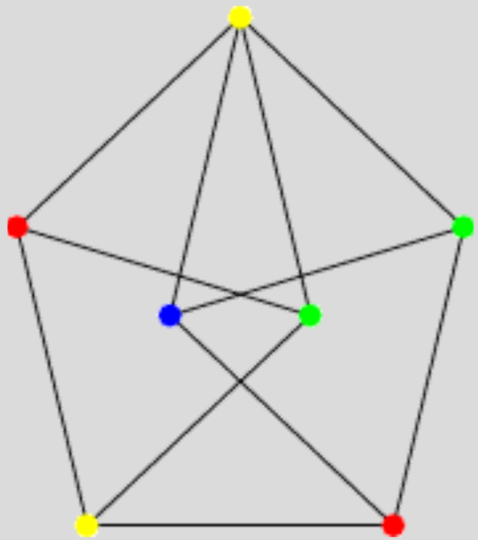
THE MOSER SPINDLE.

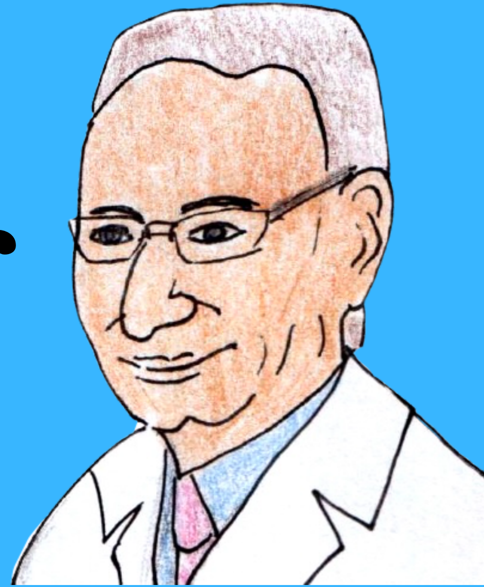


Daniel Cohen
Yiwen Wang
Golrokh Nouri
Sahar Murtaza

THE YEAR IS 1924: A young Austrian couple named Laura Feurstein and Robert Moser, decide to immigrate to Winnipeg, Canada. Little did they know that their Children, 3 year old, Leo and his soon to be younger brother William would grow up to make major contributions to many fields of mathematics, namely number theory and group theory.

