THEORY OF PREFERENCES

Agenda

- Assumptions of consumer preferences
- Notion of utility function (Cardinal vs. ordinal)
- Tradeoffs & Marginal Rate of Substitution (MRS)
- Monotone transformations

Three Key Ideas

(1) Individuals have consistent preferences

(2) Individuals seek to maximize their preference ordering.

(3) People are willing to make tradeoffs between different (economic) goods.

PREFERENCE ORDERING

Notation: 2 good case

- Consumption bundle is denoted by:

$$A = (q_1, q_2)$$

where $q_i \geq 0$ for $i = 1, 2$ are specific quantities for good i.

- Preference statements are given by a preference - indifference relation $\succsim$, which means "at least as good as."

↳ Specifically,

(i) $A \succ B$ means "prefers A to B"

(ii) $A \sim B$ means "indifferent btw A and B"

* Implicit in making choices is the ability to either (i) or (ii)

* Comparing attractiveness of 2 bundles for all possible bundles allows us to construct a complete preference ordering if 2 conditions are met:

(1) people can always make preference statements

(2) preference statements are consistent.

Minimum requirement for a theory of preferences:
assumptions which imply conditions (1) and (2).

A1: Completeness

For any pair of bundles, only one of the following is true:

(i) $A \succ B$
(ii) $B \succ A$
(iii) $A \sim B$

People can compare any 2 bundles (even ones with trivial differences)

Implication: Rules out questions about how we come to have the preferences we do.

A2: Consistency (Transitivity)

Given any 3 bundles A, B and C:

(i) if $A \succ B$ and $B \succ C$ then $A \succ C$
(ii) if $A \sim B$ and $B \sim C$ then $A \sim C$

Implication: No bundle can occupy two different places in one's preference ordering.

* A1 and A2 are the core assumptions
* A1 and A2 together are what we mean by rational behavior.

Using Utility Functions to Capture Preferences

We rarely work with preference orderings themselves. Why?

Ex: Take a list of 100 restaurants. Assume A1 and A2 hold. Compare each restaurant to the other 99 and rank them.

↳ Too much info. to deal with here.

↳ There are 100! comparisons (i.e. $100 \times 99 \times 98 \times \ldots \times 2 \times 1$

SOLUTION: Rank the restaurants in such a way that if $A \succ B$, then A is assigned a higher number than B.

↳ Now we only need to keep track of 100 numbers. (one for each restaurant).

* This is exactly how economics proceeds. We call such a ranking a utility function (a function which yields a rank order of preferences).

* Utility functions do not contain more or less info. than preference orderings themselves. But they are an easier device to work with.

<u>Definition:</u> A utility function u is a function that maps the set of all bundles into numbers such that:

(i) if $A \succ B$ then $u(A) > u(B)$

(ii) if $A \sim B$ then $u(A) = u(B)$

<u>EX</u> Let $A \equiv$ Economics classes

$B \equiv$ Math classes

$C \equiv$ Sociology classes.

If $A \succ B \succ C$ THEN $u(A) = 3$, $u(B) = 2$; $u(C) = 1$

preserves these preferences

But wait ... so does $\hat{u}(A) = (3)^2$, $\hat{u}(B) = (2)^2$, $\hat{u}(C) = (1)^2$

<u>Important Point</u>: The utility numbers themselves convey no info. Only the <u>order</u> of the numbers matter.

(we'll return to cardinal vs. ordinal utility later)

Formally,

If $u(A)$ represents a preference ordering, then so does

$$v(A) = f(u(A))$$

where f is <u>any</u> strictly increasing (Monotone) function.

A3: CONTINUITY

If $A \succ B$ and C lies within ε (epsilon) radius of B then $A \succ C$.

Implication: A3 together with A1 and A2 imply we can define a continuous utility function. Later we'll see that A3 gives rise to "well-behaved" demand curves.

* Not an essential assumption, but a practical one.

EX Let $q_1 \equiv$ onces of beer / day

$q_2 \equiv$ onces of pop / day.

