Cost Curves

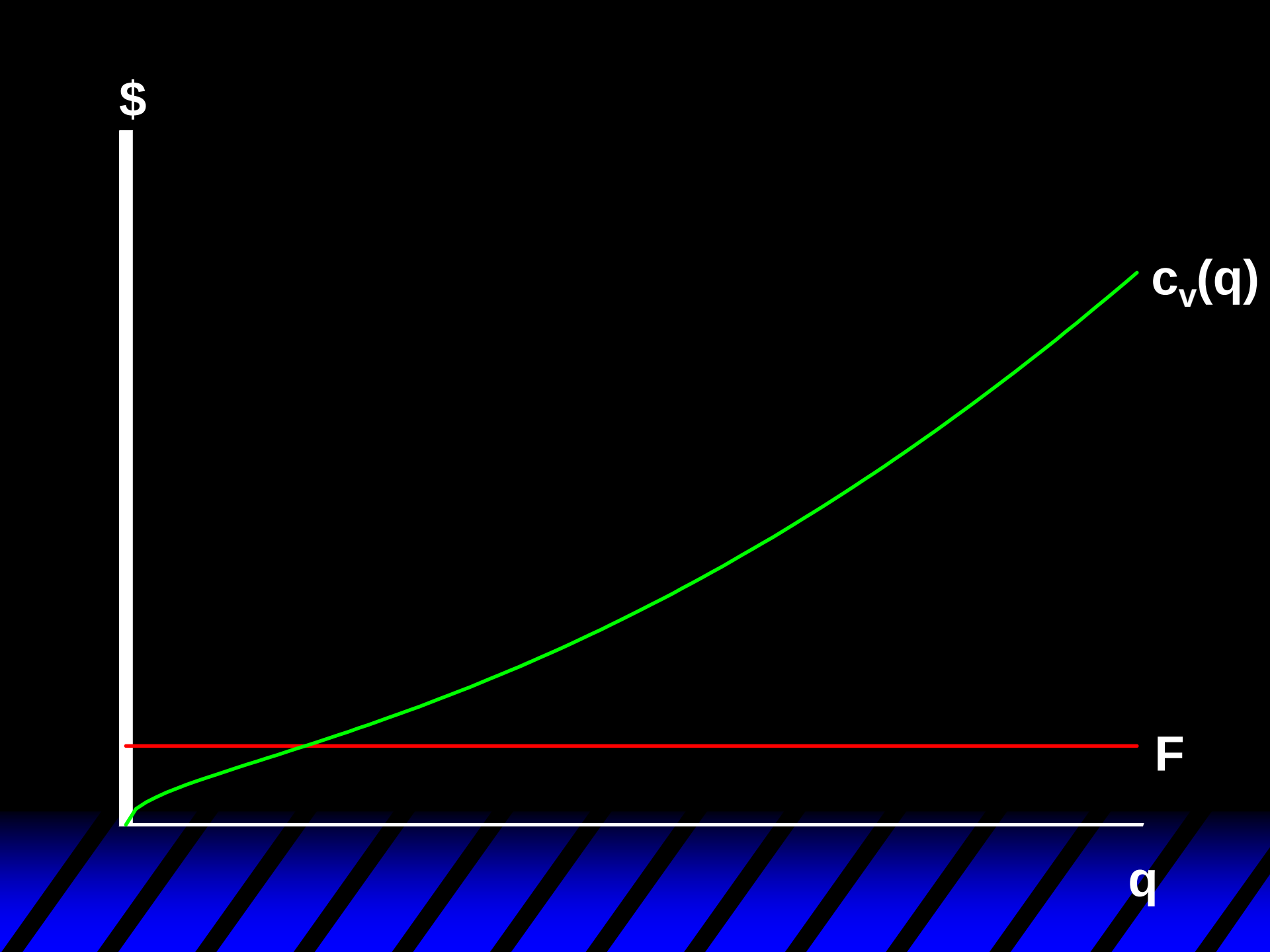
Fixed, Variable & Total Cost Functions

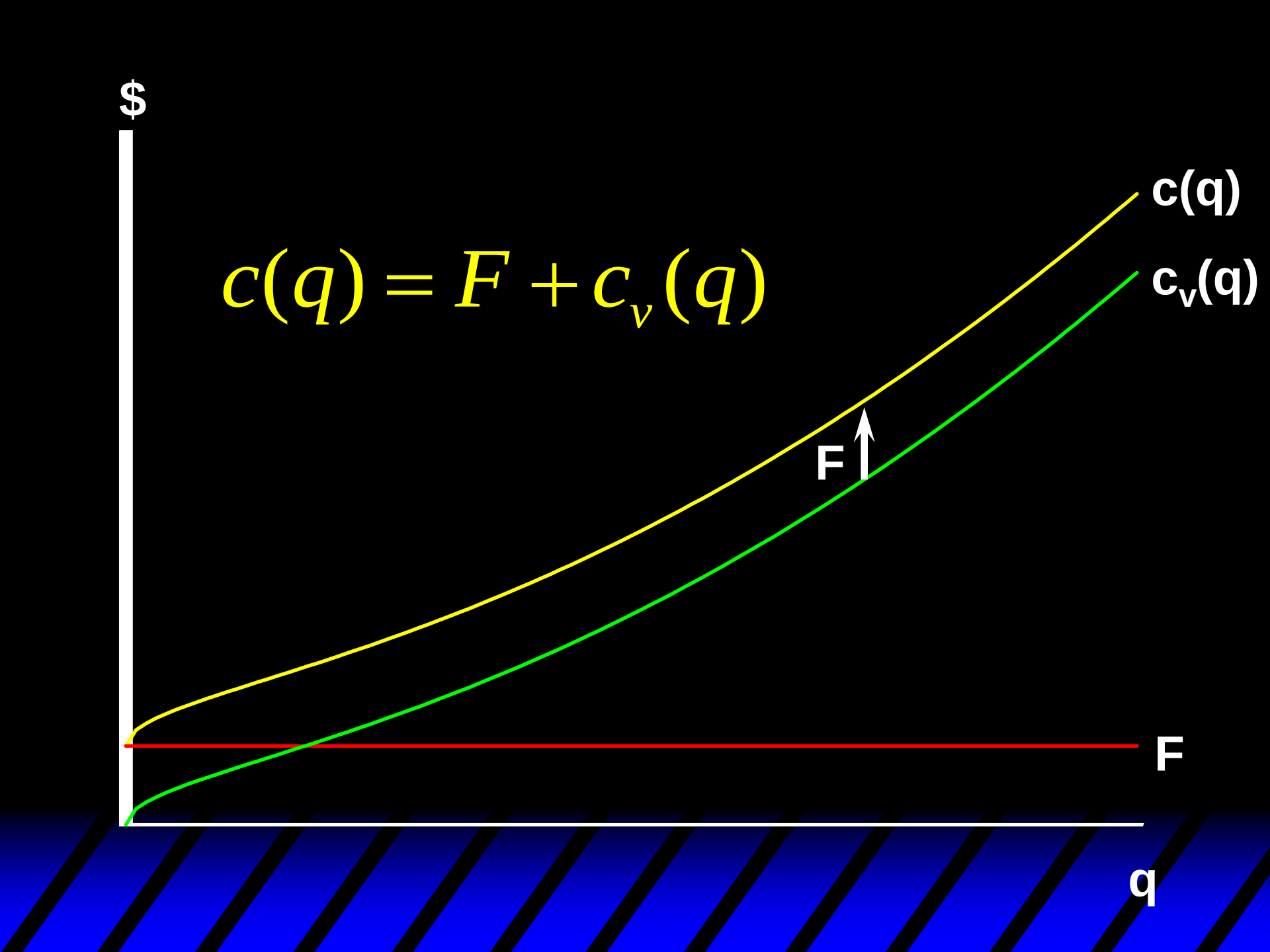
- **F is the total cost to a firm of its short-run fixed inputs. F, the firm's fixed cost, does not vary with the firm's output level.**
- **$c_v(q)$ is the total cost to a firm of its variable inputs when producing q output units. $c_v(q)$ is the firm's variable cost function.**
- **$c_v(q)$ depends upon the levels of the fixed inputs.**

Fixed, Variable & Total Cost Functions

- **c(q) is the total cost of all inputs, fixed and variable, when producing q output units. c(q) is the firm's total cost function;**

