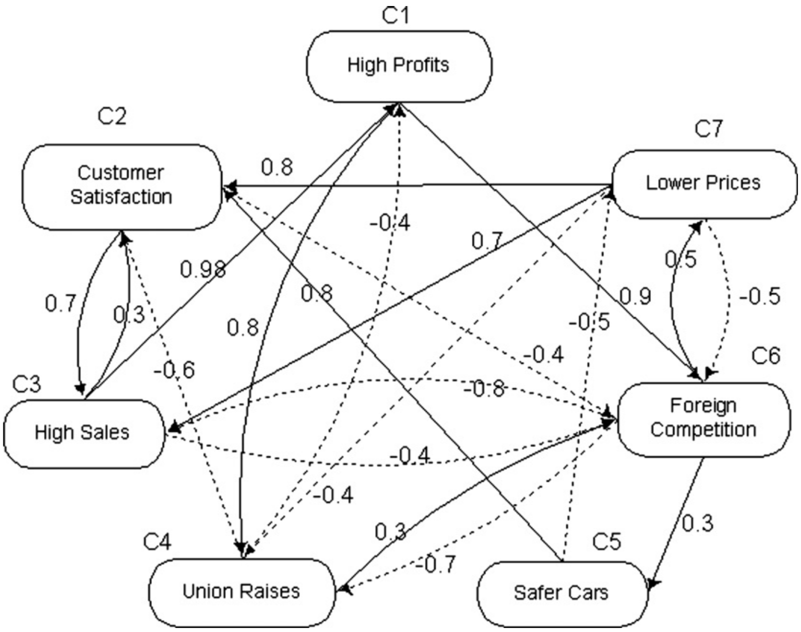

Fig. 3. FCM of a car industry (modified version of original taken from [7]).

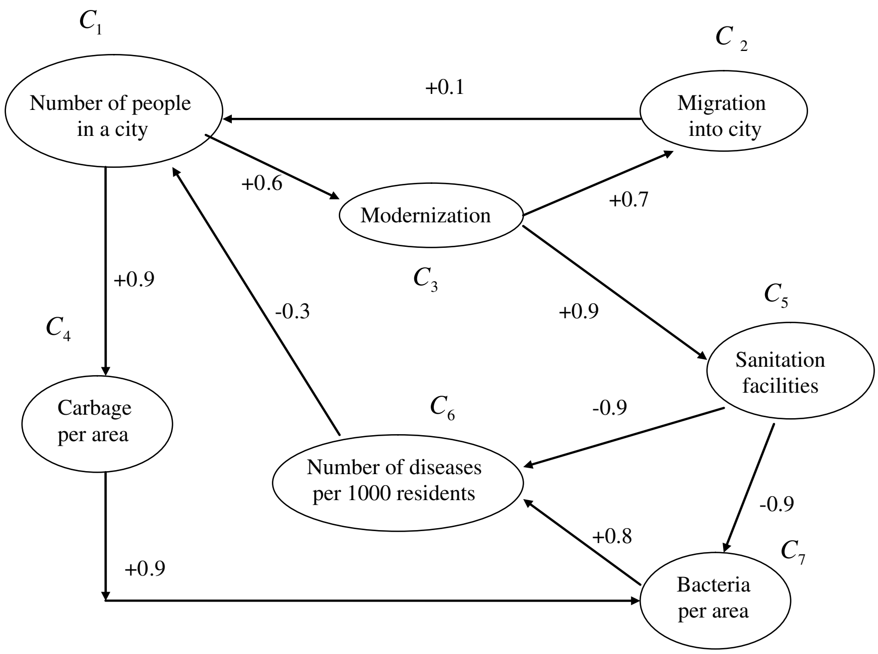

Fig. 4. A Civil Engineering FCM [18].

The weights of the arcs that connect the concepts are once again in the interval $[-1,1]$ and represent the causal relationships that exist between the concepts. So, for example, an increase in C1: the population of the city will cause (a) increase in C3: Modernization of the City to a degree 0.6 (that is, in linguistic terms, "quite high") and (b) a increase in the C4: City's Garbage to a degree 0.9 (that is, in linguistic terms, "extremely high").

3. FCM and artificial neural networks

An FCM can be considered as a type of recurrent artificial neural network having the structure of Fig. 5. Each concept/neuron C_i of the FCM graph is accompanied by a number A_i^t that represents the activation level of concept C_i at time step t. If n is the number of FCM's neurons/concepts, then the vector $\mathbf{A}^t = [A_1^t, A_2^t, \ldots, A_n^t]$ gives the state of the FCM at time step t.

The weight matrix $\mathbf{W}$ of this artificial neural network is defined by the $n \times n$ matrix, where each of its element w_{ij} equals to the weight of the arc that connects concept/neuron C_i with concept/neuron C_j. All elements w_{ii} of the weight matrix $\mathbf{W}$ equals to zero, since FCM structure does not allow any direct connection between a concept and itself. All other elements w_{ij} $(i \neq j)$ belong to the interval $[-1,1]$. The FCM is a discrete time system where the activation levels of all the concepts are simultaneously updated, that is the system has synchronous updating [9]. The new state of the system $\mathbf{A}^{t+1} = [A_1^{t+1}, A_2^{t+1}, \ldots, A_n^{t+1}]$ is calculated based on the classical operation of the ANNs, that is by evaluating each $A_i^{t+1} i = 1, \ldots, n$, according the following function that McCullock and Pitts [19] have proposed, where f is the transfer function:

$$A_i^{t+1} = f\left(\sum_{j=1}^{n} w_{ji} A_j^t\right). \tag{1}$$

Using weight matrices, Eq. (1) becomes $\mathbf{A}^{t+1} = f(\mathbf{A}^t\mathbf{W})$. Comparing FCMs with other types of ANNs the following can be noticed. Classic types of ANNs act like "black boxes" where both neurons and their interconnections do not have individually any clear meaning for the problem itself. On the contrary, all the concepts and the weights of the FCM have a precise meaning for the problem and correspond to specific variables. This means that the representation capabilities of the FCMs are superior compared to those of other ANNs. This is important advantage for the inference process since the outcome of the system can be fully explained and justified. A known disadvantage is the difficulty in estimating and incorporating in FCMs, time lags between the cause and the effect. Various attempts have been made to overcome this drawback [6,8,22].