If (16 oz beer, 12 oz pop) $\succ$ (12 oz beer, 14 oz pop)

THEN we cannot have:

(16 oz beer, 12 oz pop) $\prec$ (12.0001 oz beer, 14 oz pop)

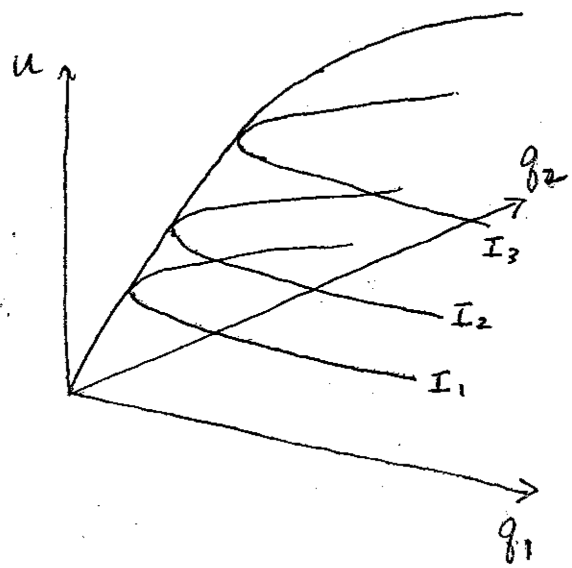
* A3 is a useful assumption. We can rank all possible combos of beer and pop by writing a continuous utility function which has quantities of beer / pop as inputs and a single number u as an output.

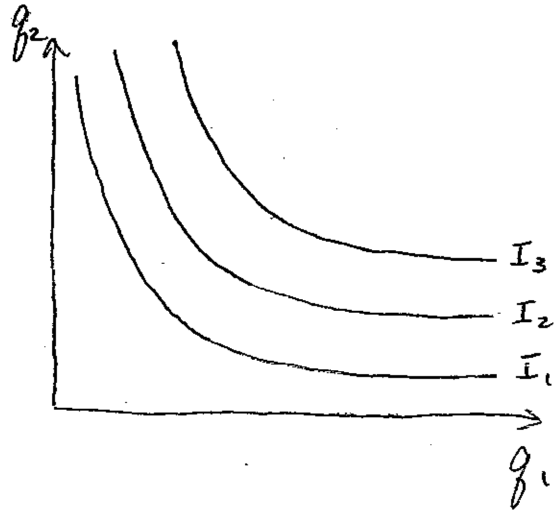
* Higher numbers mean more preferred bundles, as before.

Definition: Indifference Curves

Given a level of utility $u(A) = \bar{u}$, the indifference curve for $\bar{u}$ is the locus of bundles (q_1, q_2) that generate utility level $\bar{u}$ for utility function $u(A)$

EX Cobb-Douglas preferences





Indifference curves are the level curves of this utility function.

↳ Horizontal traces projected onto the $q_1 q_2$-plane

$$\left.\begin{array}{l} I_3 \rightarrow \text{utility level } u_3 \\ I_2 \rightarrow \text{utility level } u_2 \\ I_1 \rightarrow \text{utility level } u_1 \end{array}\right\} u_3 > u_2 > u_1$$

↳ The set of all indifference curves is called an Indifference Map.

A4: Non-satiation

Consider 2 bundles: $A = (q_1^A, q_2^A)$ and $B = (q_1^B, q_2^B)$

If $q_1^A > q_1^B$ and $q_2^A \geq q_2^B$ Then $A \succ B$.

Implications: (i) Utility is increasing in both q_1 and q_2.

↳ Does not apply to bads (garbage, pollution, work, etc.)

(ii) No upper limit to utility

↳ Indifference Map stretches out indefinitely.

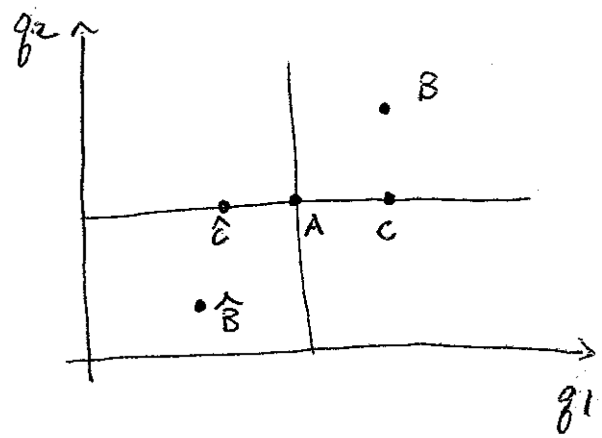
(iii) Scarcity.