$$c(q) = F + c_v(q).$$


$
$c_v(q)$
F
q


$
$c(q) = F + c_v(q)$
c(q)
c_v(q)
F
F
q

Av. Fixed, Av. Variable & Av. Total Cost Curves

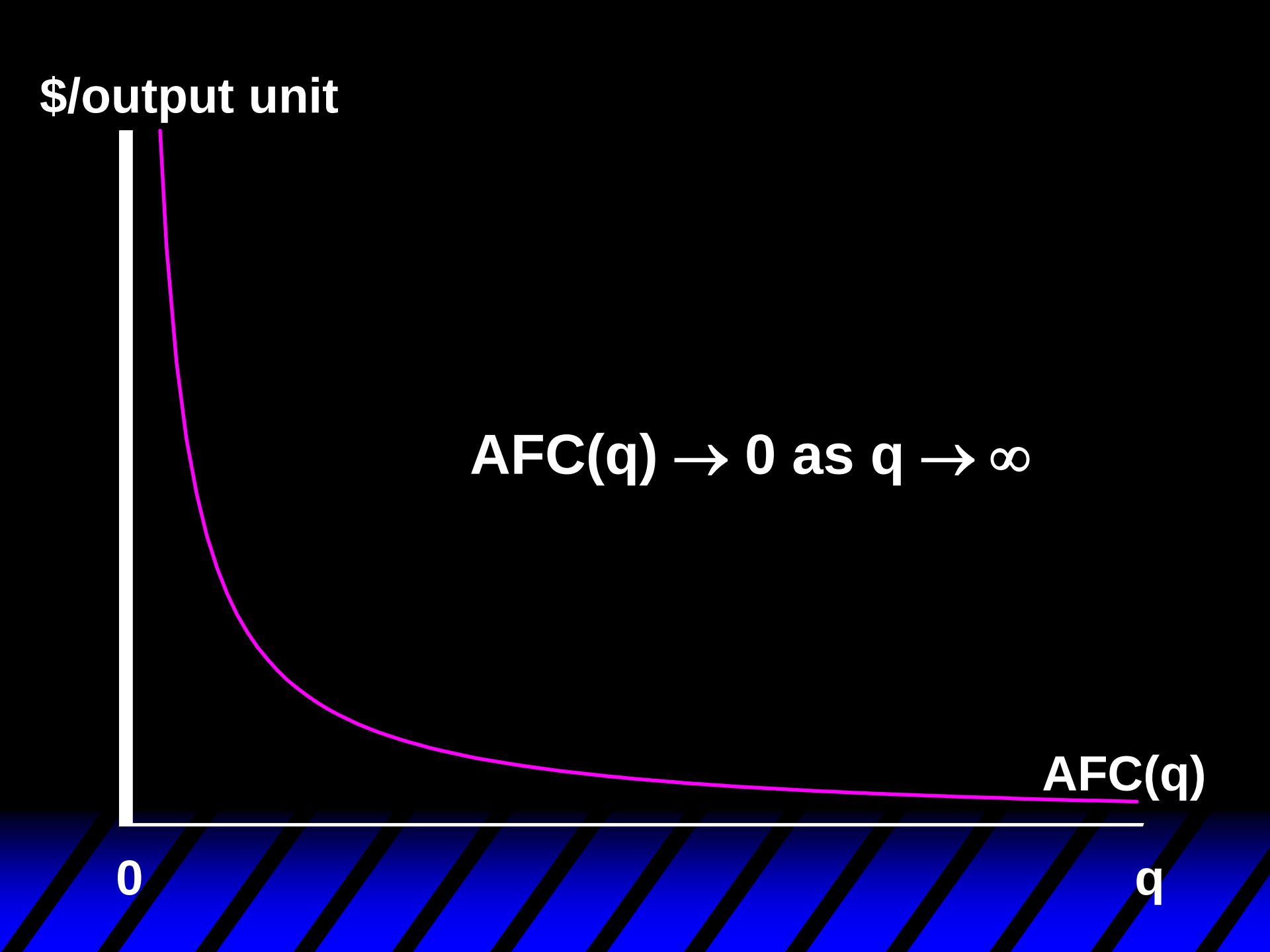
- **The firm's total cost function is**

$$c(q) = F + c_v(q).$$

For y > 0, the firm's average total cost function is

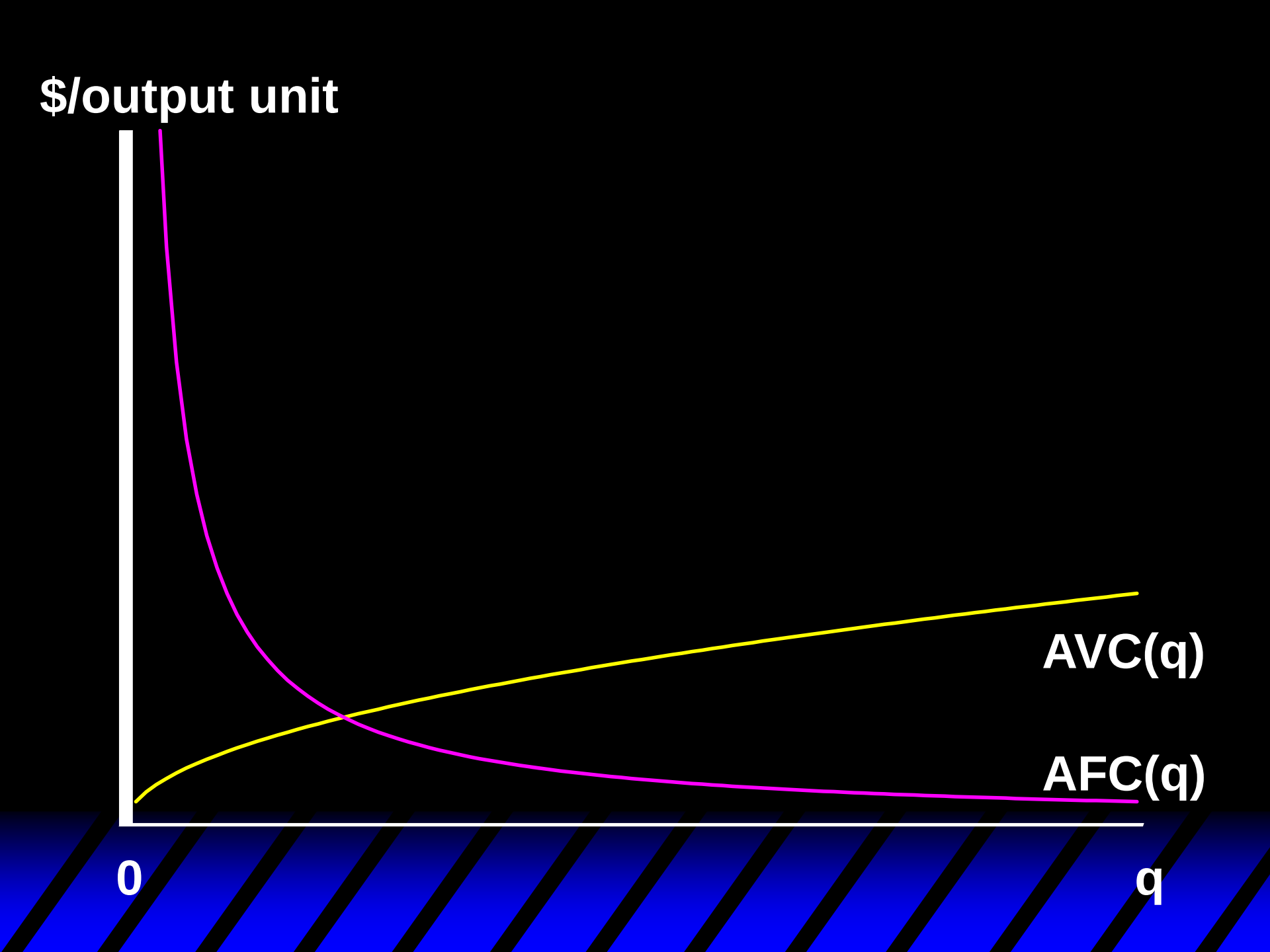
$$AC(q) = \frac{F}{q} + \frac{c_v(q)}{q}$$

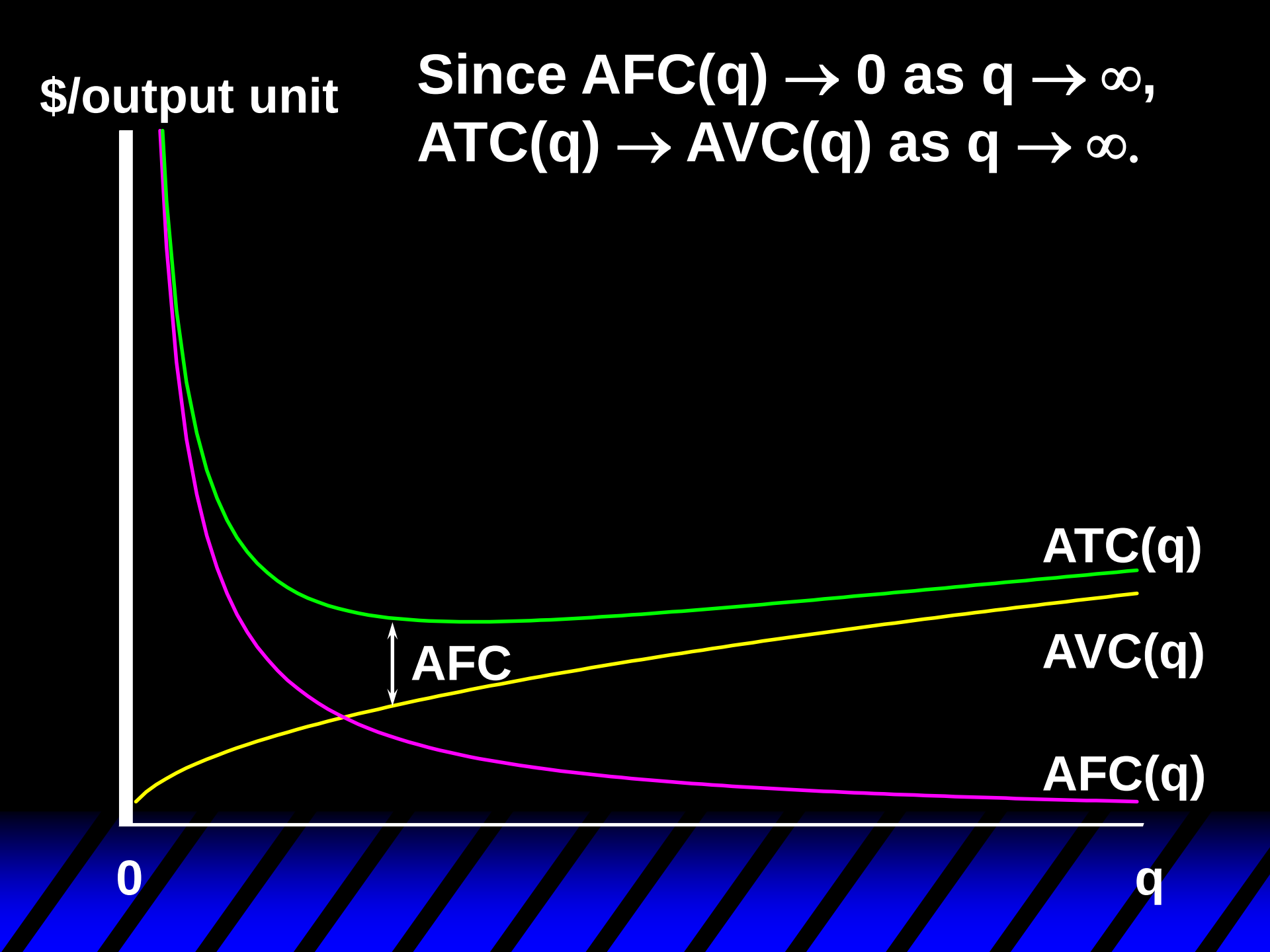
$$= AFC(q) + AVC(q).$$


$/output unit
AFC(q) → 0 as q → ∞
AFC(q)
0
q

Av. Fixed, Av. Variable & Av. Total Cost Curves

- **In a short-run with a fixed amount of at least one input, the Law of Diminishing (Marginal) Returns must apply, causing the firm's average variable cost of production to increase eventually.**


