DEMAND THEORY I

THEORY OF CONSUMER CHOICE

Ingredients:

- preferences (utility Function)
- constraint (Budget set)
- Prices

Consumer's problem:

- Max utility subject to budget constraint

characteristics of Solution:

- A set of Demand Functions that satisfy the following:
  - ↳ Budget exhaustion (Non-satiation)
  - ↳ $\underbrace{\text{Psychic tradeoff}}_{\text{MRS}} = \underbrace{\text{Monetary tradeoff}}_{\text{relative price}}$ (for most problems)

Budget Constraint

$Y \equiv$ consumer Income (per period)

$q_i \equiv$ qty of good i (per period)

$p_i \equiv$ price of good i

n-good case:

$$\boxed{p_1 q_1 + p_2 q_2 + \dots + p_n q_n = \sum_{i=1}^{n} p_i q_i \leq Y}$$

$p_i q_i$ = exp. on good i (per period)

$\sum p_i q_i$ = sum of exp. on all n-goods (per period)

$$\underbrace{\sum_{i=1}^{n} p_i q_i}_{\text{total exp. per period}} \leq \underbrace{Y}_{\text{Total Income per period}}$$

* Main drawback of n-good case: We can't draw it!

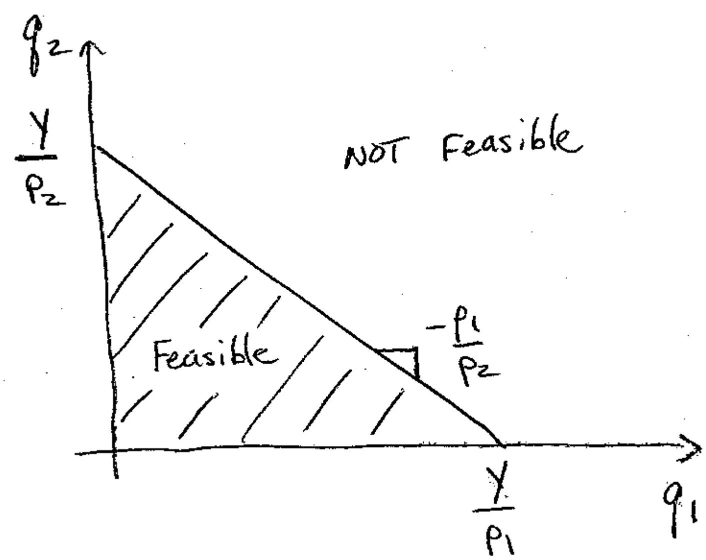
2-good case

$$\boxed{p_1 q_1 + p_2 q_2 \leq Y}$$

Total Exp. on goods 1 & 2 cannot exceed Total income devoted to goods 1 & 2.

Rewrite Budget Constraint:

$$\boxed{q_2 \leq \frac{Y}{p_2} - \frac{p_1}{p_2} q_1}$$



Intercept

$q_1 = 0 \Rightarrow q_2 = \frac{Y}{p_2}$

Slope

$$\left.\frac{dq_2}{dq_1}\right|_{Y=Y_0} = -\frac{p_1}{p_2}$$

p_1, p_2 and Y are exogenous variables

q_1 and q_2 are endogenous variables

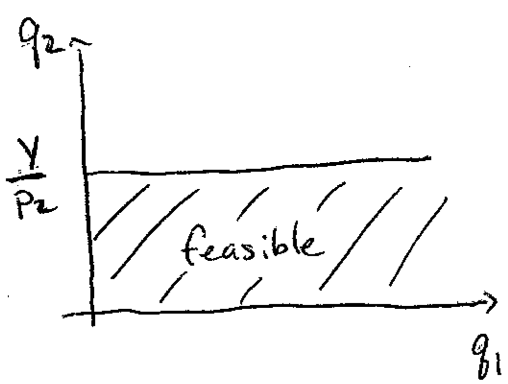
Properties

(i) Bounded

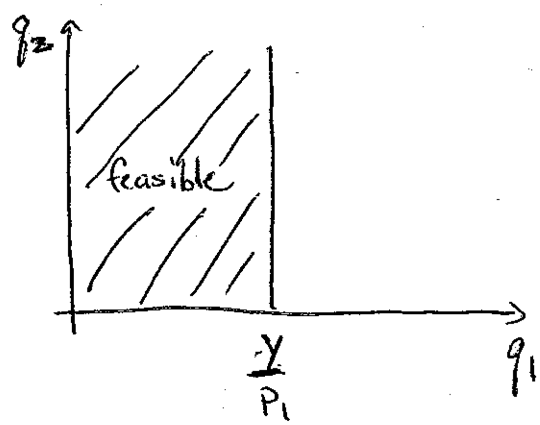
from below: $q_i \geq 0$ for $i = 1, 2$ (non-negative)

from above: $Y > 0$ and $p_i \neq 0$ for $i = 1, 2$

↳ we rule out cases where p_1 or p_2 are zero.