Techy Properties of Indifference Curves

(i) Every bundle lies on some indifference curve (A1)

(ii) Indifference Curves are downward sloping

Sketch of Proof:



- $B \succ A$ since $q_1^B > q_1^A$ and $q_2^B > q_2^A$ (reverse inequalities for $\hat{B}$)

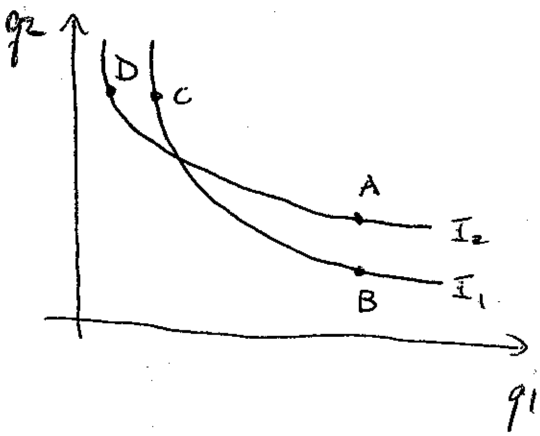
  ⟹ Ind. curves cannot have positive slope!

- $C \succ A$ since $q_1^C > q_1^A$ and $q_2^C = q_2^A$ (reverse inequalities for $\hat{C}$)

  ⟹ Ind. curves cannot have zero or infinite slope!

- Thus, Ind. curves must have negative slope!!

(iii) Indifference Curves <u>cannot</u> intersect.



<u>Sketch of Proof:</u>

According to I_1 and I_2:		According to A2:
$A \succ B$		$A \succ B$
$B \sim C$		$B \sim C$
$C \succ D$		$C \succ D$
$\boxed{D \sim A}$	$\longleftrightarrow$	$\boxed{A \succ D}$

* Thus $A \succ D$ <u>and</u> $A \sim D$ which is a <u>contradiction</u>.

<u>A5: Maximization</u>

Individuals always make choices that leave them better off.

↳ Key behavioral assumption in economics

↳ Separates economics from other social sciences

<u>Implications:</u> - restricts type of arguments we make

↳ not based on ignorance or irrationality.

- all gains from trade are fully exploited.

↳ No \$10 bills left lying on the sidewalk

↳ Think of check-out lines at supermarket.

Trade offs

- The idea that people maximize over complete and consistent preferences is one key concept in explaining human behavior.

- The idea that people are willing to make trade offs is another. This is the principle of substitution.

- There's nothing we won't tradeoff (at least on the margin)

Reason: Most decisions we make are marginal decisions

EX's How do I allocate an extra \$ here or there.

How do I allocate an extra hour here or there.

- Tradeoffs and Indifference curves are inextricably linked.

Tradeoffs & MRS

- We need a measure to quantify the rate at which two or more goods tradeoff against one another,

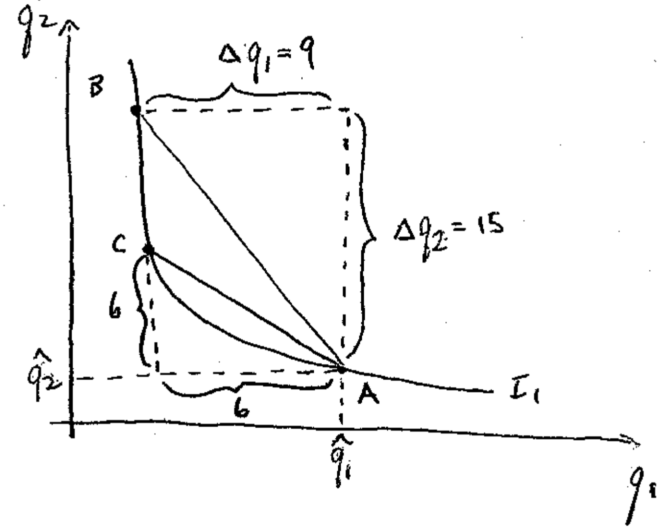
Definiton The Marginal Rate of Substitution (MRS) btw any 2 goods is the absolute value of the slope of an indifference curve at any allocation $(\hat{q}_1, \hat{q}_2)$.