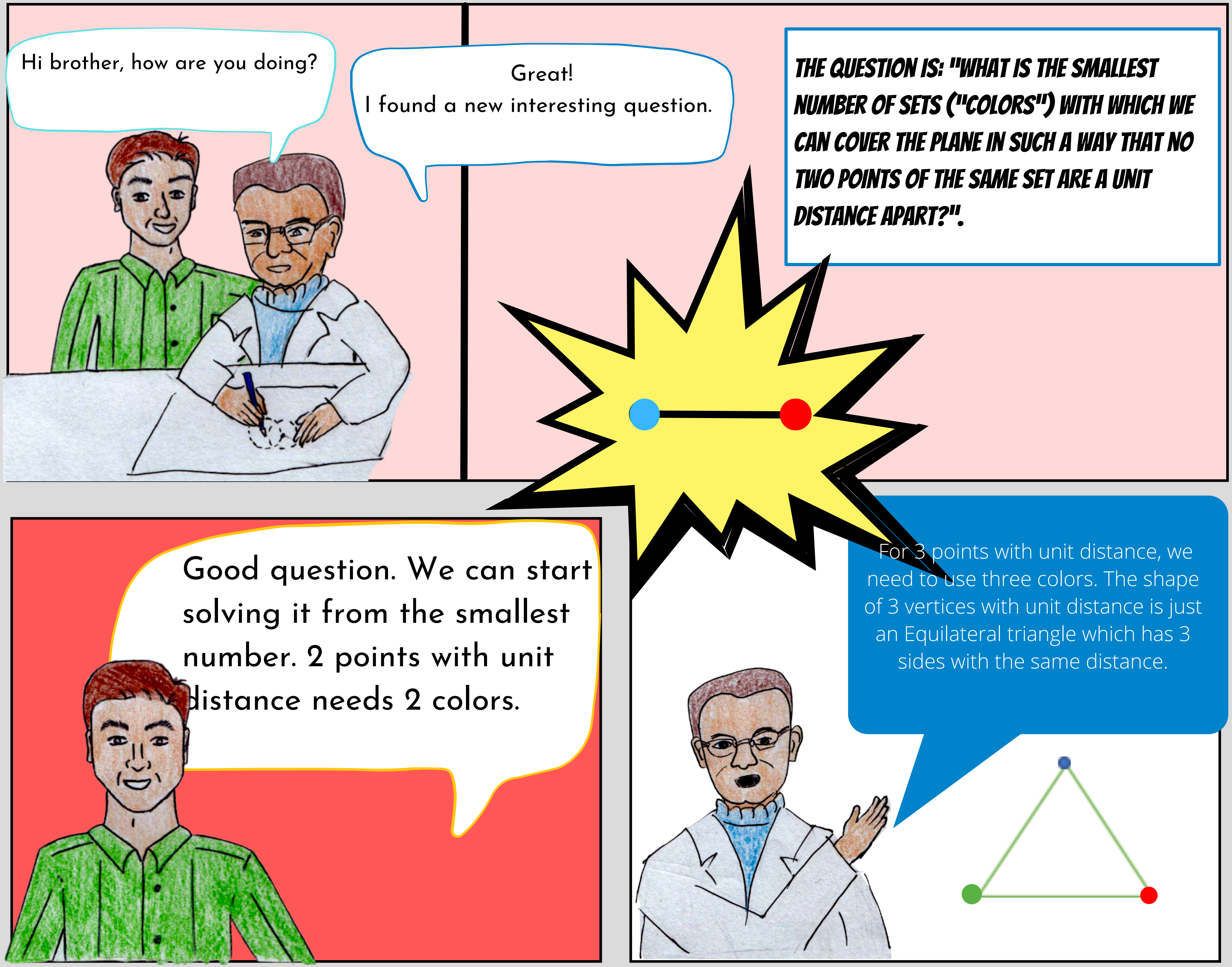
Leo Moser

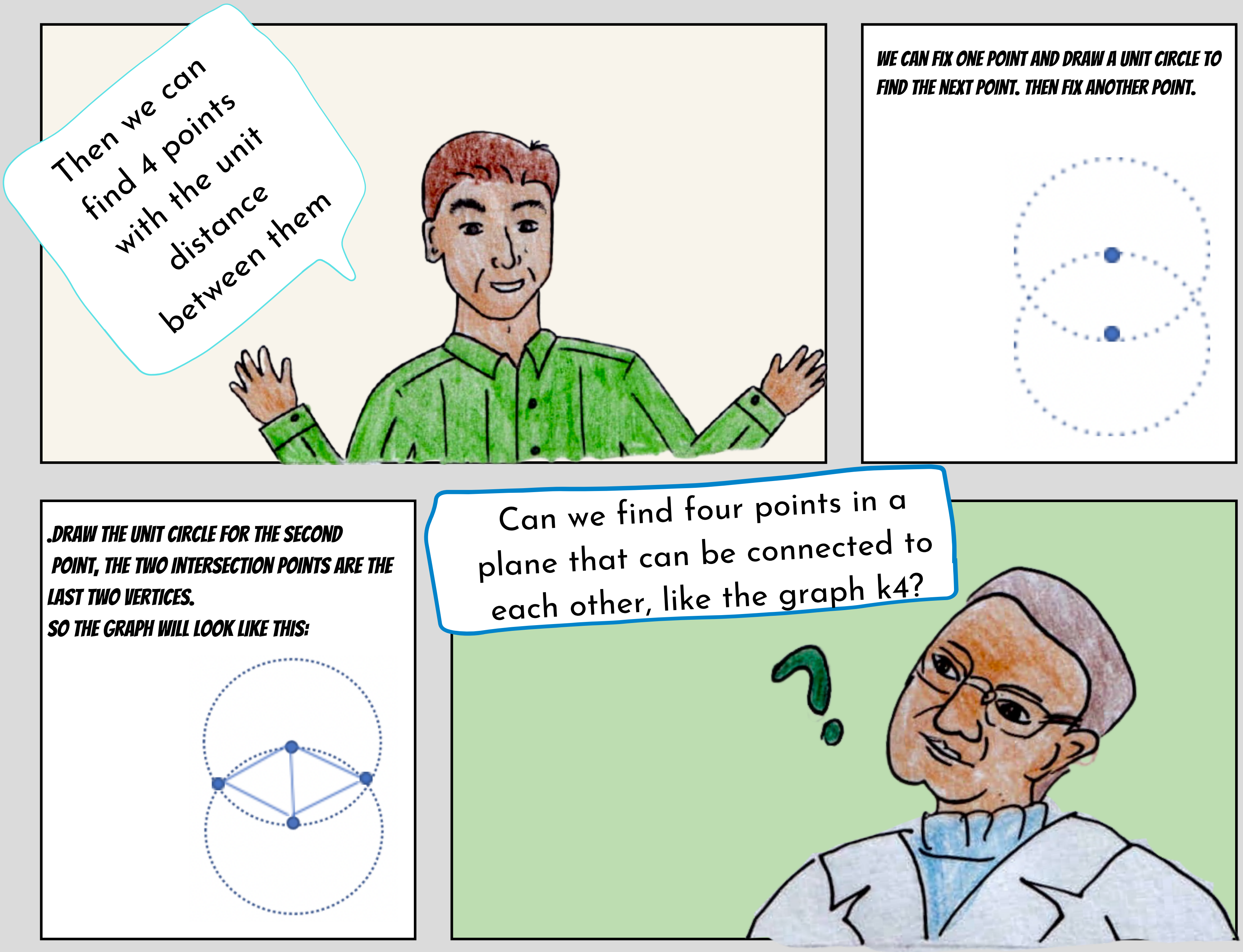
1921-1970



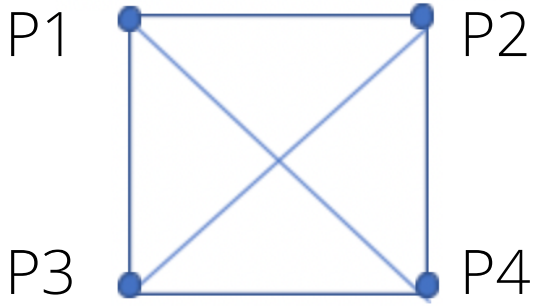
William Moser

1927-2009

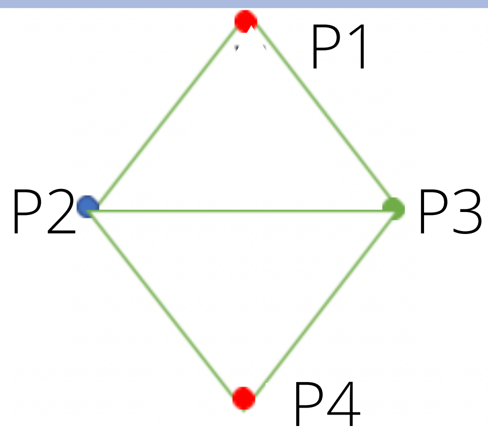

Hi brother, how are you doing?
Great! I found a new interesting question.
THE QUESTION IS: "WHAT IS THE SMALLEST NUMBER OF SETS ("COLORS") WITH WHICH WE CAN COVER THE PLANE IN SUCH A WAY THAT NO TWO POINTS OF THE SAME SET ARE A UNIT DISTANCE APART?".
Good question. We can start solving it from the smallest number. 2 points with unit distance needs 2 colors.
For 3 points with unit distance, we need to use three colors. The shape of 3 vertices with unit distance is just an Equilateral triangle which has 3 sides with the same distance.