$p_1 = 0 \Rightarrow q_1$ is "free"



$p_2 = 0 \Rightarrow q_2$ is "free"

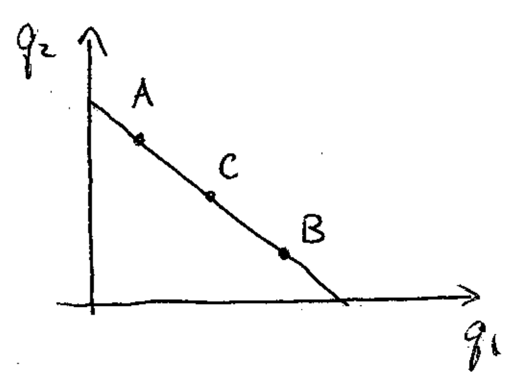
(ii) Closed

any bundle (q_1, q_2) __on__ budget line is feasible.

(iii) Weakly Convex

If bundles A and B are on budget line, then so is

$C = \alpha A + (1-\alpha) B$



Interpretation

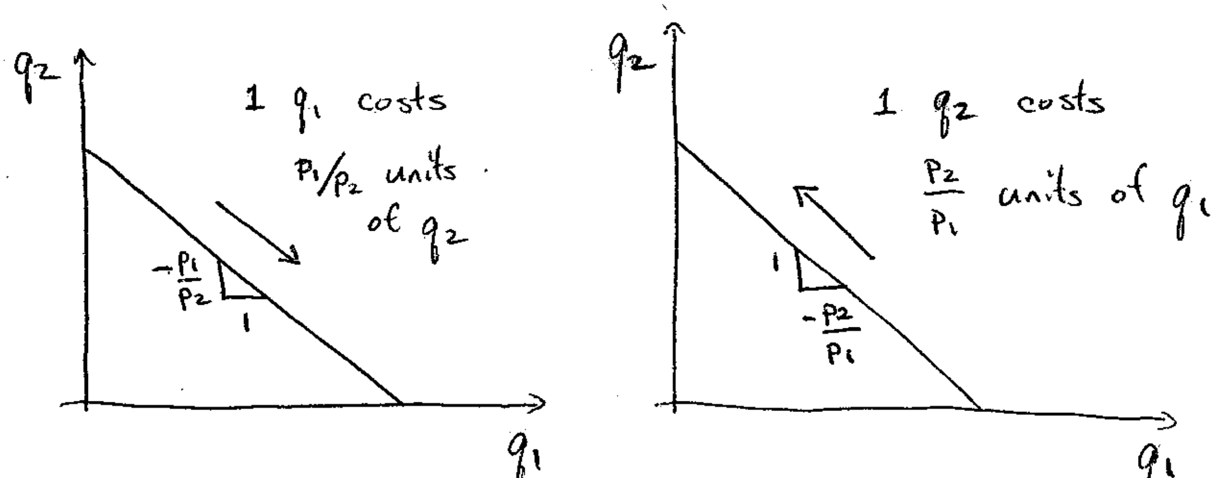
Budget line $q_2 \leq \frac{Y}{p_2} - \frac{p_1}{p_2} q_1$ tell us two important parameters for explaining behavior.

(i) $\frac{Y}{p_i}$ Real Income (measured in terms of good i)

↳ You don't eat money; only the goods money buys.