Rate of Substitution: $\left|\frac{\Delta q_2}{\Delta q_1}\right|$

$A \to B: \left|\frac{\Delta q_2}{\Delta q_1}\right| = \frac{15}{9} = 1.67$

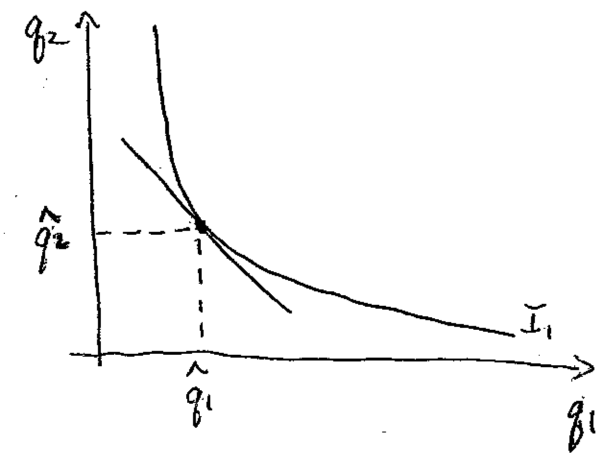
$A \to C: \left|\frac{\Delta q_2}{\Delta q_1}\right| = \frac{6}{6} = 1$

Which is it?



* To avoid ambiguity with discrete (non-marginal) changes we use the MRS

* Graphically, the MRS is the absolute value of the slope of the tangent line to I_1 at allocation $(\hat{q}_1, \hat{q}_2)$.



Formally,

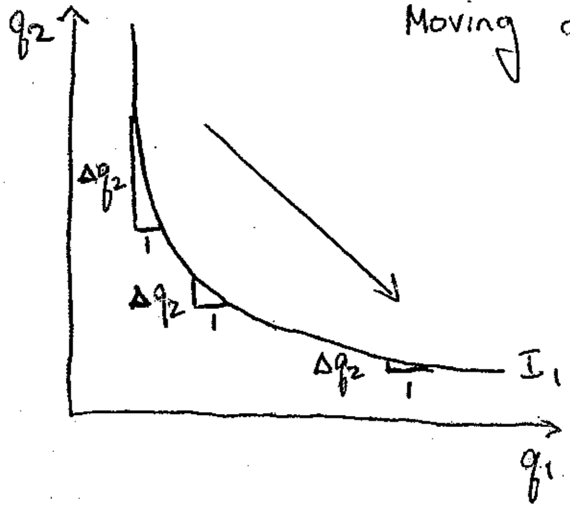
$$u(q_1, q_2) = \bar{u}$$

$$\frac{\partial u}{\partial q_1} dq_1 + \frac{\partial u}{\partial q_2} dq_2 = 0 \quad \text{(total differential)}$$

$$\frac{dq_2}{dq_1} = -\frac{\partial u/\partial q_1}{\partial u/\partial q_2}$$

$$MRS = \frac{\partial u/\partial q_1}{\partial u/\partial q_2} \quad \text{(along a level curve)}$$

Interpretation



Moving down I_1, we write: $\begin{cases} \text{MRS of } q_1 \text{ for } q_2 \\ MRS_{12} \end{cases}$

WHAT MRS MEASURES

(i) Amount of q_2 need to give up to get 1 more unit of q_1
(ii) Marginal Value (MV) of q_1 in terms of foregone q_2

* With I_1, MRS varies along indifference curve (not always true)

↳ amount of q_2 consumer is willing to give up to get 1 more q_1 decreases with the amount of q_1 the consumer has.

↳ Thus, I_1 is said to exhibit Diminishing MRS

Implication: - consumers prefer diversity in consumption

- Equivalently, mixtures of goods are prefered to extremes.