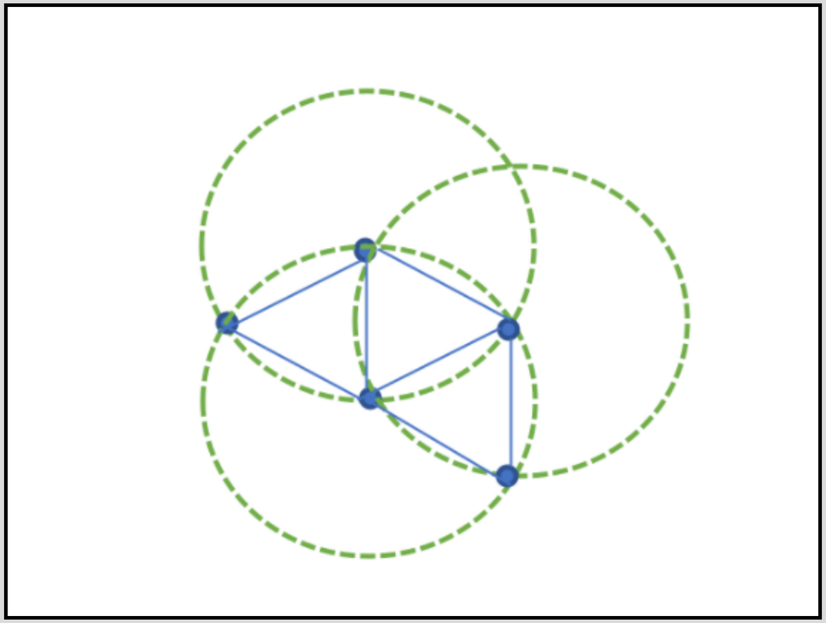

Then we can find 4 points with the unit distance between them
WE CAN FIX ONE POINT AND DRAW A UNIT CIRCLE TO FIND THE NEXT POINT. THEN FIX ANOTHER POINT.
.DRAW THE UNIT CIRCLE FOR THE SECOND POINT, THE TWO INTERSECTION POINTS ARE THE LAST TWO VERTICES.
SO THE GRAPH WILL LOOK LIKE THIS:
Can we find four points in a plane that can be connected to each other, like the graph k4?

let two points be friends if their connecting edge is one unit in length. Then we draw the complete graph K4 where we assume the edges P1P2, P2P3 P3P4 and P4P1 are one unit in length. Then by the Pythagorean theorem, the two diagonals P2P3 and P1P4 must be of length sqrt(2). If we let one diagonal be 1, the other diagonal must then be larger than 1. Therefore, 4 points in the plane cannot be friends and there must be a pair of points that are not connected.