The current research on FCMs is concentrated in two areas:

(a) *Learning of FCM*. The success of the FCM model depends heavily on the method the FCM is constructed [11,32]. Various methods for manually deriving the FCM can be found in [2]. Due to the ANN origin of FCM, various researchers attempt the automatic construction of the weight matrix of FCM in the following way. As in ANNs, training data sets are used to estimate the weights among the concepts of the FCM, thus the weight matrix of the FCM is automatically created. Genetic Algorithms and non linear Hebbian Learning are methods that are proposed for the FCM learning [12,17,23,25].

(b) *Inference and Representation Capabilities of FCM*. Having an FCM created either manually or automatically, the FCM should be simulated in order the consequences of the imposed scenarios to be found and inference to be made. In this way, the FCM provides predictions and supports decisions. The quality of the results depends heavily on the representation capabilities of the FCM. The transfer function of the FCM's neurons determines FCM's representation capabilities [21,31,33].

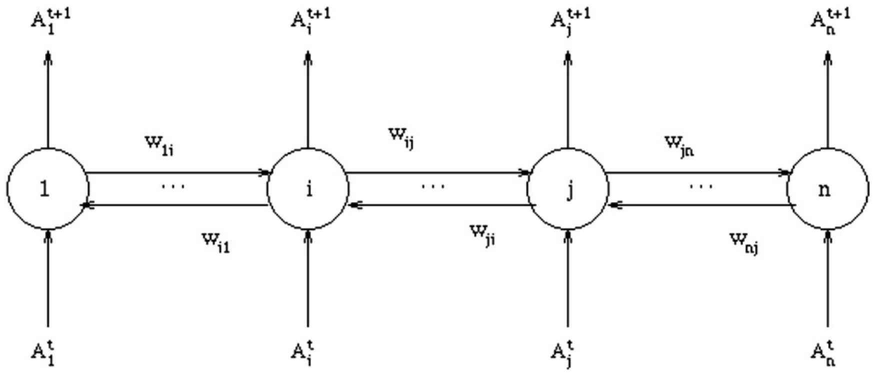

Fig. 5. FCM's structure as a Recurrent Artificial Neural Network.

In this paper, the study is concentrated to the second area, examining the inference and representation capabilities of various types of FCMs. Such a research will help the decision makers that use FCM, to choose the right type of FCM, in order the chosen FCM to have the representation capabilities that are important for the level of inference required by the specific case. It should also be mentioned that the two areas above have a close connection since the representation capabilities of FCM and the transfer function of the FCM's neurons have a direct influence to the learning process of FCM.

4. The transfer function of FCMs

In FCM structure, a design choice of the FCM is the selection of the transfer function f shown in Eq. (1) of the neurons, which determines the values that activation level A_i^t can take. The transfer functions that are most frequently used are:

(1) The sign function [5,15,28]

$$f_{\text{sign}}(x) = \begin{cases} 1, & x > 0, \\ 0, & x \leqslant 0. \end{cases} \tag{2}$$

The use of this function allows the activation level of each neuron to be either 0 or 1, leading to the development of binary FCMs, where each concept is either activated or not activated.

(2) The trivalent function [5,28]

$$f_{\text{tri}}(x) = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0. \end{cases} \tag{3}$$

In this case, three state FCMs are created. When the activation level of concept C_i equals 1, it means that this concept increases, when the activation level equals -1, it means that the concept decreases, and when the activation level equals to 0, it means that the concept remain stable.

(3) The sigmoid function with saturation levels -1 and 1 [26,28].

$$f(x) = \tanh(x) \text{ or} \tag{4}$$

$$f(x) = \frac{e^{2x} - 1}{e^{2x} + 1}. \tag{5}$$

The activation level can now take any value from the interval $[-1,1]$ and so continuous state FCMs are created.

Other transfer functions are also proposed; see for example [21,31,33].

A binary FCM of n concepts has 2^n different states $[A_1^t, A_2^t, \ldots, A_n^t]$ and moves at the corners of the $[0,1]^n$ hypercube. A trivalent FCM has 3^n different states and moves at the corners, at the middle of the edges, the center of the sides and the center of the $[-1,1]^n$ hypercube. On the contrary, if the FCM has continuous state values, like the trivalent FCM, then the system has infinite number of different states and moves freely across the whole space of the $[-1,1]^n$ hypercube. In this case, the study of the dynamic behavior of the FCM refers to the trace to the $[-1,1]^n$ hypercube that the system defines.

5. Dynamic behavior of FCMs

Conclusions about the model that the FCM represents are drawn by the study of the dynamic behavior of the FCM system. This resembles the recall phase of the ANN, where the ANN is asked to behave according to the knowledge that has been encoded on the weights of its connections. The inference mechanism of the FCM starts with the initialization of the system [14]. This is done by setting specific values to the activation level of each FCM's concept, based either on the estimations of the experts for the current state of the concepts of the FCM model, or based on a specific scenario the consequences of which we want FCM to predict. After that, the concepts are free to interact. The activation level of each concept influences the other concepts according to the weighted connections that exist between them. This interaction continues until one of the following happens:

(1) An equilibrium point is found. In this case, $\exists t_0 \in N : A_i^{t+1} = A_i^t \ \forall t \geqslant t_0, \ i = 1, \ldots, n$. The state $\mathbf{A}^{t_0}$ is the final equilibrium point of the system and it is the state at which, the various positive and negative interactions between the neurons/concepts of the FCM, have reached equilibrium.