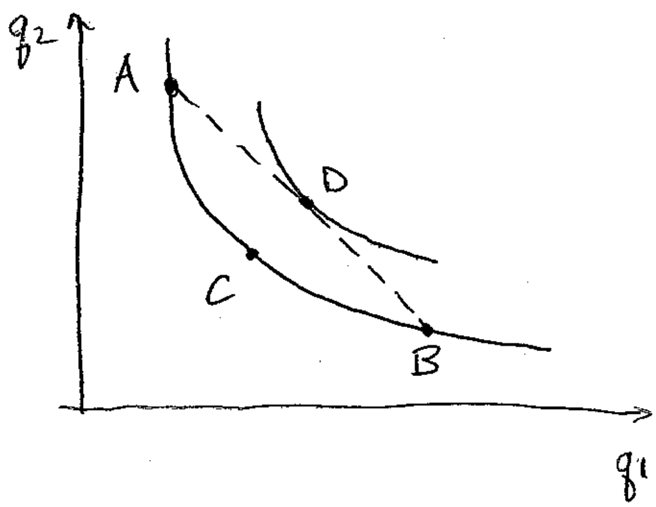
* Think of your preference for "going out" or "staying home" on a weekly basis

A6: Convexity

An alternative def'n of diminishing MRS is given by: the following:

Given $A \sim C$ and $B \sim C$ THEN $\alpha A + (1-\alpha) B \succ C$

for all $\alpha \in (0,1)$



* A (convex) combination of bundles A and B yield higher utility that bundles A, B, or C.

OPTIONAL

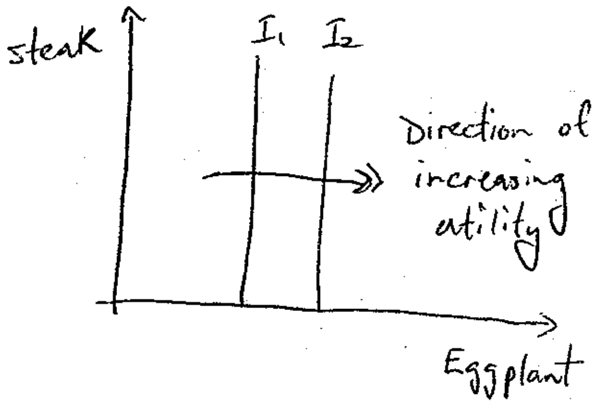
Define $D = \alpha A + (1-\alpha) B$ for all $\alpha \in (0,1)$

Bundle D contains: $q_1^D = \alpha q_1^A + (1-\alpha) q_1^B$

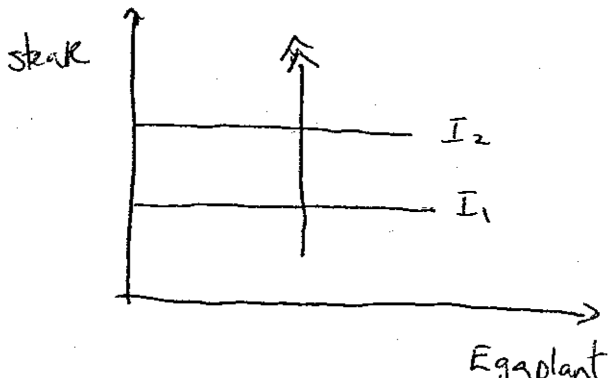
$$q_2^D = \alpha q_2^A + (1-\alpha) q_2^B$$

$$\Rightarrow u(\alpha q_1^A + (1-\alpha) q_1^B, \alpha q_2^A + (1-\alpha) q_2^B) > \alpha u(q_1^A, q_2^A) + (1-\alpha) u(q_1^B, q_2^B)$$

