OPMT 5701, Fall 2006

Multiple Constraints
Lab Assignment 11

1. Consider the case of a two-good world where both goods, x and y. are rationed. Let the consumer, Myrtle, have the utility function $U = U(x, y)$. Myrtle has a fixed money budget of B and faces the money prices P_x and P_y. Further, Myrtle has an allotment of coupons, denoted C, which can be used to purchase both x or y at a coupon price of c_x and c_y. Therefore Myrtle's maximization problem is

   Maximize
   $$U = U(x, y)$$
   Subject to
   $$\begin{aligned} B &\geq P_x x + P_y y \\ C &\geq c_x x + c_y y \end{aligned}$$
   and the non-negativity constraint $x \geq 0$ and $y \geq 0$.

   Suppose, for the budget, $B = 12$, $P_x = P_y = 1$ and for the coupons $C = 24, c_x = 4, c_y = 1$. Find the optimal x and y , value for U and which constraints are binding if Myrtle's utility function is:
   The Constraints are
   $$\begin{aligned} 12 &= x + y \\ 24 &= 4x + y \end{aligned}$$
   The Lagrange is
   $$\begin{aligned} L &= U(x, y) + \lambda_1 (12 - x - y) + \lambda_2 (24 - 4x - y) \\ L_x &= U_x - \lambda_1 - 4\lambda_2 = 0 \\ L_y &= U_y - \lambda_1 - \lambda_2 = 0 \\ L_{\lambda 1} &= 12 - x - y \geq 0 \\ L_{\lambda 2} &= 24 - 4x - y \geq 0 \end{aligned}$$

   (a) $U = xy$
   $$U_x = y, U_y = x$$
   Sub into FOC's. Solution: Both constraints are binding $x = 4, y = 8$

   (b) $U = x^2 y$
   $$U_x = 2xy, U_y = x^2$$
   Sub into FOC's. Solution: only Coupon constraint is binding $x = 4, y = 8$

   (c) $U = \ln x + 2 \ln y$
   $$U_x = 1/x, U_y = 2/y$$
   Sub into FOC's. Solution: only Budget constraint is binding $x = 4, y = 8$
   SEE FIGURE ONE: each case is illustrated showing the tangencies that Violate and the solutions

2. Skippy lives on an island where she produces two goods, x *and* y, according the the production possibility frontier $400 \geq x^2 + y^2$, and she consumes all the goods herself. Skippy also faces and environmental constraint on her total output of both goods. The environmental constraint is given by $x + y \leq 24$. Her utility function is
   $$u = xy$$

   (a) Write down the Kuhn Tucker first order conditions.

   (b) Find Skippy's optimal x *and* y. Identify which constaints are binding. **Environmental Constraint is Binding**$x = y = 12$

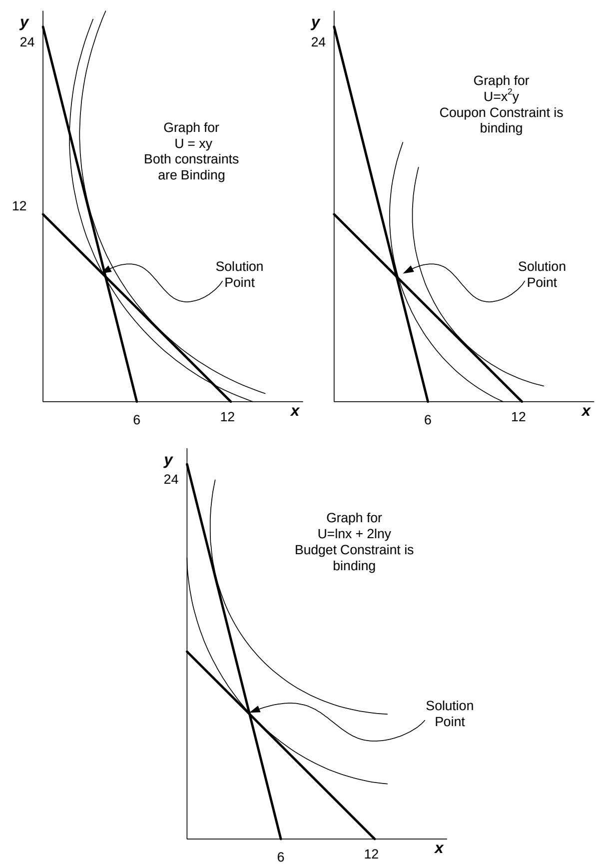

Figure 1: The 3 cases for Question 1

(c) Graph your results.

(d) On the next island lives Sparky who has all the same constraints as Skippy but Sparky's utility function is $u = xy^3$. Redo a, b, and c for Sparky

3. An electric company is setting up a power plant in a foreign country and it has to plan its capacity. The peak period demand for power is given by $p_1 = 400 - q_1$ and the off-peak is given by $p_2 = 380 - q_2$. The variable cost to is 20 per unit (paid in both markets) and capacity costs 10 per unit which is only paid once and is used in both periods.

   (a) write down the lagrangian and Kuhn-Tucker conditions for this problem

$$\begin{aligned}
L &= (400 - q_1)q_1 + (380 - q_2)\, q_2 - 20q_1 - 20q_2 - 10K + \lambda_1(K - q_1) + \lambda_2(K - q_2) \\
L_{q1} &= 400 - 2q_1 - 20 - \lambda_1 = 0 \\
L_{q1} &= 380 - 2q_2 - 20 - \lambda_2 = 0 \\
L_K &= -10 + \lambda_1 + \lambda_2 = 0 \\
L_{\lambda 1} &= K - q_1 \geq 0 \\
L_{\lambda 2} &= K - q_2 \geq 0
\end{aligned}$$

   (b) Find the optimal outputs and capacity for this problem.

$$\begin{aligned}
\lambda_2 &= 0, \lambda_1 = 10 \\
q_1 &= \frac{380 - \lambda_1}{2} = 185 = K \\
q_2 &= \frac{360 - \lambda_2}{2} = 180
\end{aligned}$$

   (c) How much of the capacity is paid for by each market (i.e. what are the values of λ_1 and λ_2)? See (a)

   (d) Now suppose capacity cost is 30 per unit (paid only once). Find quantities, capacity and how much of the capacity is paid for by each market (i.e. λ_1 and λ_2)?

$$\begin{aligned}
\lambda_2 + \lambda_1 &= 30, q_1 = q_2 = q \\
q &= \frac{380 - \lambda_1}{2} = 177.5 \\
q &= \frac{360 - (30 - \lambda_1)}{2} = 177.5 = K
\end{aligned}$$