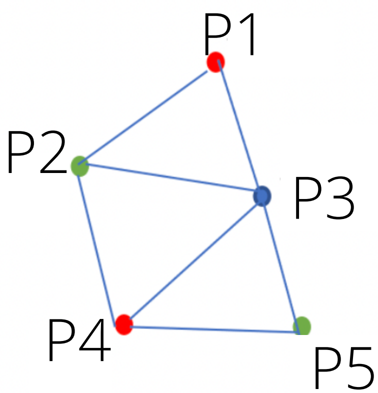


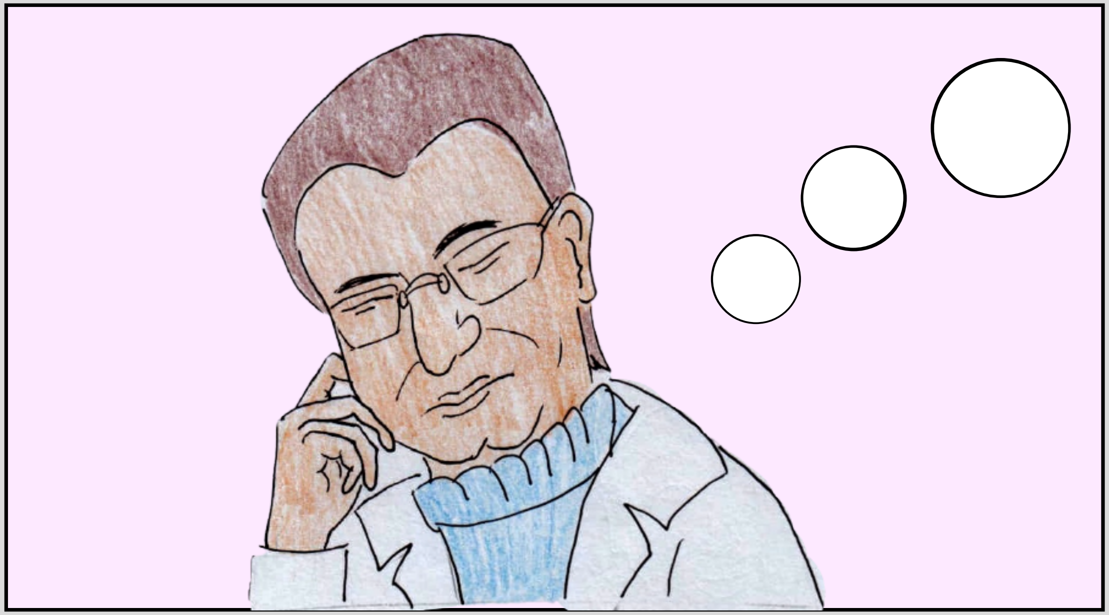
I Agree with you. If we color P1 with red, then P2 can be blue and P3 can be green. However, P4 can have the same color as P1 because P4 does not connect with P1.

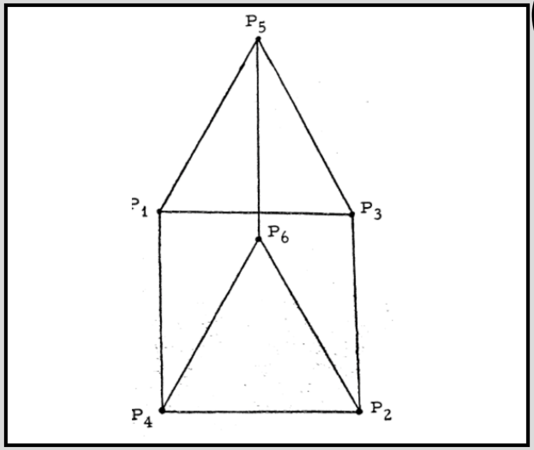




TO ADD THE FIFTH POINT IN THE 4 VERTICES GRAPH, THE FIFTH VERTEX CAN CONNECT WITH TWO OF THE 4 VERTICES.