$/output unit
AVC(q)
AFC(q)
0
q


$/output unit
Since AFC(q) → 0 as q → ∞,
ATC(q) → AVC(q) as q → ∞.
ATC(q)
AVC(q)
AFC
AFC(q)
0
q

Marginal Cost Function

- **Marginal cost is the rate-of-change of variable production cost as the output level changes. That is,**

$$MC(q) = \frac{\partial c_v(q)}{\partial q}.$$

Marginal & Average Cost Functions

- **How is marginal cost related to average variable cost?**

Marginal & Average Cost Functions

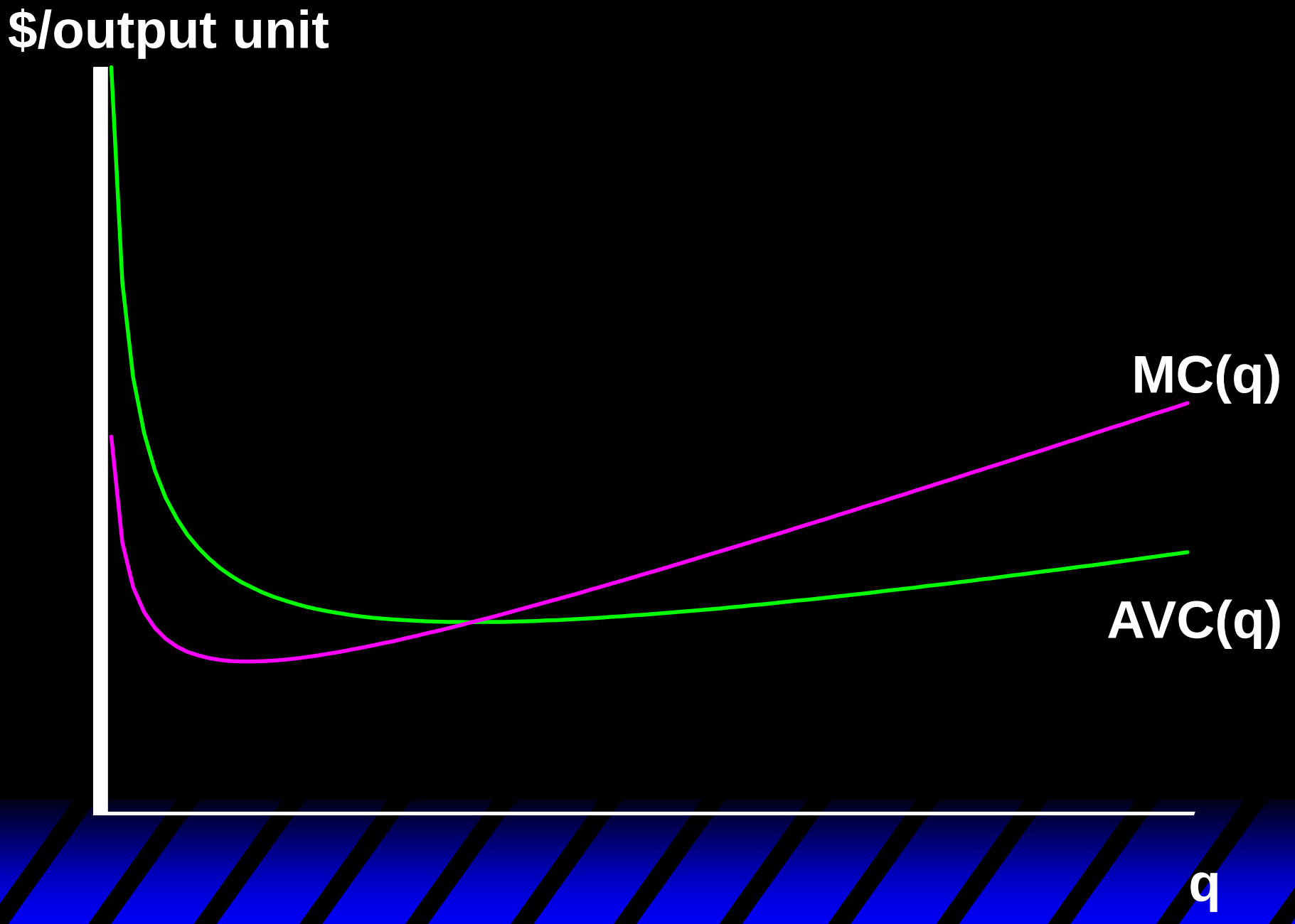
Since $AVC(q) = \frac{c_v(q)}{q},$

$$\frac{\partial AVC(q)}{\partial q} = \frac{q \times MC(q) - 1 \times c_v(q)}{q^2}.$$

Therefore,

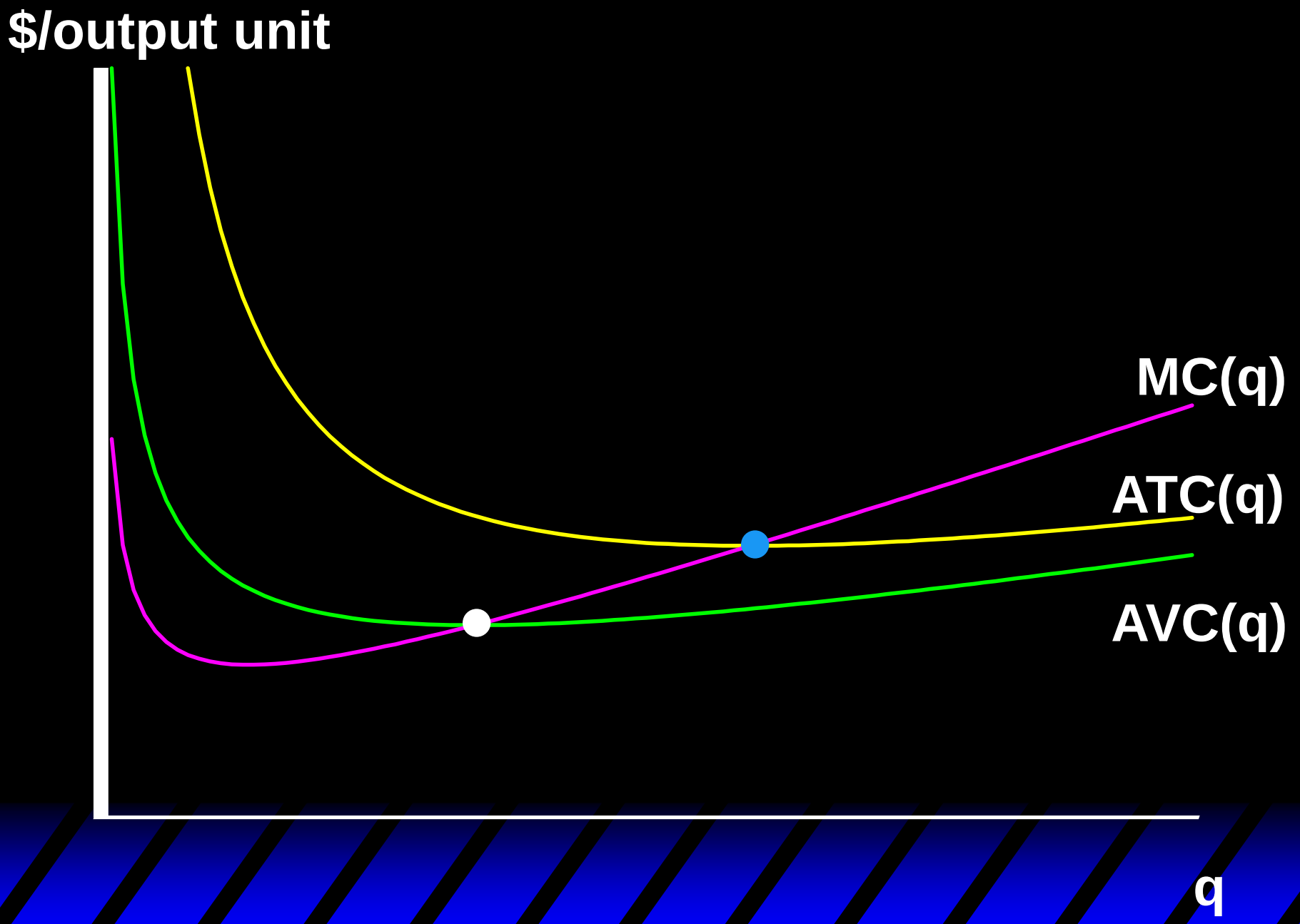
$$\frac{\partial AVC(q)}{\partial q} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \text{as} \quad q \times MC(q) \begin{matrix} > \\ = \\ < \end{matrix} c_v(q).$$

$$\frac{\partial AVC(q)}{\partial q} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \text{as} \quad MC(q) \begin{matrix} > \\ = \\ < \end{matrix} \frac{c_v(q)}{q} = AVC(q).$$