(2) A limit cycle behavior is reached [9,24]. In this case, $\exists t_0, T \in N : A_i^{t+T} = A_i^t \ \forall t \geqslant t_0, \ i = 1, \ldots, n$. The system exhibits a periodic behavior where after a certain number of time steps, that is equal to the period T of the system, the system reaches the same state.

(3) The system exhibits a chaotic behavior [3,24].

The FCM system is deterministic and so, if it reaches a state to which it has been previously, the system will enter a closed orbit which will always repeat itself. In the case of the binary FCM of n concepts, since the system has only 2^n different state, it is apparent that after a maximum of 2^n time steps, it will return to a previously visited state. So binary FCM can reach either an equilibrium point or a limit cycle behavior (of maximum period 2^n) but never chaotic behavior. For the same reason, a trivalent FCM of n concepts, can never exhibit chaotic behavior but only reach an equilibrium point or a limit cycle behavior with maximum length 3^n. On the contrary, a continuous state FCM can additionally exhibit also chaotic behavior.

The equilibrium point and the limit cycles that the FCM can reach correspond to hidden patterns that are encoded in the connections that are present at the FCM graph [15]. The encoding of these patterns in the FCM structure corresponds to the classical training procedure of the ANNs and is an open research problem.

6. Inference in binary and trivalent FCMs

6.1. Comparing binary and trivalent FCMs

FCMs can be used for quantitative problems, especially when a quick answer is needed. But the representation capabilities of FCMs and FCM's inference capabilities are fully exploited when the topic of the FCM concerns a qualitative problem where fuzziness is an additional drawback and no quantitative methods can be used. The civil engineering application of Section 2.3 is the most suitable for our study and so we will use the FCM of Fig. 4 for the study of the binary and trivalent FCMs that follows. In FCMs, inference is made by introducing to the FCM structure, specific scenarios that have the corresponding initial states. For example, we can assume that there is increase in the city's "Population" ($C_1 = 1$), increase in the "Migration" into the city ($C_2 = 1$), and increase in its "Modernization" ($C_3 = 1$). According to these, the initialization of the system must be done with the vector [1 1 1 0 0 0 0]. Imposing this data to the binary FCM, Eq. (1) of Section 3 is applied repeatedly for the update of each concept's activation level, using as transfer function, the sign function (Eq. (2) of Section 4). The dynamic behavior is the following:

Time step t	Population of city	Migration into city	Modernization	Garbage per area	Sanitation Facilities	Number of diseases per 1000 resident	Bacteria per area
0	1	1	1	0	0	0	0
1	1	1	1	1	1	0	0
2	1	1	1	1	1	0	0
3	1	1	1	1	1	0	0

It can be noticed that the system, after a short period of 3 time steps, reached an equilibrium point. At that point, there is also increase in the "Garbage per area" and the "Sanitation facilities". An explanation to the above is the following: The increase of the "Population of the city" led to an increase of the "Garbage per area". Moreover, the increase of the "Modernization" led to an increase in the provided "Sanitation Facilities". "Bacteria per area" are not increased, although there is an increase of the "Population of the city", because of the improvement to the "Sanitation Facilities". This also explains why "Number of diseases" is not increased.

It should be stressed that the value 0 at concepts "Bacteria per area" and "Number of diseases" means "no increase". Binary FCM does not allow us to separate the case where a concept is stable, from the case where a concept decreases. A solution comes with the use of trivalent FCMs.

Initializing the trivalent FCM with the same values, the system evolves in the following way:

Time step t	Population of city	Migration into city	Modernization	Garbage per area	Sanitation Facilities	Number of diseases per 1000 resident	Bacteria per area
0	1	1	1	0	0	0	0
1	1	1	1	1	1	0	0
2	1	1	1	1	1	−1	0
3	1	1	1	1	1	−1	0
4	1	1	1	1	1	−1	0

Once again, Eq. (1) of Section 3 is applied repeatedly for the update of each concept's activation level, but now the trivalent function (Eq. (3) of Section 4) is used as transfer function. In a similar way, the system reached quickly an equilibrium point. The final stable state is similar with that of the binary FCM with the difference that concept "Number of diseases" has activation value −1 and so it appears to decrease. So the additional conclusion to those of the binary FCM is that the "Number of diseases" is decreasing while the "Bacteria per area" remain stable.

The increased representation capabilities of the trivalent FCM when compared with the binary FCM, are also illustrated with the following scenario. Let us assume that due to some external to the FCM systems reasons, there is an increase in the "Number of diseases per 1000 resident" of the city ($C_6 = 1$). Using the binary FCM, no increase is predicted to any other concept of the FCM, since the system reaches immediately the state $A^1 = [0000000]$, where all concepts are stable and not increasing. On the contrary, the trivalent FCM has the following behavior:

Time step t	Population of city	Migration into city	Modernization	Garbage per area	Sanitation Facilities	Number of diseases per 1000 resident	Bacteria per area
0	0	0	0	0	0	1	0
1	−1	0	0	0	0	0	0
2	0	0	−1	−1	0	0	0
3	0	−1	0	0	−1	0	−1
4	−1	0	0	0	0	1	1
5	−1	0	1	−1	0	1	0
6	−1	−1	−1	−1	−1	0	−1
7	−1	−1	−1	−1	−1	1	0
8	−1	−1	−1	−1	−1	1	0

We see that after 8 time steps, the system reached an equilibrium point that is different from that reached by the binary FCM. The trivalent FCM predicts decrease of the "Population of the city", leading also to a decrease of the "Modernization" of the city, which in turns leads to a decrease of the "Migration into city". The decrease of the "Modernization" also tends to increase the "Bacteria per area". But the decrease of the "Population" leads to a decrease of the "Garbage per area" which in turn leads to the increase of the "Bacteria per area". These two opposite influences neutralize one another and as a result the "Bacteria per area" remain stable. Although this happens, the "Number of diseases" is increasing because of the decrease of the "Sanitation facilities". All the above can not be predicted by binary FCM since it can not separate the decrease of a concept from the case where the concept remains stable.