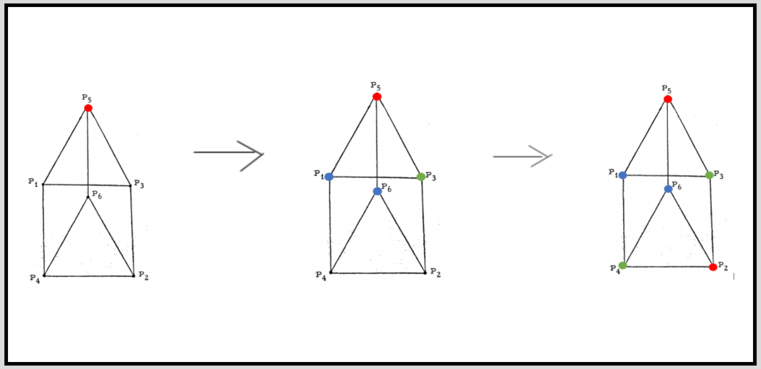




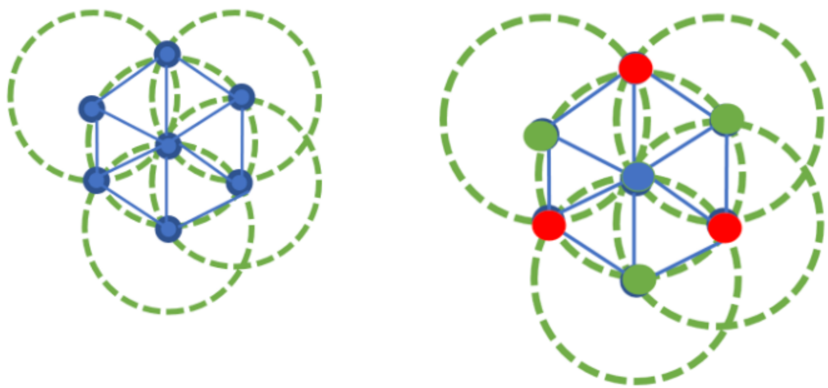


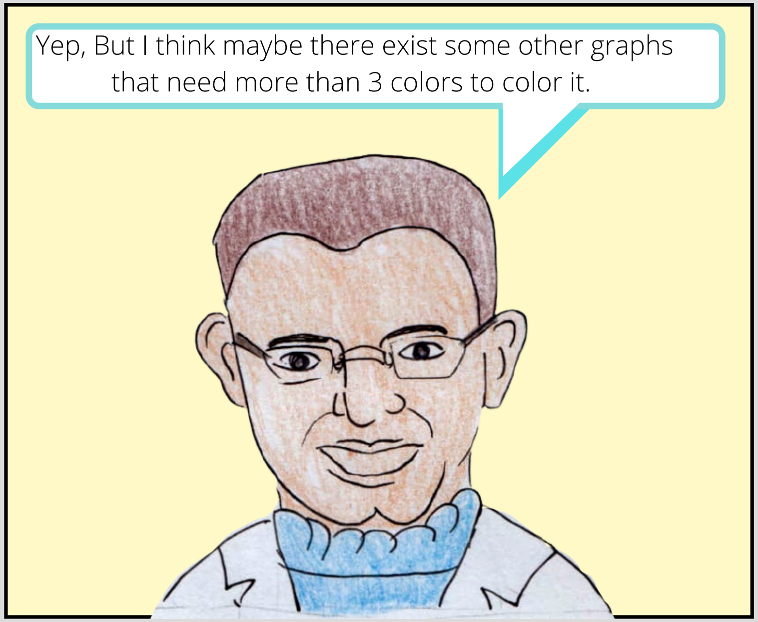
How about 6 vertices in the plane?
We can draw the graph like that.

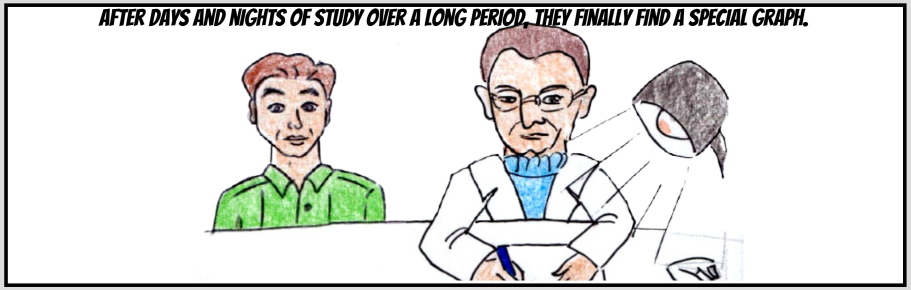
Since the graph is symmetric, we can start coloring the graph from P5. If we color P5 red, P1 blue, P6 blue, and P3 must be green. Then P4 can be either green or red. However, if P4 is red, then P2 should be colored with a new color. So we color P4 green, and P2 can be colored red. So any set of six points permits a proper 3-coloring.