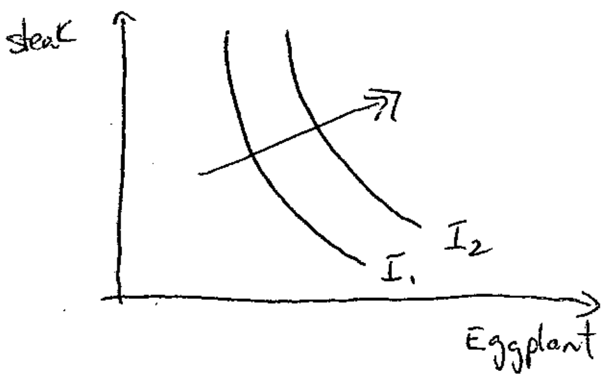
DIFFERENT PREFERENCE



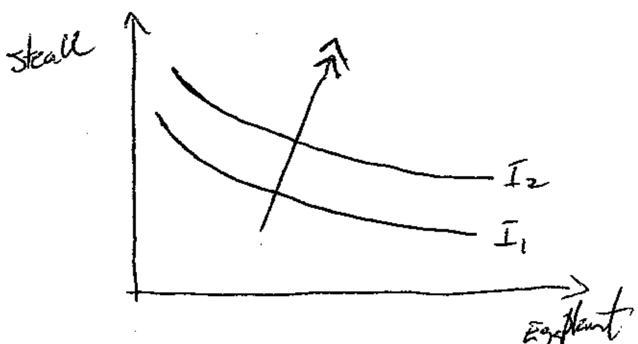
* Vegetarian (hates meat)
  - will not give up eggplant for steak at all.



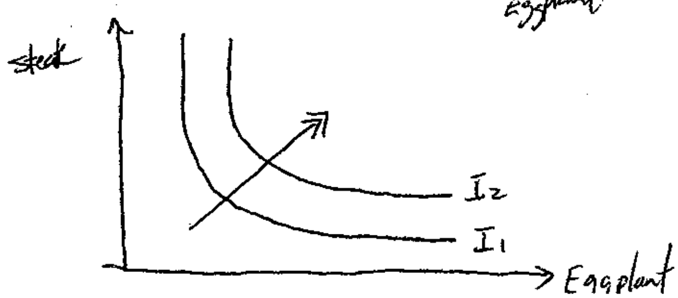
* Meat-lover
  - will not give up steak for eggplant at all.



* Strong preference for Eggplant
  - It takes alot of steak for each small reduction in eggplant

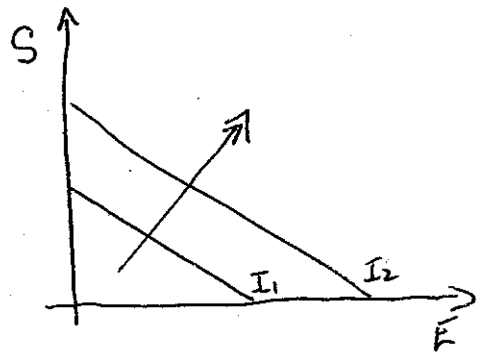


* Strong preference for steak
  - requires alot of Eggplant for each small reduction in steak.



* No strong preference
  - prefers diversity in diet.

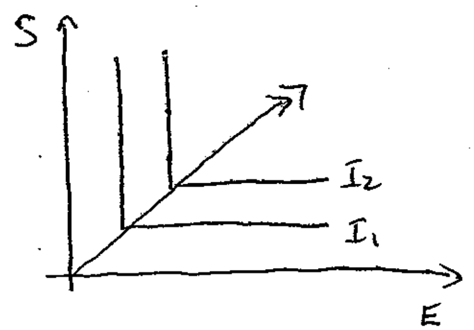
Other Possibilities



* Perfect Substitutes

↳ tradeoff of steak for eggplant (or vice versa) is constant no matter how much of each good you have.

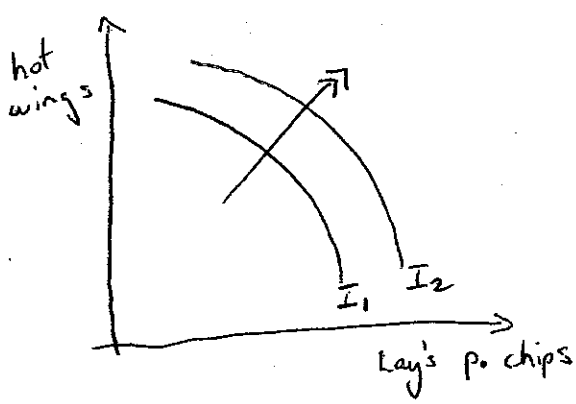
↳ only total amount matters.