$/output unit
MC(q)
AVC(q)
q

Marginal & Average Cost Functions

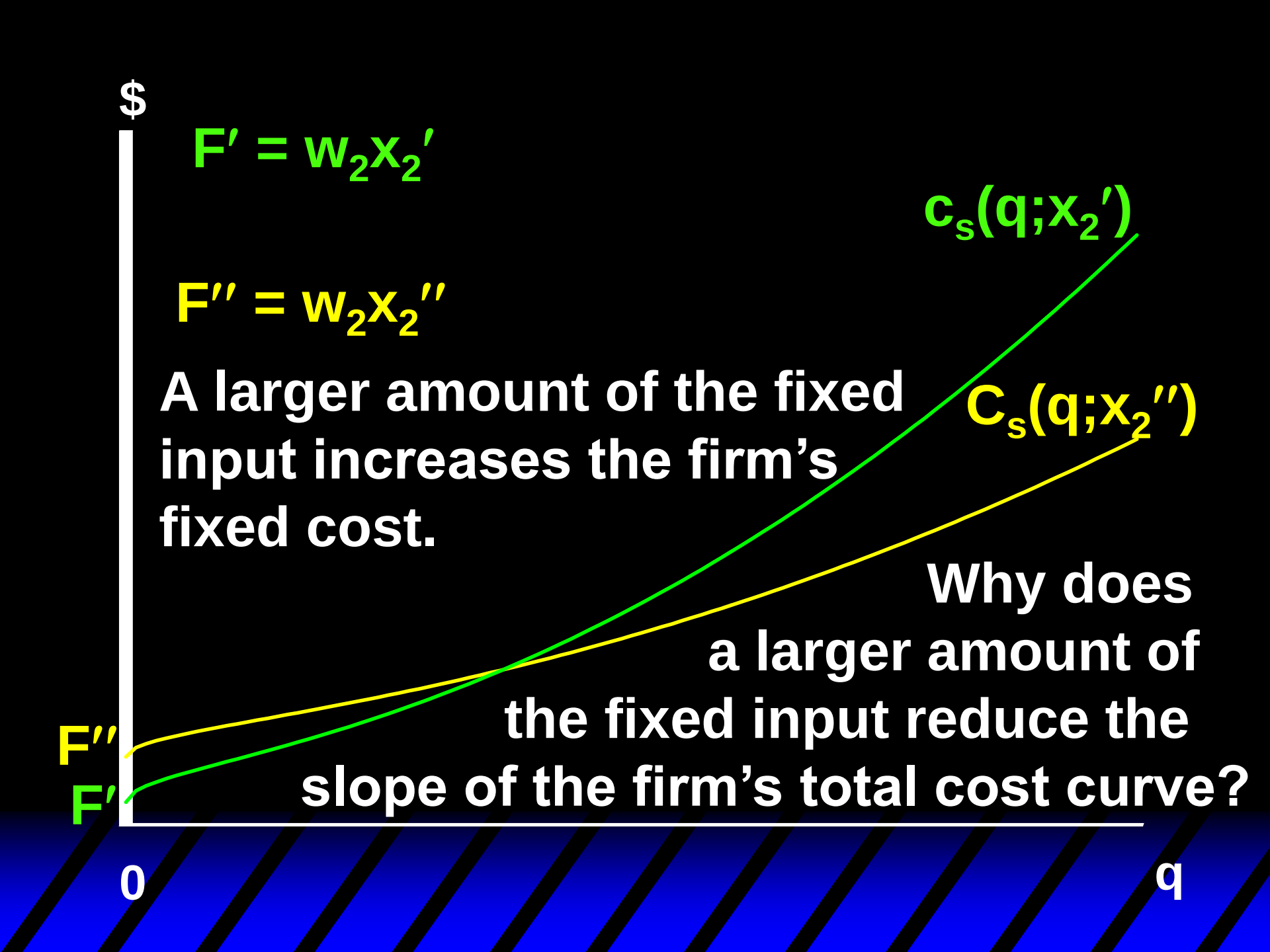
- **The short-run MC curve intersects the short-run AVC curve from below at the AVC curve's minimum.**
- **And, similarly, the short-run MC curve intersects the short-run ATC curve from below at the ATC curve's minimum.**


$/output unit
MC(q)
ATC(q)
AVC(q)
q

Short-Run & Long-Run Total Cost Curves

- A firm has a different short-run total cost curve for each possible short-run circumstance.
- Suppose the firm can be in one of just three short-runs;

$x_2 = x_2'$
or $x_2 = x_2''$ $\qquad x_2' < x_2'' < x_2'''$.
or $x_2 = x_2'''$.


$
$F' = w_2x_2'$
$c_s(q;x_2')$
$F'' = w_2x_2''$
A larger amount of the fixed input increases the firm's fixed cost.
$C_s(q;x_2'')$
Why does a larger amount of the fixed input reduce the slope of the firm's total cost curve?
F''
F'
0
q

Short-Run & Long-Run Total Cost Curves

MP_1 is the marginal physical productivity of the variable input 1, so one extra unit of input 1 gives MP_1 extra output units. Therefore, the extra amount of input 1 needed for 1 extra output unit is $1/MP_1$ units of input 1.

Each unit of input 1 costs w_1, so the firm's extra cost from producing one extra unit of output is

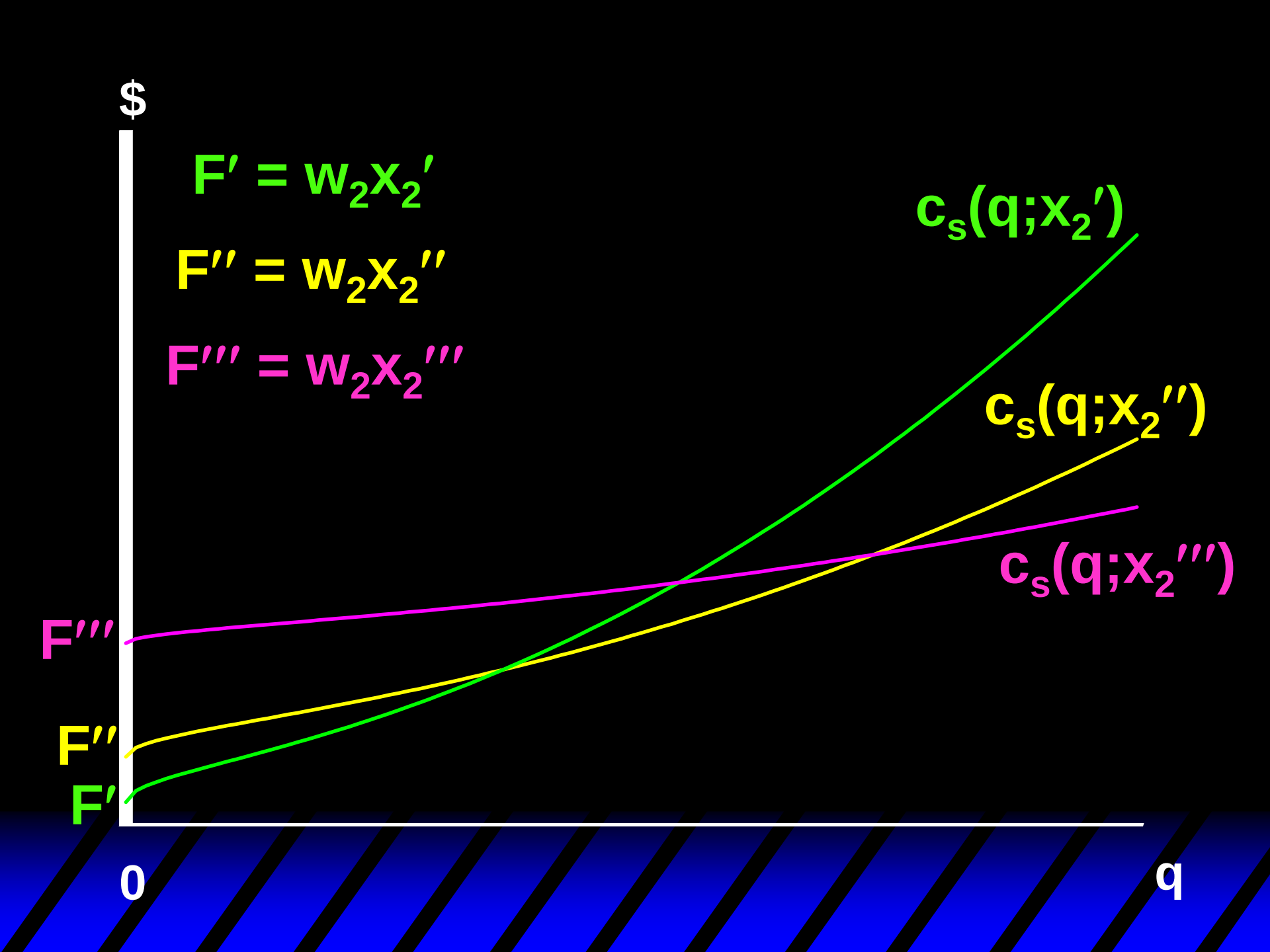
$$MC = \frac{w_1}{MP_1}.$$

Short-Run & Long-Run Total Cost Curves

$MC = \frac{w_1}{MP_1}$ is the slope of the firm's total cost curve.