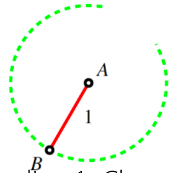
I am interested in 7 vertices now. How can we draw a unit distance graph on seven vertices? We can have 6 vertices around 1 vertex. Since 3 vertices can create an Equilateral triangle, with 3 angles are all 60 degrees. 6 Equilateral triangle can cover 6*60 = 360 degree. The graph is valid and we can use 3 colors to color it.







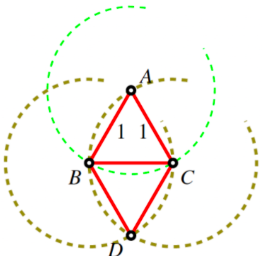
step 1



Start by drawing a circle with centre A and radius 1. Choose a point B on the perimeter of this circle.

step 2

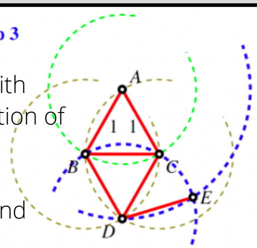
Draw a circle with centre B and radius 1. Let C be the intersection of these circles. Then draw a circle with centre C and radius 1.



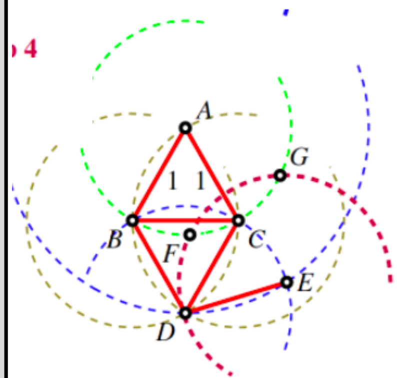
step 3

Contuing this way, drawing circle with radius 1 and centre at the intersection of the two newest cirles...

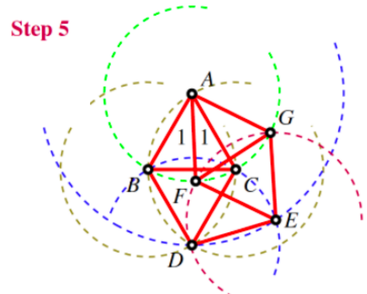
Draw a circle with the center at A and passing through the point D