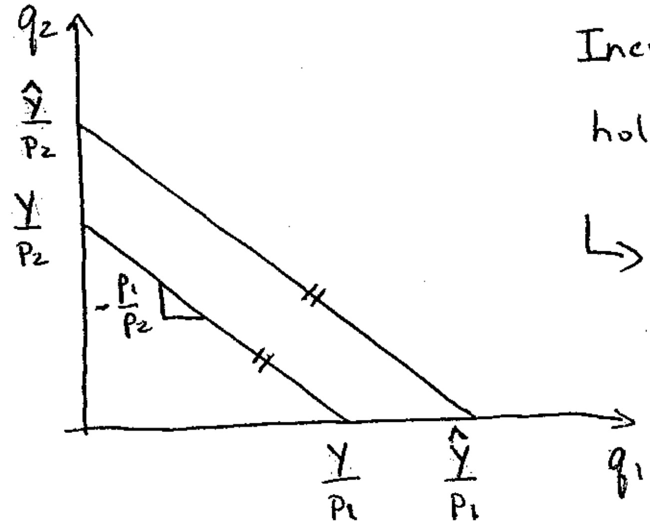
(ii) $\frac{p_i}{p_j}$, $i \neq j$ Relative Prices

↳ measures opportunity cost of q_i in terms of units of q_j



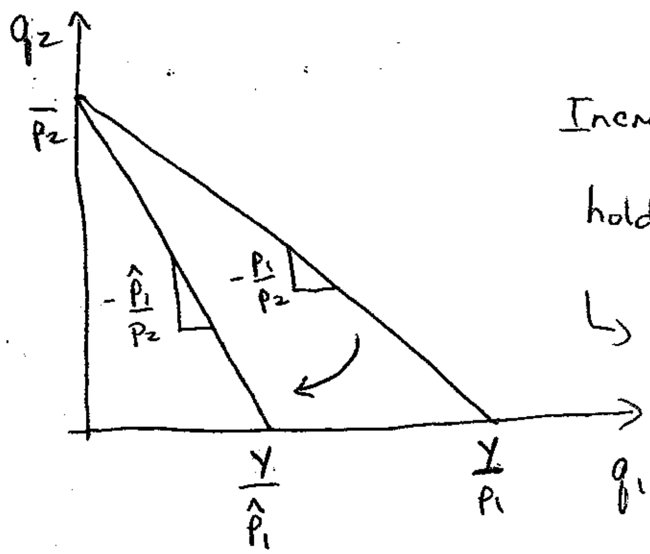
Important Point: It's real income and relative prices that matter, not nominal income and nominal prices.

Change in Exogenous Variables



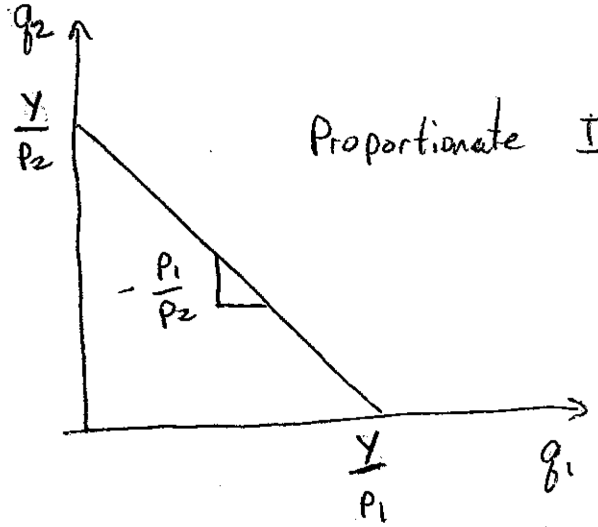
Increase in income ($\hat{Y} > Y$) holding p_1 and p_2 constant.

↳ feasible set expands.



Increase in price of good 1 ($\hat{p}_1 > p_1$) holding Y and p_2 constant.

↳ feasible set contracts (along q_1)



Proportionate Increase in p_1, p_2 and Y.

↳ feasible set unaffected

↳ No money illusion.

Macro Example

Let $h = 24 \equiv$ hours of available time / day

$n \equiv$ hours employed / day

$\ell \equiv$ hours of leisure / day

$\boxed{\text{Time Constraint: } 24 = n + \ell}$

Let $c \equiv$ consumption / period

$w \equiv$ wage rate / hour worked

$\boxed{\text{Budget Constraint: } c \leq wn}$

* Combine the Time and Budget Constraint as follows:

Since $24 = n + \ell \Rightarrow n = 24 - \ell$

Thus,

$$\boxed{c \leq w(24 - \ell)}$$

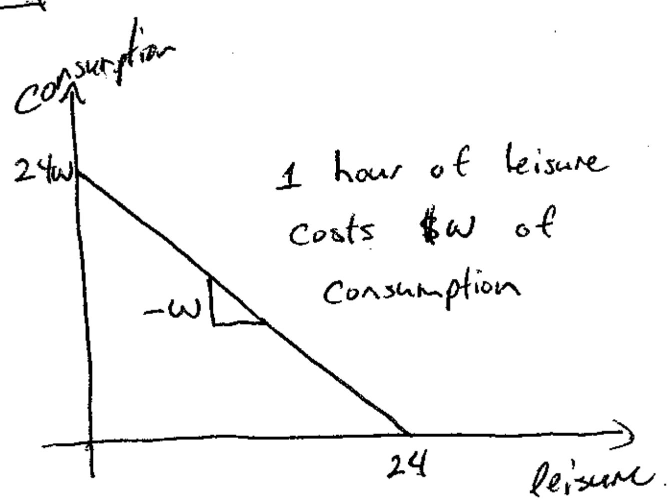
intercepts:

if $\ell = 0 \Rightarrow c \leq 24w$

if $c = 0 \Rightarrow \ell \leq 24$

slope: $c \leq 24w - w\ell$

$\Rightarrow -w$

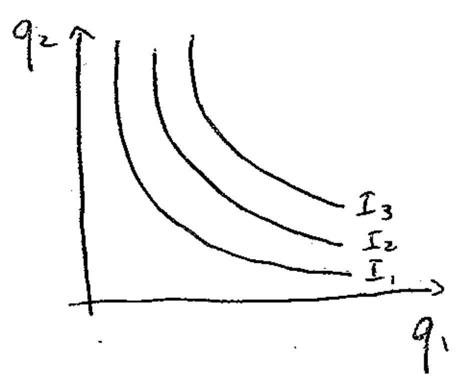


Utility Maximization

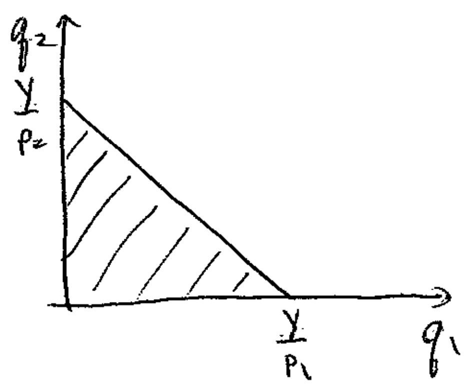
* Consumer's problem is to choice the best bundle of goods he/she can afford.

* The best bundle is the one which maximizes utility over all affordable bundles.

Graphical Solution

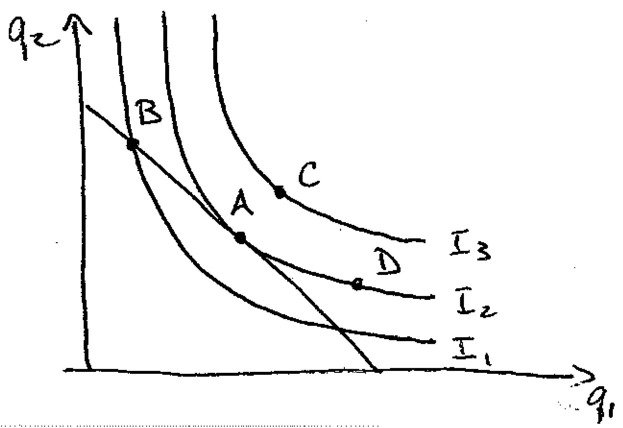


Preferences are defined in $q_1 q_2$-plane



Budget set is defined in $q_1 q_2$-plane.

* Put Indifference Map together with Budget constraint to find utility maximizing bundle.



Preference ordering:

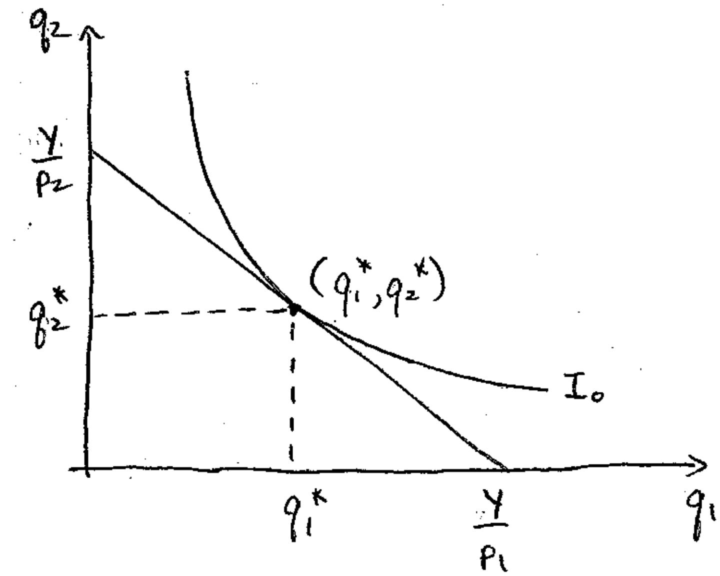
$C \succ A, \; A \sim D, \; D \succ B$

Solution is Bundle A!