Moreover, in trivalent FCMs, we can ask the system to predict the consequences of a decrease of a concept. For example, we can examine the predictions for the case where there is an increase in the "Population of the city" and a decrease in the "Sanitation Facilities". The evolution of the system is the following:

Time step t	Population of city	Migration into city	Modernization	Garbage per area	Sanitation Facilities	Number of diseases per 1000 resident	Bacteria per area
0	1	0	0	0	−1	0	0
1	0	0	1	1	0	1	1
2	−1	1	0	0	1	1	1
3	−1	0	−1	−1	0	−1	−1
4	1	−1	−1	−1	−1	−1	−1
5	1	−1	1	1	−1	1	0
6	−1	1	1	1	1	1	1
7	−1	1	−1	−1	1	−1	0
8	1	−1	−1	−1	−1	−1	−1
9	1	−1	1	1	−1	1	0
10	−1	1	1	1	1	1	1
11	−1	1	−1	−1	1	−1	0
12	1	−1	−1	−1	−1	1	0

It can be noticed that the state of the trivalent FCM at time steps 4, 8 and 12 is the same. The same applies for time steps 5 and 9, time steps 6 and 10 and also time steps 7 and 11. In other words, the system reached a limit cycle behavior with period of 4 time steps which will never stop, unless it is influenced by an external, to the FCM system, factor. The four successive states of the limit cycle are the following:

$\mathbf{A^1} = [1\ {-1}\ {-1}\ {-1}\ {-1}\ {-1}\ {-1}]$
$\mathbf{A^2} = [1\ {-1}\ 1\ 1\ {-1}\ 1\ 0]$
$\mathbf{A^3} = [{-1}\ 1\ 1\ 1\ 1\ 1\ 1]$
$\mathbf{A^4} = [{-1}\ 1\ {-1}\ {-1}\ 1\ {-1}\ 0]$

We can notice that all concepts in some of the states above increase and in some decrease. Moreover, state $\mathbf{A^3}$ is the opposite of state $\mathbf{A^1}$, and $\mathbf{A^4}$ is the opposite of state $\mathbf{A^2}$. These mean that the city does not reach a stable situation but changes periodically reaching again and again the same or the exact opposite states. It can be concluded that an increase in the "Population of the city" and a decrease in the "Sanitation Facilities" will lead the city to a series of periodic increases and decreases of the FCM's concepts, meaning cycles in the status of the city.

A technique that is frequently used by FCM researchers [5,15,16] is that of the restrain of the value of a concept to a certain degree. This technique resembles the existence of bias to ANN, since having a concept constantly activated to certain level, influences all other concepts that are connected with it, in a similar manner with that of a steady bias. In this case, we study the consequences of the constant increase or constant decrease of a concept, to all other concepts of the FCM. We can apply this technique in our FCM, assuming that we want to examine what the system predicts for a scenario where the "Sanitation Facilities" of the city constantly decreases and the "Population" of the city initially was increasing. To do that, the concept "Sanitation Facilities" is set to -1 for the whole transition phase of the system towards equilibrium. The transition phase is the following:

Time step t	Population of city	Migration into city	Modernization	Garbage per area	Sanitation Facilities	Number of diseases per 1000 resident	Bacteria per area
0	1	0	0	0	-1	0	0
1	0	0	1	1	-1	1	1
2	-1	1	0	0	-1	1	1
3	-1	0	-1	-1	-1	1	1
4	-1	-1	-1	-1	-1	1	0
5	-1	-1	-1	-1	-1	1	0

As it is shown above, the system reaches an equilibrium point and not a limit cycle. From the state of the system at equilibrium, we can conclude that the constant decrease of the "Sanitation Facilities" of the city leads to a decrease of the "Population", the "Modernization", the "Migration into city" and the "Garbage per area". The decrease of the "Garbage" does not lead to a decrease of the number of "Bacteria" which remains stable because of the influence of the decrease of the "Sanitation Facilities". Only the "Number of diseases per 1000 residents" increases.

6.2. Finding all final states of binary and trivalent FCMs

Both binary and trivalent FCMs can exhibit a different number of limit cycles or equilibrium points (which can also be considered as limit cycles of period 1). To determine all the different limit cycles that such FCMs can reach, we must examine the behavior of the system for all different initial states of the FCM. For the 7-concept FCM of Fig. 4, there are $2^7 = 128$ different initial states for the binary FCM and $3^7 = 2187$ for the trivalent FCM. To examine all these cases, we developed a computer program written in C programming language that gives the different limit cycles that the FCM reaches, accompanied with the number of different initial states that reach each of them. For the FCM of Fig. 4, the computer program gives the following results:

Behavior	Final State	Length of limit cycle	Number of initial states that reach this final state
A	[0 0 0 0 0 0 0]	1	36
B	[1 1 1 1 1 0 0]	1	6
C	[1 0 0 0 0 0 0]	3	54
	[0 0 1 1 0 0 0]		
	[0 1 0 0 1 0 1]		
D	[0 1 1 1 1 0 1]	3	32
	[1 1 0 0 1 0 0]		
	[1 0 1 1 0 0 0]		

From the above, we see that there are two equilibrium point and two limit cycles of period 3. For the case of the trivalent FCM, the corresponding results are the following:

Behavior	Final State	Length of limit cycle	Number of initial states that reach this final state
A	[0 0 0 0 0 0 0]	1	1
B	[1 −1 −1 −1 −1 1 0]	1	279
C	[1 1 1 1 1 −1 0]	1	279
D	[−1 −1 1 1 −1 −1 −1]	2	104
	[1 1 −1 −1 1 1 1]		
E	[−1 1 −1 −1 1 1 1]	4	470
	[1 −1 1 1 −1 1 0]		
	[1 −1 −1 −1 −1 −1 −1]		
	[−1 −1 −1 −1 1 0]		
F	[1 1 1 1 1 1 1]	4	470
	[1 1 −1 −1 1 −1 0]		
	[1 −1 1 1 −1 −1 −1]		
	[−1 1 1 1 1 −1 0]		
G	[−1 1 1 1 1 1 1]	4	584
	[1 −1 1 1 −1 1 0]		
	[1 −1 −1 −1 −1 −1 −1]		
	[−1 1 −1 −1 1 −1 0]		

In this case there are three equilibrium points, one limit cycle with period 2 and 3 limit cycles with period 4. We notice that, as we were expecting, trivalent FCM has more complex and wealthy dynamic behavior than that of binary FCM. We should also mention that the above results are useful, since they show all the different dynamic behavior that the FCM can exhibit. In this way, we can lead the system to the desired behavior, choosing the correct initial state, leading to a strategic plan for the problem that FCM models. Moreover, we can identify the various limit cycles that the FCM systems can reach and so examine the cycles that for a system can enter.

It should be noticed that a similar computer program cannot be built for a Sigmoid FCM because its initial states are infinite since each of the n concepts can take any value from the whole interval $[-1,1]$. An, as close as possible, computer program would have to take properly distributed samples of all possible Sigmoid FCM initial states and statistically examine the results. Such an approach can be found in [30].

7. Inference in sigmoid FCMs

In the case of the Sigmoid FCM, the activation level of the concepts are in the interval $[-1,1]$ and so the initialization of the system can be done with values from the whole interval $[-1,1]$. Imposing the first scenario that we used for the binary and trivalent FCM, we initialize sigmoid FCM with a "big" increase of city's "Population" ($A_1^0 = 0.8$), "medium" increase in "Migration into city" ($A_2^0 = 0.5$) and a "small" increase of the "Modernization" ($A_3^0 = 0.3$). This means that instead of having the initial vector equal to $A^0 = [1,1,1,0,0,0,0]$ as it was in Binary and Trivalent FCM, now for the Sigmoid FCM, the initial vector is $A^0 = [0.8,0.5,0.3,0,0,0,0]$. The system, after a transition phase that is shown in Fig. 6, reaches an equilibrium point. The initial and the final states of the system are the following:

Time step t	Population of city	Migration into city	Modernization	Garbage per area	Sanitation Facilities	Number of diseases per 1000 resident	Bacteria per area
0	0.8	0.5	0.3	0	0	0	0
1	0.124	0.482	0.834	0.947	0.588	0.000	0.000
2	0.120	0.897	0.184	0.273	0.954	−0.868	0.668
...	...	...	...	...	...	...	...
17	0.742	0.888	0.805	0.931	0.948	−0.976	−0.038
18	0.742	0.888	0.805	0.931	0.948	−0.976	−0.037
19	0.742	0.888	0.805	0.931	0.948	−0.976	−0.037

We can notice that qualitatively, the conclusion is the same with that of the trivalent FCM, with all concepts to be positively activated, except the "Number of diseases" that is negative and "Bacteria per area" that is close to zero. The advantage of the use of the sigmoid FCM is that now we have an indication of the degree of increase or decrease of the FCM's concepts.

We can also impose the scenario that we have imposed to trivalent FCM and reached a limit cycle behavior. In that scenario, we had increase in the "Population of the city" and decrease in the "Sanitation facilities". Since in sigmoid FCMs, also the degree of these increases and decreases can be expressed, we introduce the scenario that initially there is a "big" increase in the "population" ($A_1^0 = 0.8$) and "medium" decrease in the "Sanitation facilities" ($A_5^0 = -0.5$). The dynamic behavior of the

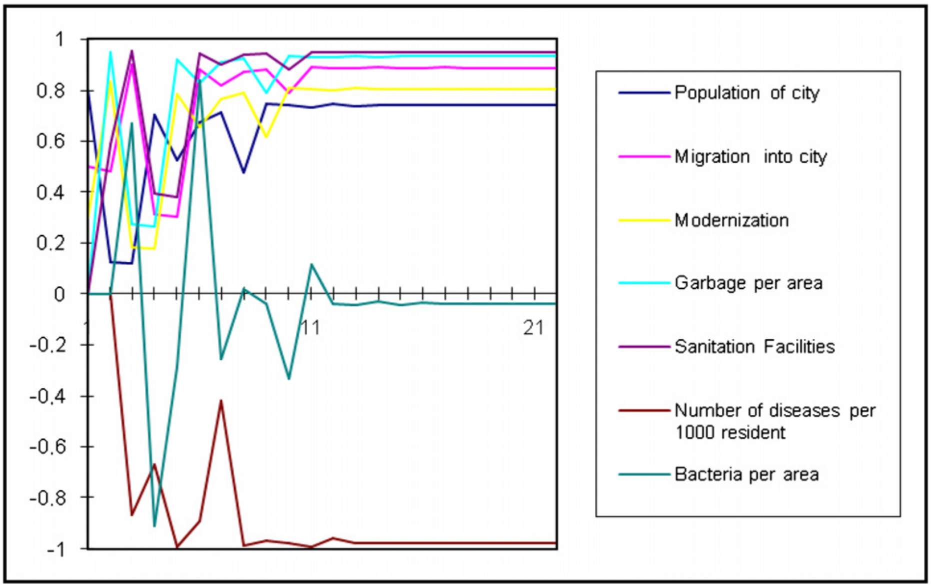

Fig. 6. Transition phase of sigmoid FCM having initial state. Population of city: $A_1^0 = 0.8$, Migration into city: $A_2^0 = 0.5$ and Modernization: $A_3^0 = 0.3$.

sigmoid FCM is shown in Fig. 7, where we can notice that, after 38 time steps, it finally reached an equilibrium point. The initial and final states of the sigmoid FCM are the following:

Time step t	Population of city	Migration into city	Modernization	Garbage per area	Sanitation Facilities	Number of diseases per 1000 resident	Bacteria per area
0	0.8	0	0	0	−0.5	0	0
1	0.000	0.000	0.834	0.947	0.000	0.809	0.809
2	−0.542	0.897	0.000	0.000	0.954	0.924	0.972
...	...	...	...	...	...	...	...
36	−0.742	−0.887	−0.805	−0.931	−0.948	0.976	0.038
37	−0.742	−0.887	−0.805	−0.931	−0.948	0.976	0.037
38	−0.742	−0.887	−0.805	−0.931	−0.948	0.976	0.037

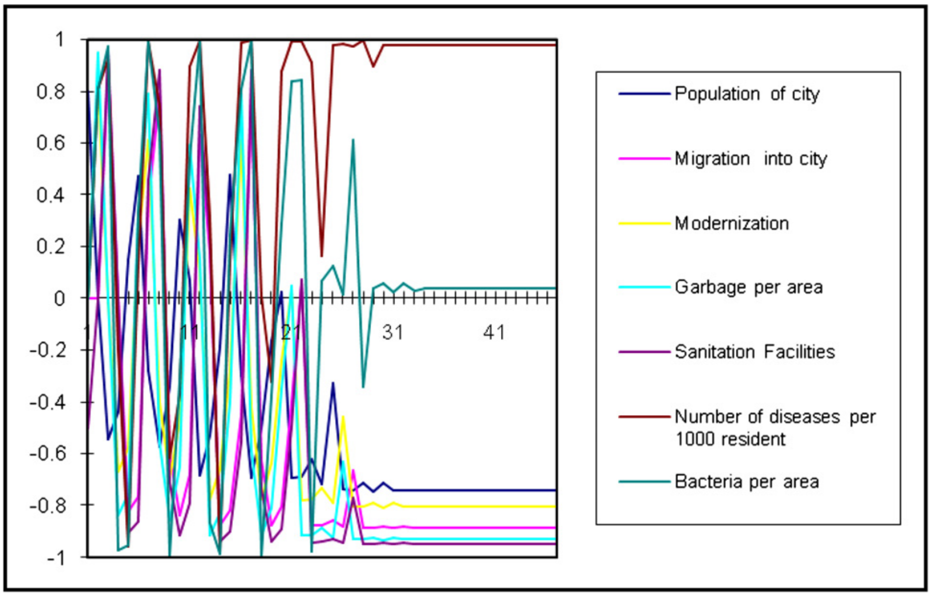

Fig. 7. Transition phase of sigmoid FCM having initial state. Population of city: $A_1^0 = 0.8$, Sanitation Facilities: $A_5^0 = -0.5$.

This dynamic behavior is different from the periodic behavior that the trivalent FCM exhibited. We can also notice that it reaches the exactly opposite equilibrium point from that of Fig. 6. This happens because now the weights of the arcs between the neurons of the FCM play a much more important role than that in binary or trivalent FCMs, since these weights can, in more detail, lead or not to the neutralization of the positive and negative influences a concept/neuron receives from all other neurons.

To illustrate that, we can impose the same scenario of Fig. 7, with only one change. The initial decrease of the "Sanitation facilities" from "medium" ($A_5^0 = -0.5$) is changed to "very small" ($A_5^0 = -0.1$). The system exhibits the dynamic behavior of Fig. 8.

After a transition phase of 54 time steps, sigmoid FCM reached an equilibrium point. The initial and the final states of the sigmoid FCM are the following:

Time step t	Population of city	Migration into city	Modernization	Garbage per area	Sanitation Facilities	Number of diseases per 1000 resident	Bacteria per area
0	0.8	0	0	0	−0.1	0	0
1	0.000	0.000	0.834	0.947	0.000	0.221	0.221
2	−0.164	0.897	0.000	0.000	0.954	0.416	0.972
...	...	...	...	...	...	...	...
52	0.742	0.887	0.805	0.931	0.948	−0.976	−0.038
53	0.742	0.887	0.805	0.931	0.948	−0.976	−0.037
54	0.742	0.887	0.805	0.931	0.948	−0.976	−0.037

It can be noticed that this equilibrium point is the opposite of that reached in the case of Fig. 7. This shows that a small change in the initialization of the system can dramatically change its final equilibrium state and also proves that the advanced representation capabilities of sigmoid FCMs leads to better prediction capabilities, with the neuron's/concept's activation to be in the whole interval $[-1,1]$.

The capabilities of the FCM technique are fully exploited in the following way. Instead of just testing scenarios by setting the initial values of FCM's concepts, we can test scenarios by changing the values of the weights of arcs of the FCM. Such scenarios are checking the consequences in the cases where the causal relationships of the FCM are modified and correspond to strategic planning studies. For example, someone would like to see the consequences of changing the way that C1: "Population of city" influences C3: "Modernization" from w_{13} = 0.6 to w_{13} = 0.4. Such a change in the causal relationship could be a matter of city's strategy. Now the scenarios with initial values of Figs. 6 and 7 can be examined again, to check possible changes in the new predicted outcomes.

In Fig. 9, we see the transition phase of the modified sigmoid FCM ($w_{13} = 0.4$) having the same initial state with that of Fig. 6, (Population of city: $A_1^0 = 0.8$, Migration into city: $A_2^0 = 0.5$ and Modernization: $A_3^0 = 0.3$).