* Perfect Complements

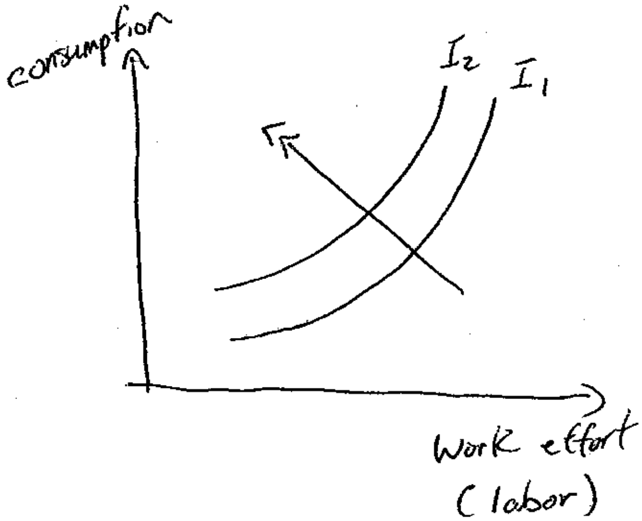
↳ goods must be consumed in fixed proportions

↳ More steak is no good unless it comes with more eggplant.



Hot wings and Lay's chips may have same property: The more you consume the more you want.

↳ No diversity ; Not convex



Labor is a "bad".

↳ Less labor increases utility.

↳ More labor can only come with more consumption (to remain indifferent)

Monotonic Transformations

Utility Functions are defined up to a "monotonic transformation."

Definition:

If $u(A)$ represents a preference ordering, then so does

$$V(A) = f(u(A))$$

where f is any strictly increasing function.

EX $u(q_1, q_2) = q_1 q_2$

Now multiply $u(\cdot)$ by 2. Does this change the ordering of the utility numbers? NO.

$\hat{u}(q_1, q_2) = 2 q_1 q_2$ represents same preferences as $u(q_1, q_2)$

EX Suppose we have: $$u(q_1, q_2) = \left[\frac{(\ln(q_1 + q_2))^2}{4}\right]^{\frac{1}{2}}$$

Messy to work with!

(1) square it $\Rightarrow$ $u(q_1, q_2) = \dfrac{[\ln(q_1 + q_2)]^2}{4}$

(2) Multiply u by 4 $\Rightarrow$ $u(q_1, q_2) = [\ln(q_1 + q_2)]^2$

(3) square root it $\Rightarrow$ $u(q_1, q_2) = \ln(q_1 + q_2)$

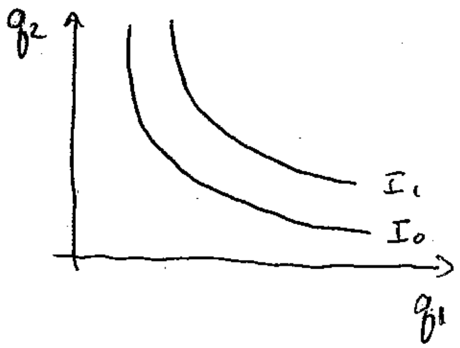
(4) un-log it $\Rightarrow$ $\boxed{u(q_1, q_2) = q_1 + q_2}$ Gives same preference ordering.

Property: If $u_2(\cdot)$ is a monotone transformation of $u_1(\cdot)$; i.e., $u_2(\cdot) = f(u_1(\cdot))$, then:

(i) $u_2(\cdot)$ and $u_1(\cdot)$ exhibit identical preference rankings

(ii) MRS of $u_2(\cdot)$ and $u_1(\cdot)$ are equivalent (for consumer theory)

EX



$$\boxed{u_1(q_1, q_2) = q_1 q_2}$$

MRS along I_0:

$$MRS = -\frac{dq_2}{dq_1}\Big|_{u=u_1} = \frac{q_2}{q_1}$$

$$\boxed{u_2(q_1, q_2) = \ln(u_1(\cdot)) = \ln q_1 + \ln q_2}$$

MRS of u_2 along indifference curve such that $u_2 = \ln u_1$:

$$MRS \doteq -\frac{dq_2}{dq_1}\Big|_{u=u_2} = \frac{q_2}{q_1}$$

The MRS of $u_2(\cdot)$ and $u_1(\cdot)$ are equivalent!

↳ Monotone transformation not only preserve preference rankings but also preserves the rate at which q_2 is substituted for q_1 (or vice versa) in order to satisfy tastes.