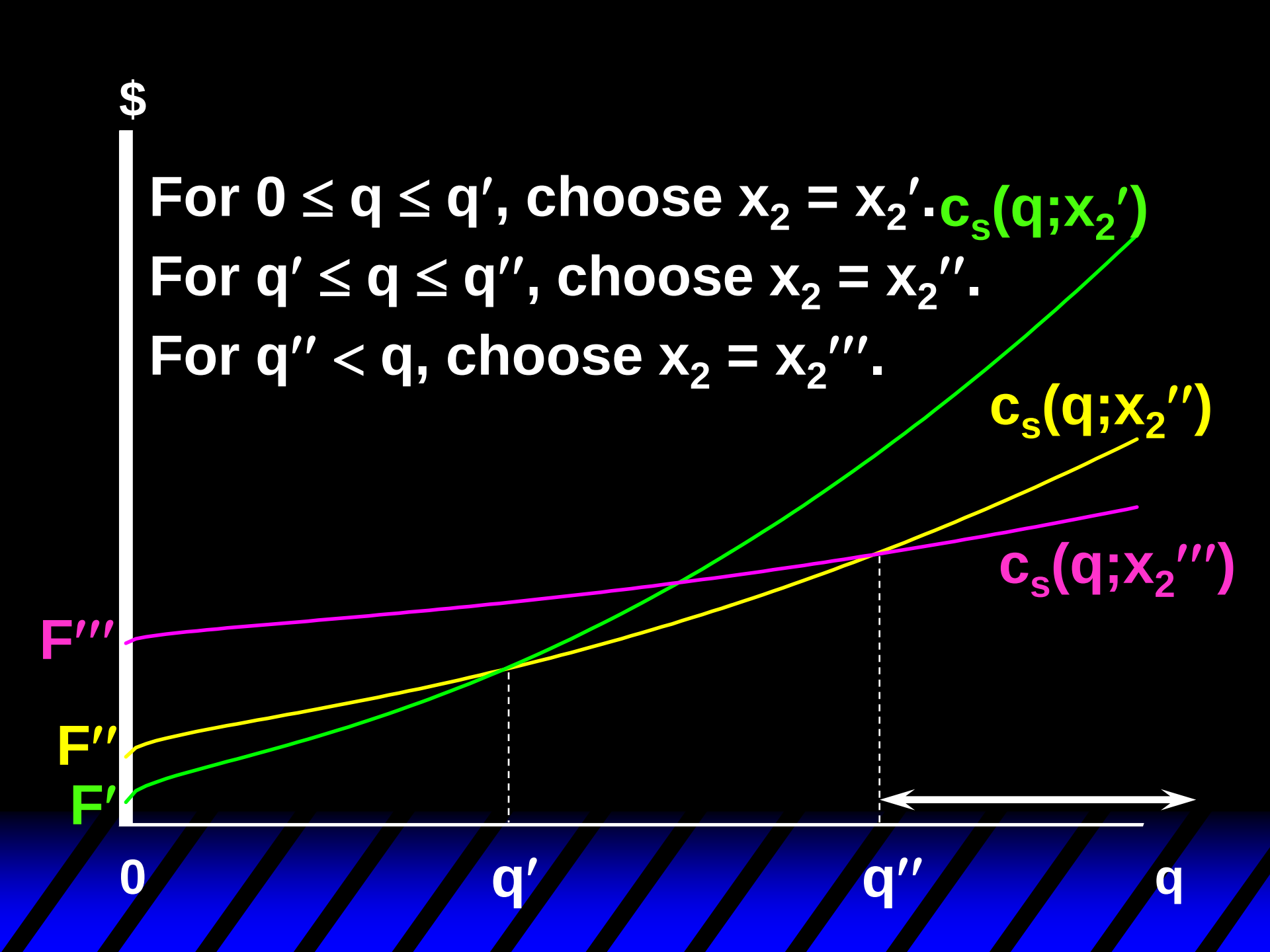
If input 2 is a complement to input 1 then MP_1 is higher for higher x_2.
Hence, MC is lower for higher x_2.

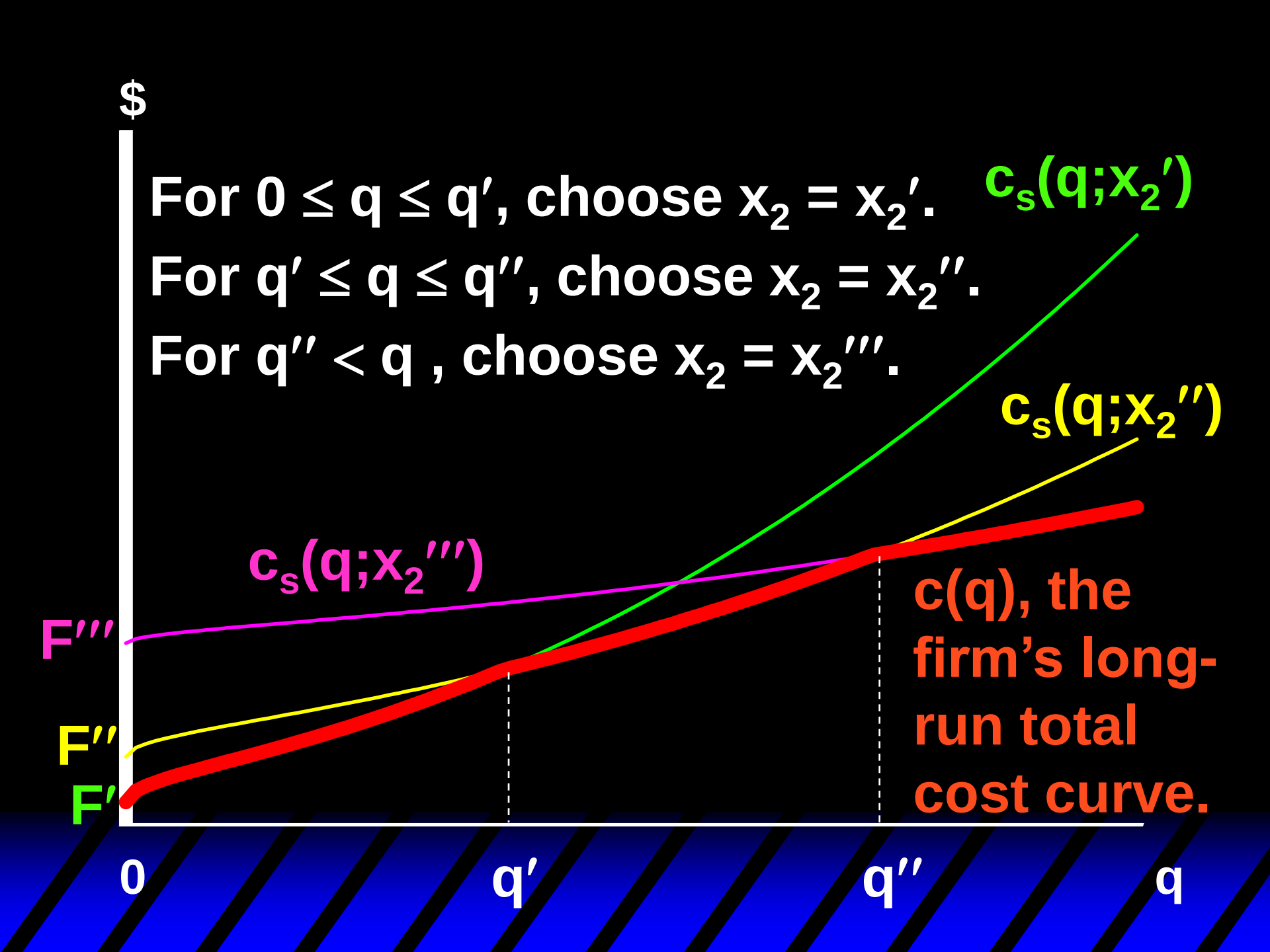
That is, a short-run total cost curve starts higher and has a lower slope if x_2 is larger.


$
F′ = w2x2′
F″ = w2x2″
F‴ = w2x2‴
cs(q;x2′)
cs(q;x2″)
cs(q;x2‴)
F‴
F″
F′
0
q

Short-Run & Long-Run Total Cost Curves

- The firm has three short-run total cost curves.
- In the long-run the firm is free to choose amongst these three since it is free to select x_2 equal to any of x_2', x_2'', or x_2'''.
- How does the firm make this choice?


$
For 0 ≤ q ≤ q′, choose x2 = x2′.
For q′ ≤ q ≤ q″, choose x2 = x2″.
For q″ < q, choose x2 = x2‴.
cs(q;x2′)
cs(q;x2″)
cs(q;x2‴)
F‴
F″
F′
0
q′
q″
q


$
For 0 ≤ q ≤ q′, choose x2 = x2′.
For q′ ≤ q ≤ q″, choose x2 = x2″.
For q″ < q , choose x2 = x2‴.
cs(q;x2′)
cs(q;x2″)
cs(q;x2‴)
F‴
F″
F′
c(q), the firm's long-run total cost curve.
0
q′
q″
q

Short-Run & Long-Run Total Cost Curves

- **The firm's long-run total cost curve consists of the lowest parts of the short-run total cost curves. The long-run total cost curve is the lower envelope of the short-run total cost curves.**

Short-Run & Long-Run Total Cost Curves

- **If input 2 is available in continuous amounts then there is an infinity of short-run total cost curves but the long-run total cost curve is still the lower envelope of all of the short-run total cost curves.**

Short-Run & Long-Run Average Total Cost Curves

- For any output level q, the long-run total cost curve always gives the lowest possible total production cost.
- Therefore, the long-run av. total cost curve must always give the lowest possible av. total production cost.
- The long-run av. total cost curve must be the lower envelope of all of the firm's short-run av. total cost curves.