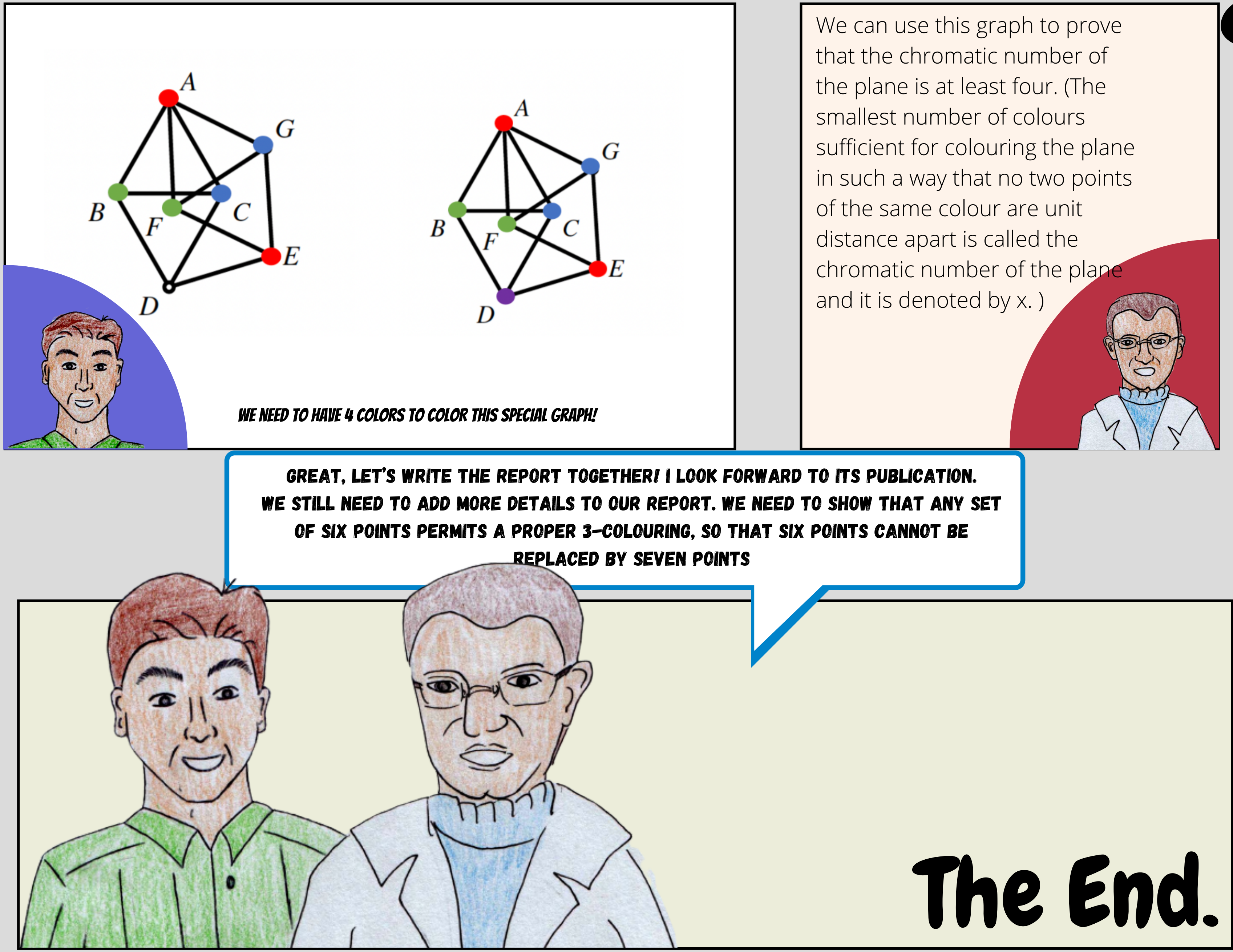
step 4



step 5



We construct a new graph where all the edges coloured in red are unit distance apart.


A G B F C E D
A G B F C E D
WE NEED TO HAVE 4 COLORS TO COLOR THIS SPECIAL GRAPH!
We can use this graph to prove that the chromatic number of the plane is at least four. (The smallest number of colours sufficient for colouring the plane in such a way that no two points of the same colour are unit distance apart is called the chromatic number of the plane and it is denoted by x.)
GREAT, LET'S WRITE THE REPORT TOGETHER! I LOOK FORWARD TO ITS PUBLICATION. WE STILL NEED TO ADD MORE DETAILS TO OUR REPORT. WE NEED TO SHOW THAT ANY SET OF SIX POINTS PERMITS A PROPER 3-COLOURING, SO THAT SIX POINTS CANNOT BE REPLACED BY SEVEN POINTS
The End.

Reference:

Leo Moser. (2021, November 8). Wikipedia. Retrieved November 29, 2021, from https://en.wikipedia.org/wiki/Leo_Moser

Moser spindle. (2021, July 6). Wikipedia. Retrieved November 29, 2021, from https://en.wikipedia.org/wiki/Moser_spindle

O'Connor, J.J., & Robertson, E.F. (2006, November). *Leo Moser*. Maths History. Retrieved November 29, 2021, from https://mathshistory.st-andrews.ac.uk/Biographies/Moser_Leo/

O'Connor, J.J., & Robertson, E.F. (2008, July). *William Moser*. Maths History. Retrieved November 29, 2021, from https://mathshistory.standrews.ac.uk/Biographies/Moser_William/

Jungić, V., (2020), *An introduction of the problem of finding the chromatic number of the plane*, Part I, Crux Mathematicorum, Vol. 45(8), 390-396.