What's wrong with other bundles?

Interior v. Corner Solutions

Interior Solution – Typical case (Essential Goods)



Characteristics of solution (q_1^*, q_2^*):

(1) $\boxed{p_1 q_1^* + p_2 q_2^* = Y}$

↳ solution is on the budget line.

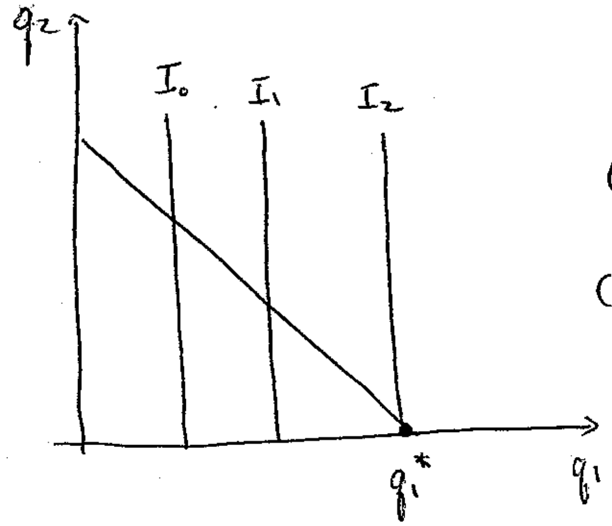
↳ follows from Non-satiation (More is preferred to less) and Maximization (More chosen over less)

(2) $$\boxed{\left.\frac{dq_2}{dq_1}\right|_{u=u_0} = \left.\frac{dq_2}{dq_1}\right|_{Y=Y_0}}$$

↳ slope of tangent line to I_0 (MRS) equals slope of budget line (price ratio p_1/p_2)

↳ rate at which consumer is willing to exchange q_1 for q_2 equals rate at which they can exchange q_1 for q_2 (determined in the market)

Corner Solutions (Inessential Goods)



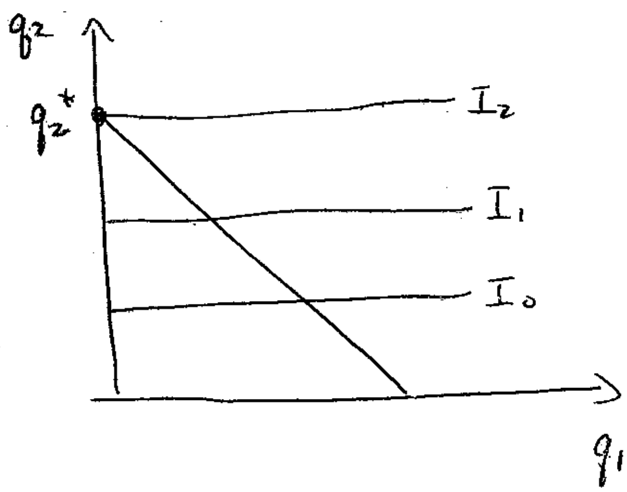
characteristics for $(q_1, q_2) = (q_1^*, 0)$

(1) $\boxed{P_1 q_1^* = Y}$

(2) $\boxed{\left.\dfrac{dq_2}{dq_1}\right|_{u=u_0} < \left.\dfrac{dq_2}{dq_1}\right|_{Y=Y_0}}$

These are -'ve #'s
If we take absolute values it reverses the inequality

Condition (2) can be rewritten as: $MRS(q_1, q_2) > P_1/P_2$



(1) $\boxed{P_2 q_2^* = Y}$

(2) $\boxed{\left.\dfrac{dq_2}{dq_1}\right|_{u=u_0} > \left.\dfrac{dq_2}{dq_1}\right|_{Y=Y_0}}$

↳ $MRS < P_1/P_2$

* preference for one good over the other is sufficiently strong so that we never buy one of the goods. Must be the case for many goods we don't buy.