Short-Run & Long-Run Average Total Cost Curves

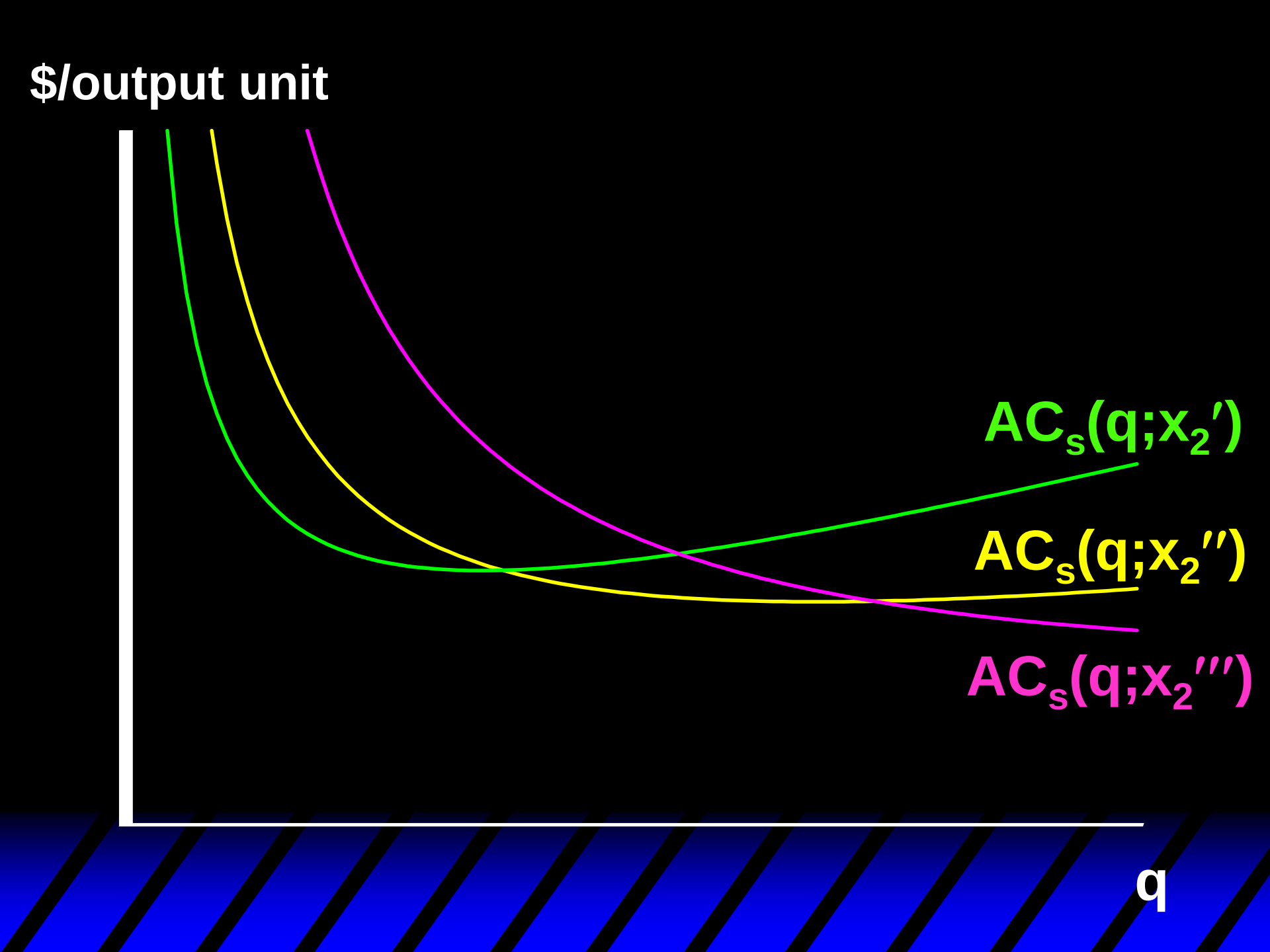
- **E.g. suppose again that the firm can be in one of just three short-runs;**