After 27 time steps, the system reached an equilibrium point. The initial and final states of the sigmoid FCM are the following:

Time step t	Population of city	Migration into city	Modernization	Garbage per area	Sanitation Facilities	Number of diseases per 1000 resident	Bacteria per area
0	0.800	0.500	0.300	0.000	0.000	0.000	0.000
1	0.124	0.482	0.664	0.947	0.588	0.000	0.000
2	0.120	0.822	0.124	0.273	0.904	−0.868	0.668
...	...	...	...	...	...	...	...
25	0.720	0.793	0.617	0.925	0.883	−0.947	0.095
26	0.720	0.793	0.617	0.925	0.883	−0.947	0.094
27	0.720	0.793	0.617	0.925	0.883	−0.947	0.094

Comparing this result with that of Fig. 6, we can conclude that the change of weight w_{13} from 0.6 to 0.4 only cause minor changes to the predicted outcomes of the scenario, having only a change in the sign of C7: "Bacteria per area", from extremely small negative to extremely small positive.

If we introduced to this modified FCM the initial state of Fig. 7, we get the following transition phase towards equilibrium (see Fig. 10).

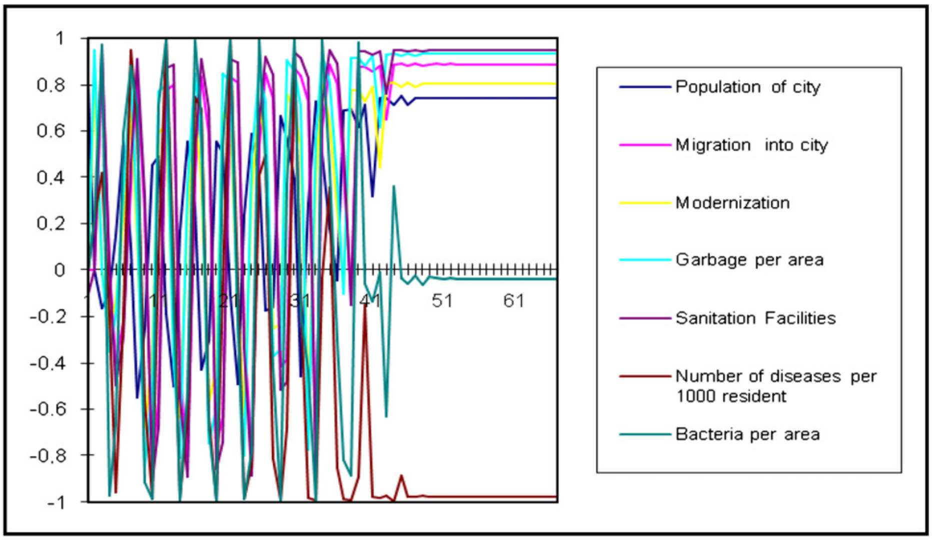

Fig. 8. Transition phase of sigmoid FCM having initial state. Population of city: $A_1^0 = 0.8$, Sanitation Facilities: $A_5^0 = -0.1$.

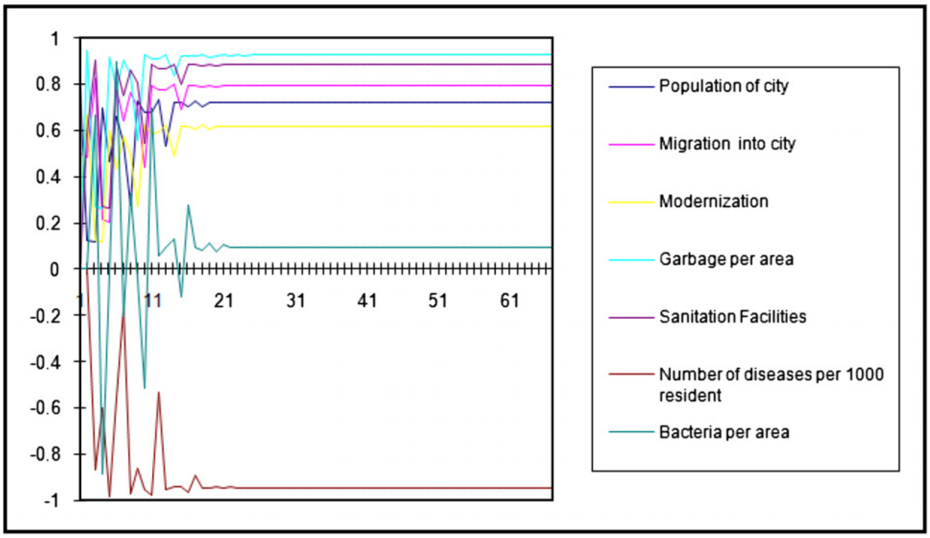

Fig. 9. Transition phase of modified sigmoid FCM ($w_{13} = 0.4$) having initial state. Population of city: $A_1^0 = 0.8$, Migration into city: $A_1^0 = 0.5$ and Modernization: $A_3^0 = 0.3$.

After a longer transition phase of 48 time steps, the system reached once again an equilibrium point. The initial and final states of the sigmoid FCM are the following:

Time step t	Population of city	Migration into city	Modernization	Garbage per area	Sanitation Facilities	Number of diseases per 1000 resident	Bacteria per area
0	0.800	0.000	0.000	0.000	−0.500	0.000	0.000
1	0.000	0.000	0.664	0.947	0.000	0.809	0.809
2	−0.542	0.822	0.000	0.000	0.904	0.924	0.972
...	...	...	...	...	...	...	...
47	−0.720	−0.793	−0.617	−0.925	−0.883	0.946	−0.093
48	−0.720	−0.793	−0.617	−0.925	−0.883	0.947	−0.094
49	−0.720	−0.793	−0.617	−0.925	−0.883	0.947	−0.094

