

SUMMARY OF CONSUMER THEORY

AXIOMS OF CONSUMER PREFERENCES

$$\begin{aligned} \text{MAX } U(q_1, q_2) \\ \text{s.t. } p_1 q_1 + p_2 q_2 \leq Y \end{aligned}$$

SOLVE

MARSHALLIAN DEMAND

$$q_i = d_i(p_i, p_j, Y) \quad i \neq j$$

SUBSTITUTE

INDIRECT UTILITY FUNCTION

$$V = v(p_i, p_j, Y) \quad i \neq j$$

ROY'S IDENTITY

$$\left( \frac{\partial v / \partial p_i}{\partial v / \partial Y} \right)$$

$$\begin{aligned} \text{MIN } p_1 q_1 + p_2 q_2 \\ \text{s.t. } U(q_1, q_2) \geq U_0 \end{aligned}$$

SOLVE

HICKSIAN DEMAND

$$q_i = h_i(p_i, p_j, U) \quad i \neq j$$

SHEPARD'S LEMMA  
( $\partial E / \partial p_i$ )

EXPENDITURE FUNCTION

$$E = E(p_i, p_j, U) \quad i \neq j$$

SUBSTITUTE

SLUTSKY EQUATION

INVERSION

- Prices and Income are OBSERVABLE
- UTILITY IS NOT OBSERVABLE

- Prices and Expenditure OBSERVABLE
- UTILITY NOT OBSERVABLE.

## WELFARE EVALUATION

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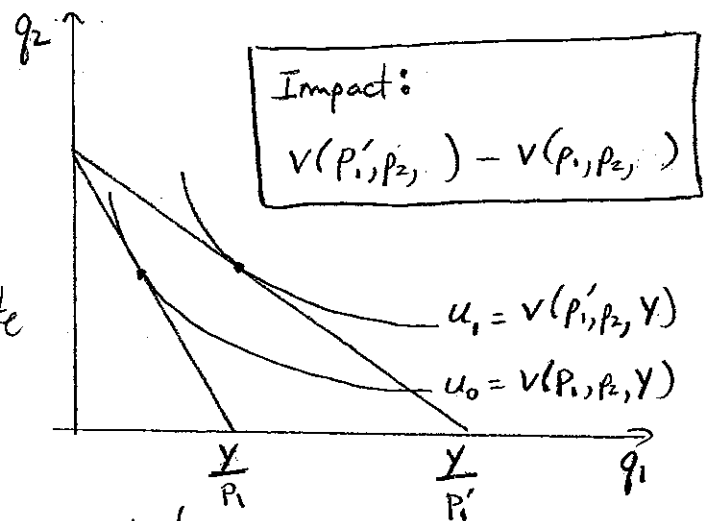
Underlying our approach to consumer theory has been the goal of developing a tool for the welfare evaluation of policy changes.

Recall that:

// The utility Max problem gives  $q_i = d_i(p_i, p_j, Y)$  and  $v(p_i, p_j, Y)$  which are observable (at least, in principle)

↳ Consider  $\Delta p_i$  from  $p_i$  to  $p_i'$ . What's the impact?

- Change in Demand is observable
- One way to measure impact is to use the solutions under old and new prices to compute the change in utility.



\* But this measure is essentially useless!

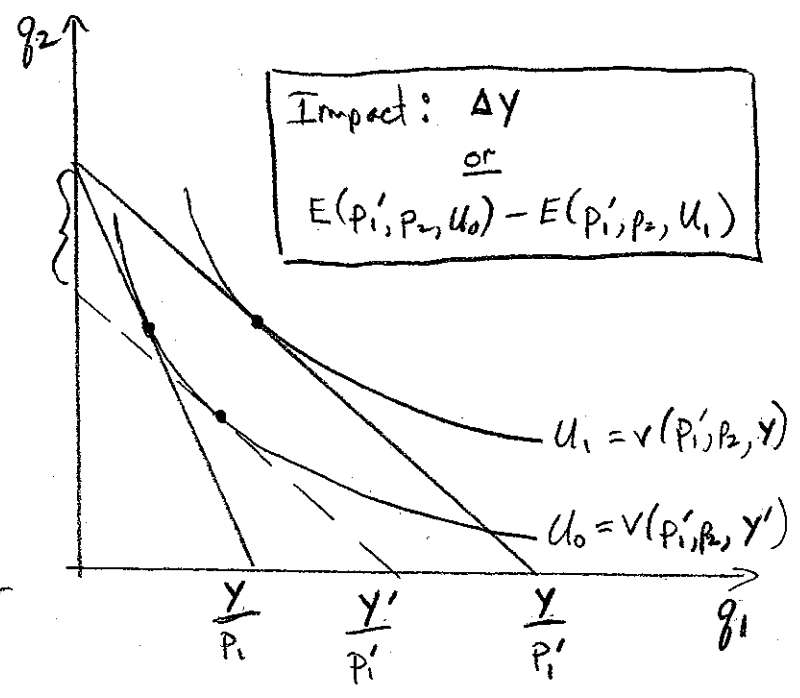
↳ utility is ordinal, not cardinal. so  $v(p_1', Y) - v(p_1, Y)$  does not tell us anything.