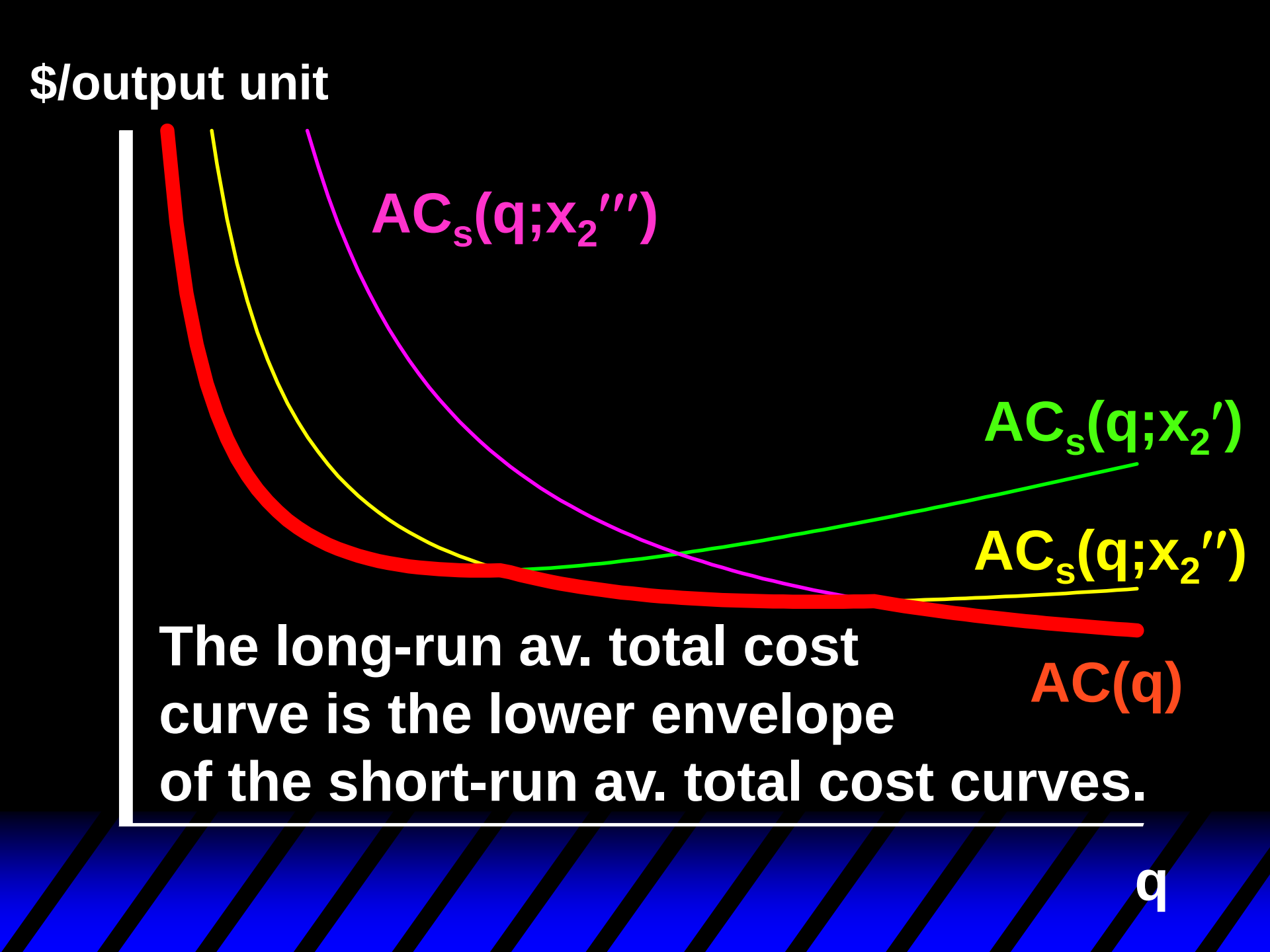
  $x_2 = x_2'$

  or $x_2 = x_2''$ $\qquad (x_2' < x_2'' < x_2''')$

  or $x_2 = x_2'''$

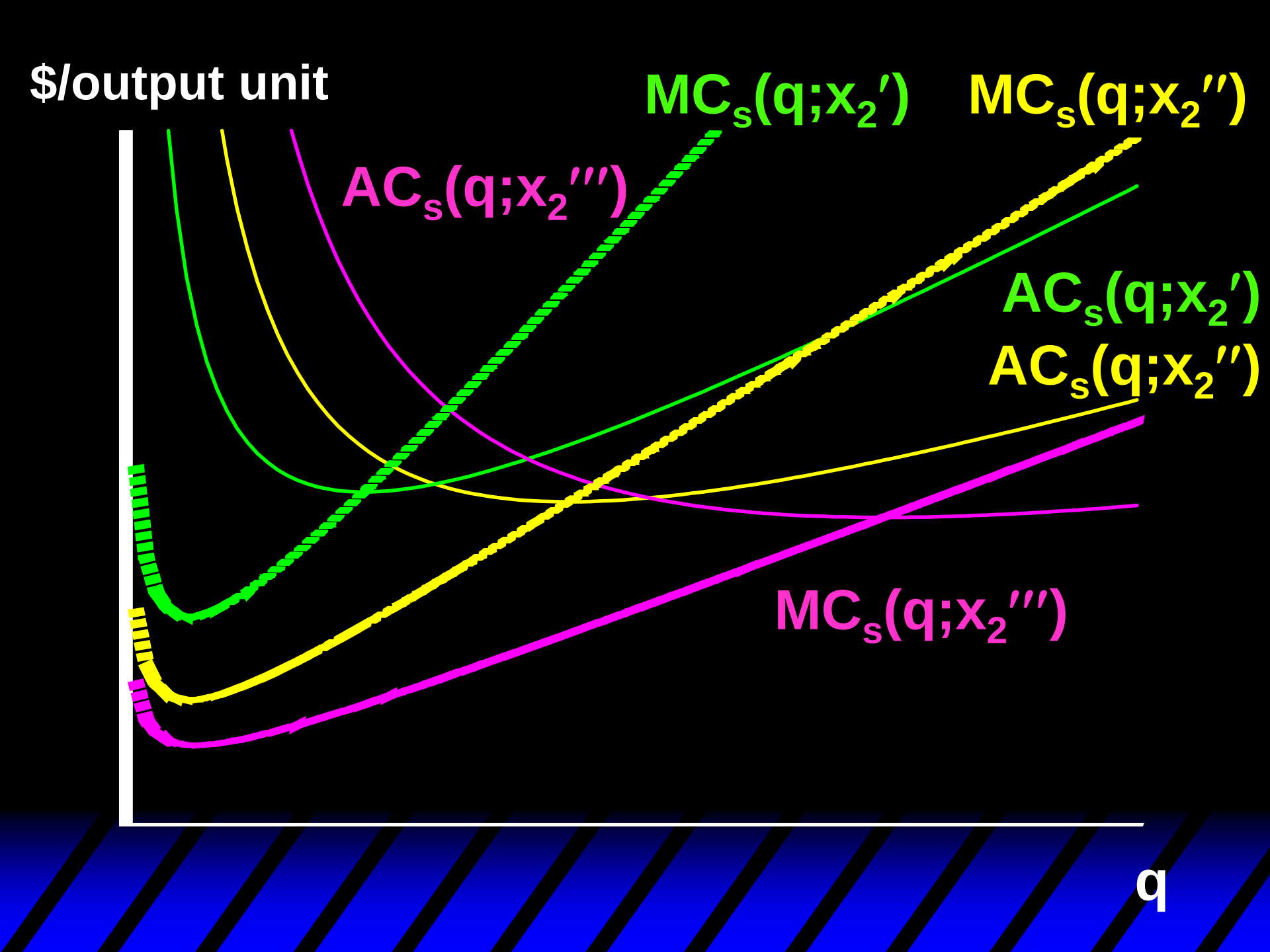
  then the firm's three short-run average total cost curves are ...

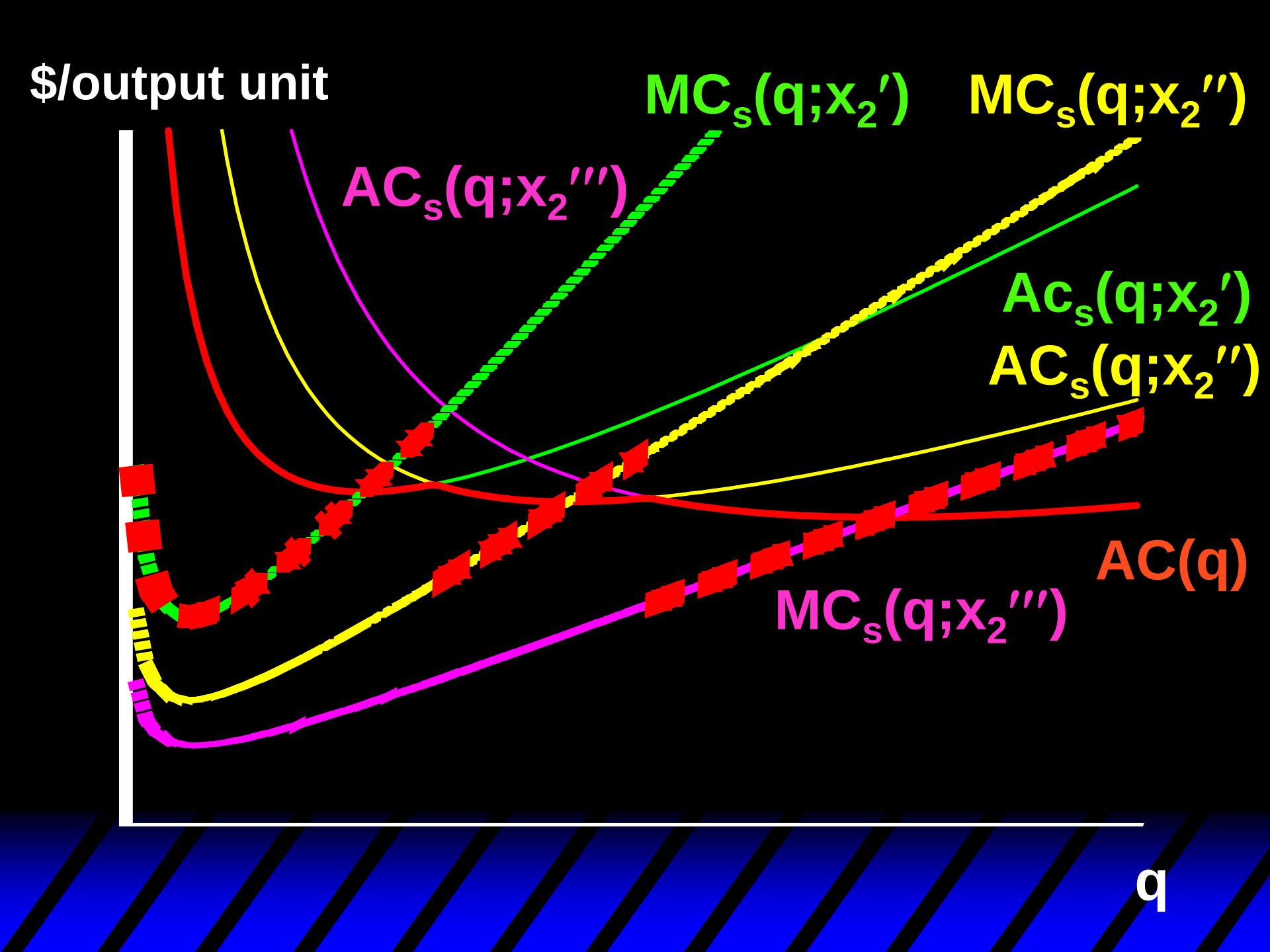

$/output unit
$AC_s(q;x_2')$
$AC_s(q;x_2'')$
$AC_s(q;x_2''')$
q


$/output unit
$AC_s(q;x_2''')$
$AC_s(q;x_2')$
$AC_s(q;x_2'')$
AC(q)
The long-run av. total cost curve is the lower envelope of the short-run av. total cost curves.
q

Short-Run & Long-Run Marginal Cost Curves

- **Q: Is the long-run marginal cost curve the lower envelope of the firm's short-run marginal cost curves?**
- **A: No.**


$/output unit
$MC_s(q;x_2')$
$MC_s(q;x_2'')$
$AC_s(q;x_2''')$
$AC_s(q;x_2')$
$AC_s(q;x_2'')$
$MC_s(q;x_2''')$
q


$/output unit
$MC_s(q;x_2')$
$MC_s(q;x_2'')$
$AC_s(q;x_2''')$
$Ac_s(q;x_2')$
$AC_s(q;x_2'')$
$AC(q)$
$MC_s(q;x_2''')$
q

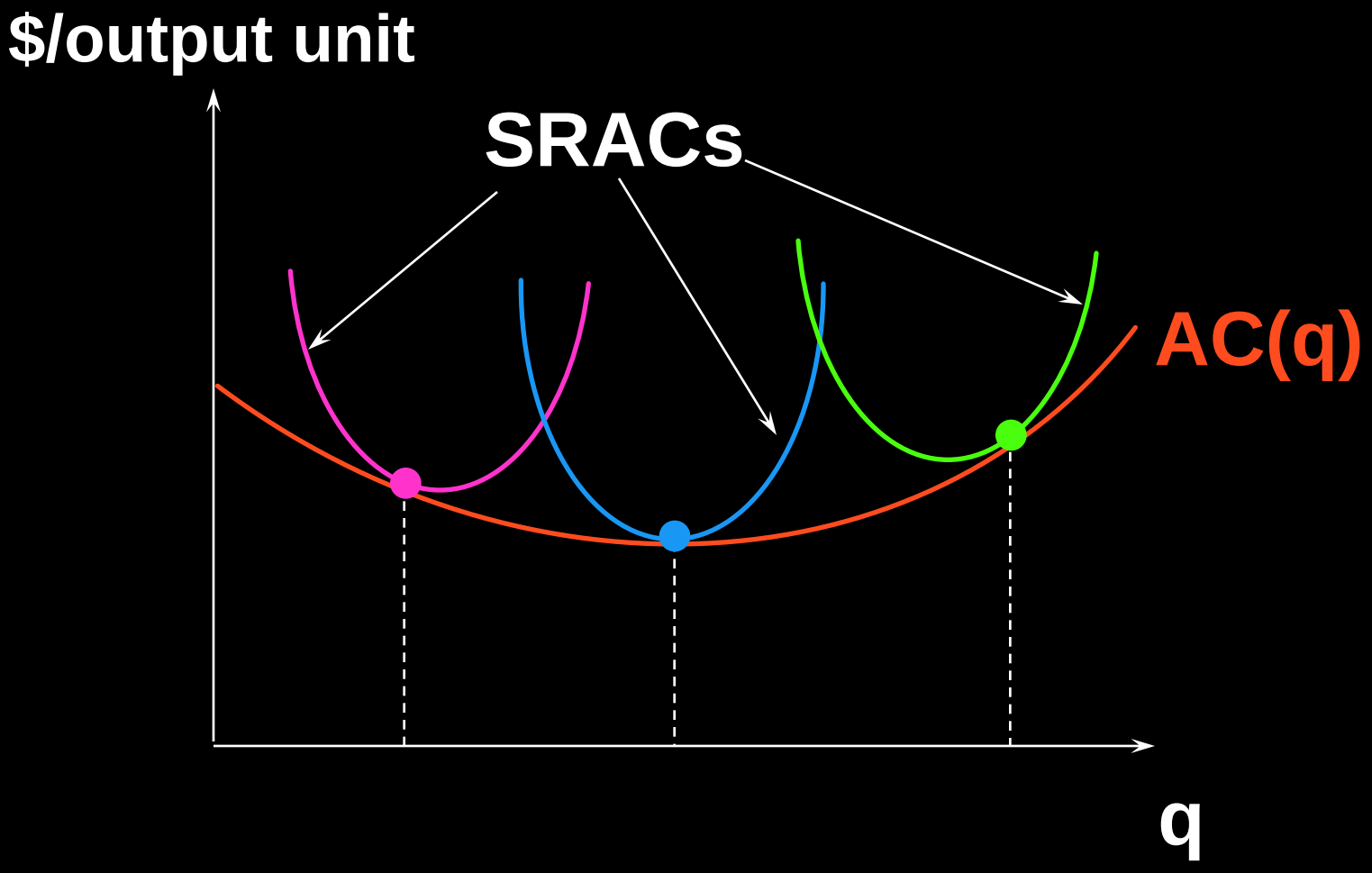
Short-Run & Long-Run Marginal Cost Curves