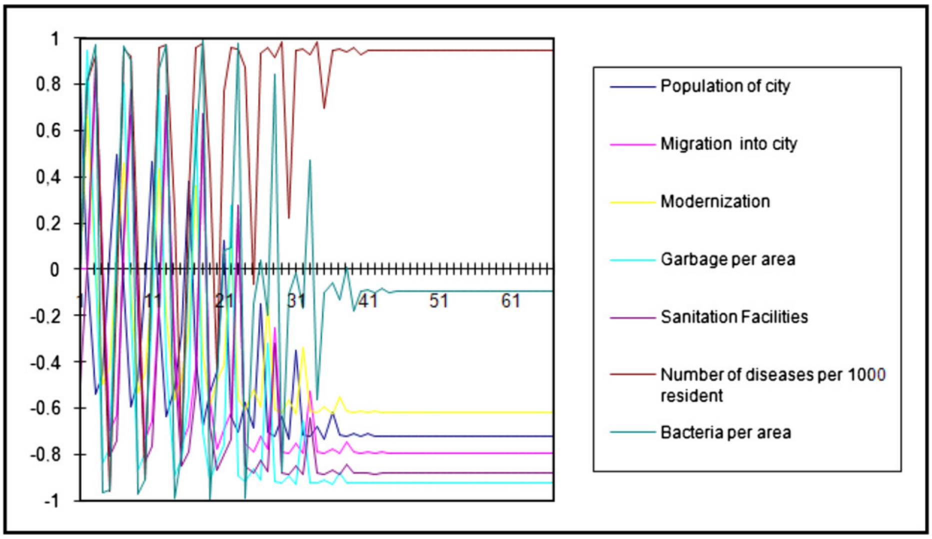

Fig. 10. Transition phase of modified sigmoid FCM (w_{13} = 0.4) having initial state. Population of city: $A_1^0 = 0.8$, Sanitation Facilities: $A_5 = -0.5$.

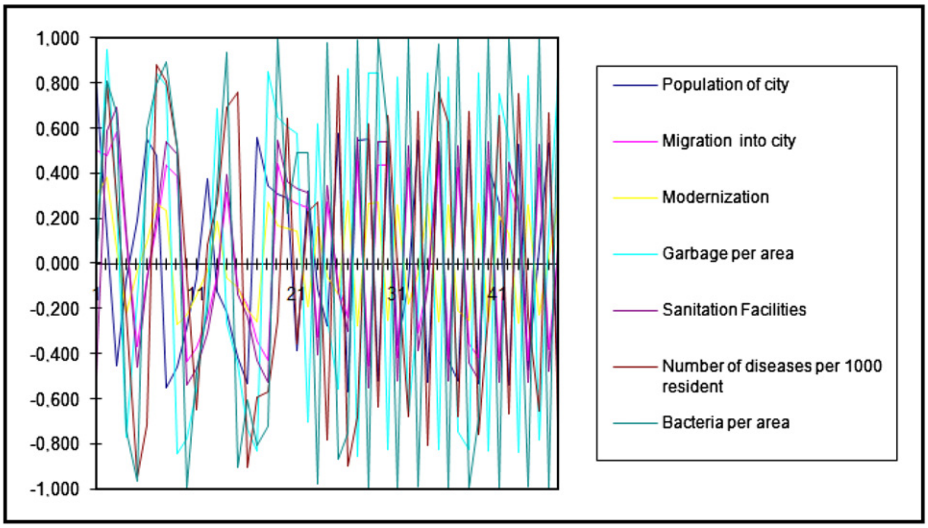

Fig. 11. Transition phase of modified sigmoid FCM (w_{13} = 0.2) having initial state. Population of city: $A_1^0 = 0.8$, Migration into city: $A_2^0 = 0.5$ and Modernization: $A_3^0 = 0.3$.

This final state is exactly the opposite of that of scenario of Fig. 9, in the same way that the final state of Fig. 7 is the exactly the opposite of that of scenario of Fig. 6.

If we change the weight w_{13} from 0.4 to 0.2 and introduce the scenario of Fig. 6, the dynamic behavior of FCM is that Fig. 11.

Now the system does not reach an equilibrium point but it enters into a periodic/limit cycle behavior with all the concepts – variables of the problem to periodically increase and decrease. This means that if strategically, the degree that C1: "Population of city" influences C3: "Modernization" is decreased from w_{13} = 0.6 to w_{13} = 0.2, then the city will get into a periodic change of the variables represented in the FCM, indicating disorder in the concepts of the FCM concerning the city.

8. Summary – conclusions

After introducing FCMs and presenting some industrial/engineering FCM applications, their dynamic behavior is discussed. The inference procedure for Binary, Trivalent and Sigmoid FCMs is examined thoroughly. Various scenarios have been imposed and simulated and conclusions are drawn based on the final state that FCM reached. The FCM structure was proved useful for making inference, especially in cases of problems with increased uncertainty and fuzziness.

Comparing the inference capabilities of the three types of FCMs, the following conclusions can be drawn:

(i) Binary, trivalent and sigmoid FCMs have inference capabilities.
(ii) Binary FCMs can only represent an increase of a concept or represent a stable concept but lack the capability of representing a decrease of a concept.
(iii) Trivalent and sigmoid FCMs can represent increase or decrease of a concept and also represent a stable concept.
(iv) Binary and trivalent FCMs can not represent the degree of an increase or a decrease of a concept.
(v) Sigmoid FCMs, by allowing neuron's activation level to take values from the whole interval $[-1,1]$, can represent also the degree of an increase or a decrease of a concept.
(vi) In sigmoid FCMs, small changes to their initial state can lead to a dramatic change to the final state of the FCM.
(vii) In sigmoid FCMs, small changes to the weight of an arc connecting two FCM's concepts, can lead to a change of the dynamic behavior of FCM from that of a reaching a equilibrium point to that of reaching a limit cycle behavior and vice versa.

Furthermore, these rules can be followed in order the right type of FCM to be chosen:

(i) Binary FCMs are suitable for highly qualitative problems where only representation of increase or stability of a concept is required.
(ii) Trivalent FCMs are suitable for qualitative problems where representation of increase, decrease or stability of a concept is required.
(iii) Sigmoid FCMs are suitable for qualitative and quantitative problems where representation of a degree of increase, a degree of decrease or stability of a concept is required and strategic planning scenarios are going to be introduced.

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