Utility Maximization

Mathematical Solution

The consumer's problem is to choose a pair of quantities of good's 1 and 2 in order to maximize utility given income and prices.

Notation

Consumer's choice problem:

$$\max_{\{q_1, q_2\}} u(q_1, q_2) \quad \text{s.t.} \quad p_1 q_1 + p_2 q_2 = Y$$

Indicates what consumer has choice over.

"subject to" or "such that"

Invoking non-satiation

* $u(q_1, q_2)$ is called the objective function

* $p_1 q_1 + p_2 q_2 = Y$ is the constraint.

The solution to this choice problem is a pair of individual demand functions, denoted by $q_1^*(p_1, p_2, Y)$ and $q_2^*(p_1, p_2, Y)$

→ For a graph of this problem in $\mathbb{R}^3$, please see Math notes.

Solution to Choice Problem

(1) Set up the Lagrangian:

$$\boxed{L(q_1, q_2, \lambda) = u(q_1, q_2) + \lambda\,(Y - p_1 q_1 - p_2 q_2)}$$

(2) Write down the set of first-order conditions (FOC's) and set them equal to zero.

$$\left.\begin{array}{ll} \text{(i)} & L_{q_1}(q_1, q_2, \lambda) = U_{q_1} - \lambda p_1 = 0 \\ \text{(ii)} & L_{q_2}(q_1, q_2, \lambda) = U_{q_2} - \lambda p_2 = 0 \\ \text{(iii)} & L_{\lambda}(q_1, q_2, \lambda) = Y - p_1 q_1 - p_2 q_2 = 0 \end{array}\right\}$$

System of 3 equations in 3 unknowns.

(3) Solve for λ in (i) and (ii) and set them equal.

This gives:

$$\frac{U_{q_1}}{p_1} = \frac{U_{q_2}}{p_2} \quad \text{or} \quad \boxed{\frac{U_{q_1}}{U_{q_2}} = \frac{p_1}{p_2}} \text{ (iv)}$$

* The LHS is MRS (or psychic tradeoff) of q_1 for q_2
* The RHS is the price ratio, or monetary tradeoff of q_1 for q_2.

Equations (iv) and (iii) comprise a system of 2 equations in 2 unknowns that can be solved by <u>substitution</u>.

Interpretation of λ:

At the solution of the consumer's problem (i.e., an interior solution), the following condition holds:

$$\boxed{\frac{u_{q_1}}{p_1} = \frac{u_{q_2}}{p_2} = \lambda}$$

says at utility MAX pair (q_1^*, q_2^*), the next dollar spent on each good yields the same marginal utility.

So, what is $\frac{\partial L}{\partial Y}$? Return to the Lagrangian:

$$L = u(q_1, q_2) + \lambda(Y - p_1 q_1 - p_2 q_2)$$

$$\frac{\partial L}{\partial Y} = \lambda$$

* λ equals the "shadow price" of the budget constraint. It expresses the quantity of "utils" that could be obtained with the next dollar devoted to ~~comption~~ consumption.

Properties: λ is not uniquely defined. It is defined only up to a monotonic transformation.

For the following specific utility functions, we want to set up the problem, solve it, graph it, and interpret the results.

Ex 1 $\underset{\{q_1,q_2\}}{\text{MAX}}\ u(q_1,q_2) = q_1 + q_2 \quad \text{s.t. } p_1 q_1 + p_2 q_2 = Y$

Ex 2 $\underset{\{q_1,q_2\}}{\text{MAX}}\ u(q_1,q_2) = \text{MIN}\{q_1, q_2\} \quad \text{s.t. } p_1 q_1 + p_2 q_2 = Y$

Ex 3 $\underset{\{q_1,q_2\}}{\text{MAX}}\ u(q_1,q_2) = q_1^{\alpha}\, q_2^{1-\alpha} \quad \text{s.t. } p_1 q_1 + p_2 q_2 = Y$

Ex 4 $\underset{\{q_1,q_2\}}{\text{MAX}}\ u(q_1,q_2) = \ln q_1 + q_2 \quad \text{s.t. } p_1 q_1 + p_2 q_2 = Y$

Ex 5 $\underset{\{q_1,q_2\}}{\text{MAX}}\ u(q_1,q_2) = q_1 q_2 \quad \text{s.t. } p_1 q_1 + p_2 q_2 = Y$

↳ where $p_1 = \$4$, $p_2 = \$3$, $Y = \$10$

and assume consumer cannot buy fractions of a good.