- **For any output level q > 0, the long-run marginal cost is the marginal cost for the short-run chosen by the firm.**
- **This is always true, no matter how many and which short-run circumstances exist for the firm.**

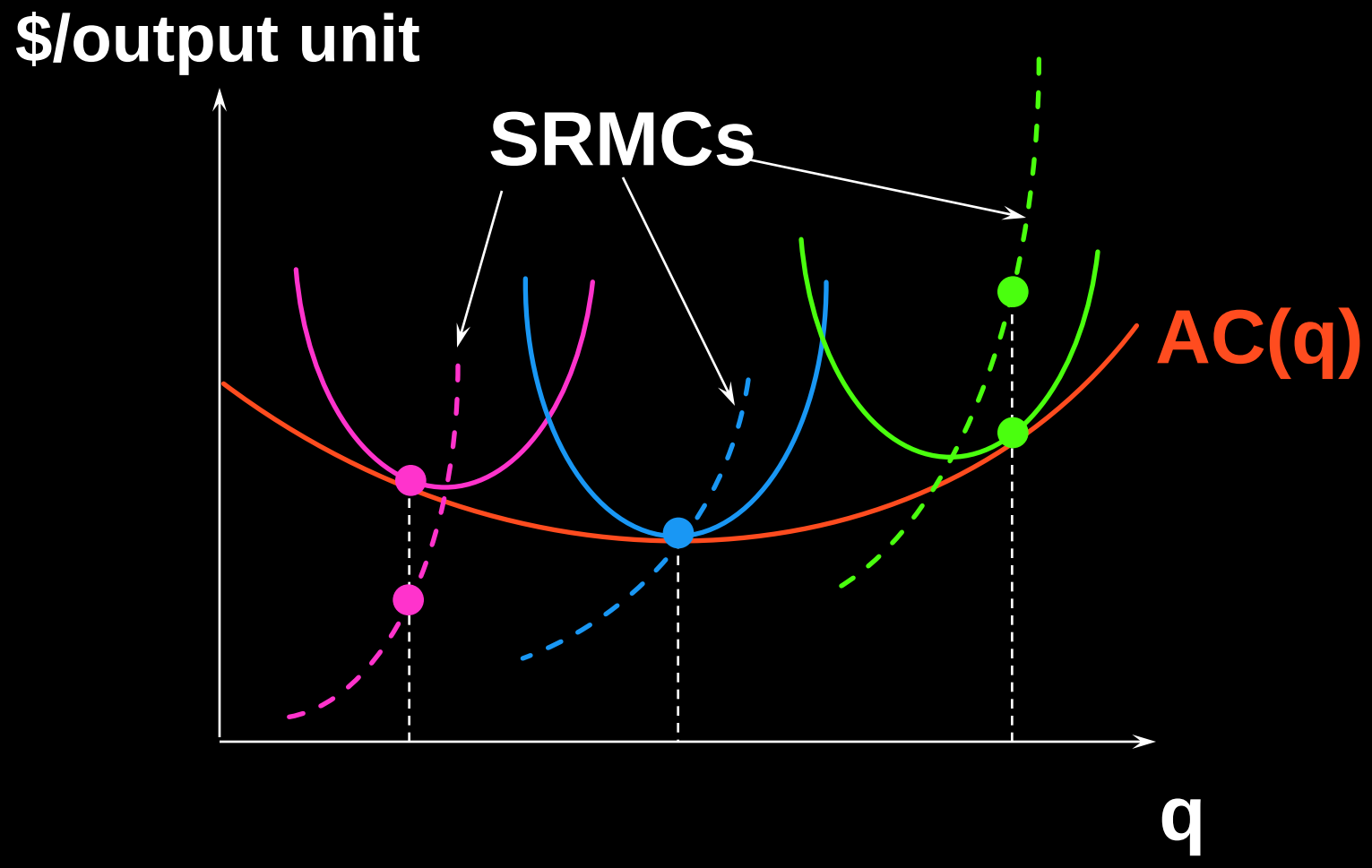
Short-Run & Long-Run Marginal Cost Curves

- **So for the continuous case, where x_2 can be fixed at any value of zero or more, the relationship between the long-run marginal cost and all of the short-run marginal costs is ...**

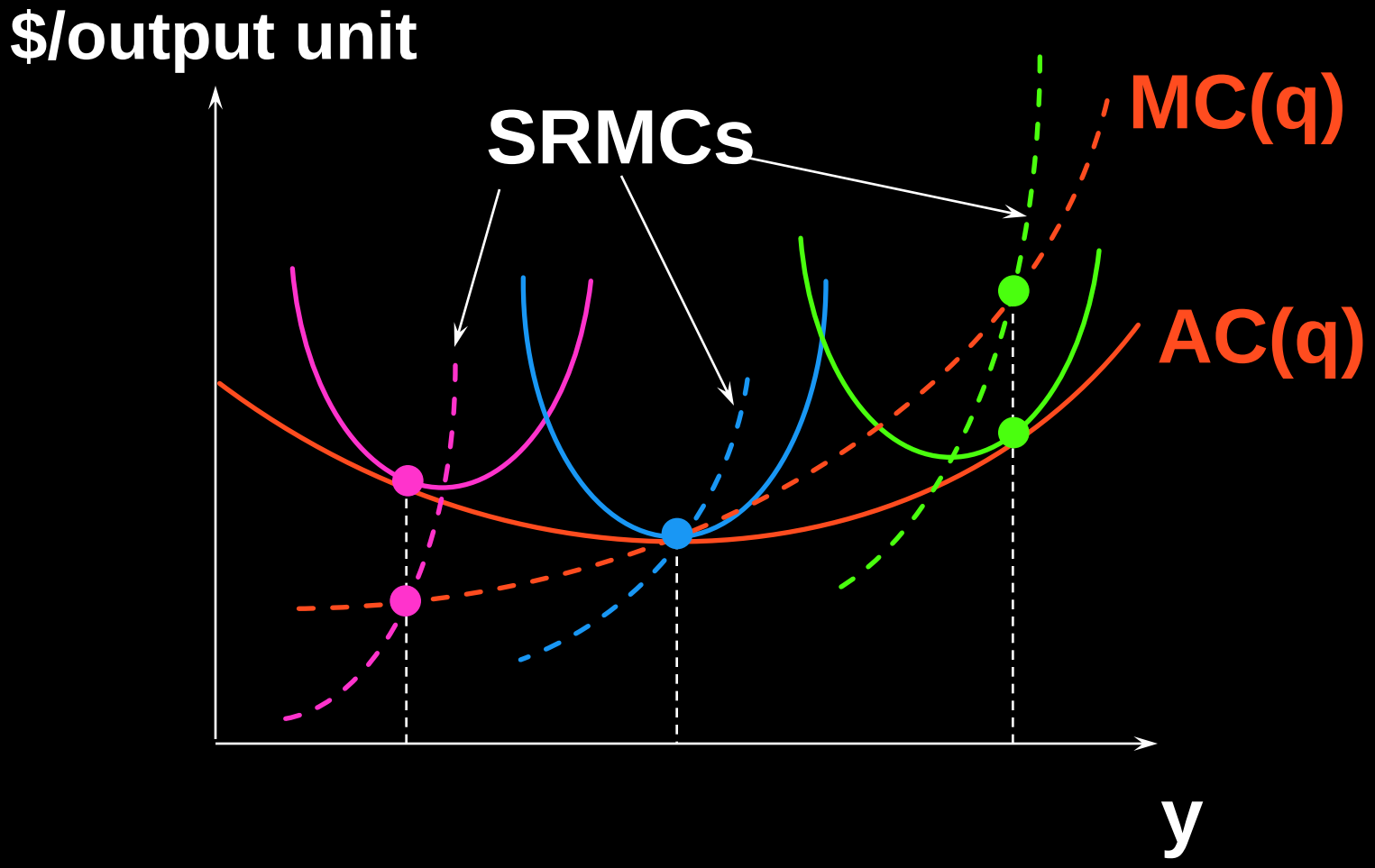
Short-Run & Long-Run Marginal Cost Curves



Short-Run & Long-Run Marginal Cost Curves



Short-Run & Long-Run Marginal Cost Curves



◆For each q > 0, the long-run MC equals the MC for the short-run chosen by the firm.