OPMT 7701 Formula sheet

if $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\ln(AB) = \ln A + \ln B, \qquad \ln(A/B) = \ln A - \ln B, \qquad \ln A^b = b \ln A$

$\frac{x^a}{x^b} = x^{a-b} \qquad \frac{1}{\sqrt{x}} = x^{-1/2} \qquad \left(\frac{xy}{z}\right)^{1/2} = \frac{x^{1/2}y^{1/2}}{z^{1/2}}$

If $y = f(x) \cdot g(x)$ then $y' = f'g + fg'$

Profits: $\pi = TR - TC$

if $y = \frac{f(x)}{g(x)}$ then $y' = \frac{f'g - fg'}{g^2}$

$MP_L = \partial Q/\partial L$ and $MP_K = \partial Q/\partial K$

if $y = f(g(x))$ then $y' = f'(g) \cdot g'(x)$

$TR = P \times Q$ and $MR = dTR/dQ$

If $y = e^{f(x)}$ then $y' = e^{f(x)} \cdot f'(x)$

$TC = AC \times Q$ and $MC = dTC/dQ$

if $y = \ln(f(x))$ then $y' = \dfrac{1}{f(x)} \cdot f'(x)$

$u = \ln f(x, y)$
$u_x = \frac{1}{f(x,y)} f_x$ and $u_y = \frac{1}{f(x,y)} f_y$

$u = e^{f(x,y)}$
$u_x = e^{f(x,y)} f_x$ and $u_y = e^{f(x,y)} f_y$

MRTS: if $Q = f(K, L)$ then $MRTS = \frac{Q_L}{Q_K} = \dfrac{\partial Q/\partial L}{\partial Q/\partial K}$

MRCS: if $U = U(x, y)$ then $MRCS = \dfrac{U_x}{U_y} = \dfrac{\partial U/\partial x}{\partial U/\partial y}$

If $u = f(x,y)g(x,y)$ then $u_x = f_x g + f g_x$ and $u_y = f_y g + f g_y$

if $u = \dfrac{f(x,y)}{g(x,y)}$ then $u_x = \dfrac{f_x g - f g_x}{g^2}$ and $u_y = \dfrac{f_y g - f g_y}{g^2}$

if $u = f(g(x,y))$ then $u_x = f'(g(x,y))g_x$ and $u_y = f'(g(x,y))g_y$

if $u = [f(x,y)]^n$ then $u_x = n[f(x,y)]^{n-1} f_x$ and $u_y = n[f(x,y)]^{n-1} f_y$

Two variable optimization:
$U = f(x, y)$

First Order Conditions $f_x = 0$ and $f_y = 0$

Second Order Conditions

Max:
$f_{xx} < 0, \quad f_{yy} < 0$

$f_{xx} \cdot f_{yy} - (f_{xy})^2 > 0$

Min:
$f_{xx} > 0, \quad f_{yy} > 0$

$f_{xx} \cdot f_{yy} - (f_{xy})^2 > 0$

Cobb-Douglas

$$\begin{aligned} Q &= AK^a L^b \\ Q_K &= aAK^{a-1} L^b \\ Q_L &= bAK^a L^{b-1} \end{aligned}$$