↳ Even more meaningless when comparing across people.

2// The EXP. MIN problem gives us  $q_i = h_i(p_i, p_{-i}, u)$  and  $E(p_i, p_{-i}, u)$  which are based in part on unobservables.

↳ Consider  $\Delta p_i$  from  $p_i$  to  $p_i'$ . What's the impact?

- A good measure is the change in income necessary to make consumer as well off as before the price change.



- By how much does  $Y$  have to change so that consumer is indifferent btw  $(p_1', p_2, Y)$  and  $(p_1', p_2, Y')$ ?

↳ For what value  $\Delta Y$  does  $v(p_1', p_2, Y) = v(p_1', p_2, Y')$ ?

\*  $\Delta Y = Y' - Y$  gives a monetary value for impact of  $\Delta p_i$ !

↳ This measure is better b/c it's comparable (observable).

↳ we can elicit amount of \$ people need to be "compensated" for a change in policy (through experiments, surveys, estimation)

\*\* BUT this approach suffers from a problem; it too depends on utility!

Summary

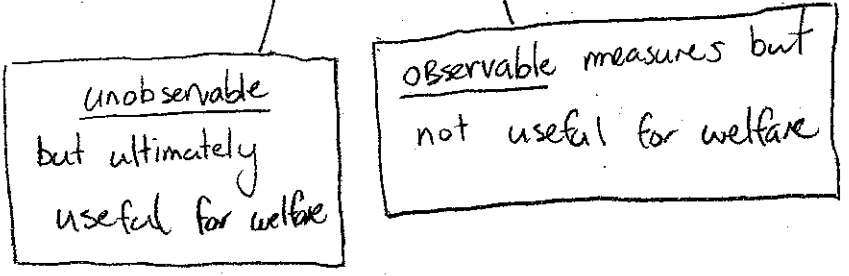
- ①  $q = d(p, Y)$  and  $V(p, Y)$  depend on observable concepts (prices and income) but cannot be used for welfare comparisons.
- ②  $q = h(p, u)$  and  $E(p, u)$  can be used for welfare comparisons but are based on unobservable concepts (utility).

SOLUTION: To somehow derive  $q = h(p, u)$  from  $q = d(p, Y)$

\*\* Fortunately, we are able to do exactly that!  
We use the Slutsky Equation to provide the link.

$$\underbrace{\frac{\partial d_i}{\partial p_i}}_{(- \text{ or } +)} = \underbrace{\frac{\partial h_i}{\partial p_i}}_{(-)} - \underbrace{\frac{\partial d_i}{\partial Y}}_{(- \text{ or } +)} q_i \quad \text{or} \quad \underbrace{\frac{\partial h_i}{\partial p_i}}_{\substack{\uparrow \\ \text{unobservable} \\ \text{but ultimately} \\ \text{useful for welfare}}} = \underbrace{\frac{\partial d_i}{\partial p_i} + \frac{\partial d_i}{\partial Y} q_i}_{\substack{\uparrow \\ \text{observable measures but} \\ \text{not useful for welfare}}}$$

Slutsky provides a link btw observable concepts and useful concepts. for welfare.



# Cost / Benefit Analysis

We've seen how individuals respond to price changes:

- $q = d(p, Y)$
- IE and SE
- Ordinary vs. Giffen Goods
- Etc...

How much do price changes hurt/help consumers?

↳ We would like a monetary measure.

↳ Fortunately, we have one - the expenditure function.

Ex : Pain killer

Initial Price = \$1/pill and I consume 4/week

New Price = \$2/pill and I consume 3/week

Total Income = \$20/week

How much worse off am I?

# Story from Principles class

Total Value of 4 pills is the entire shaded area

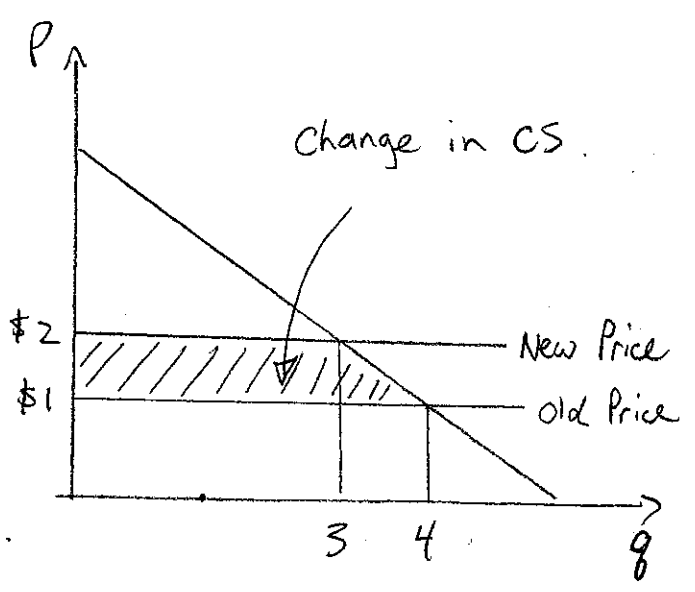
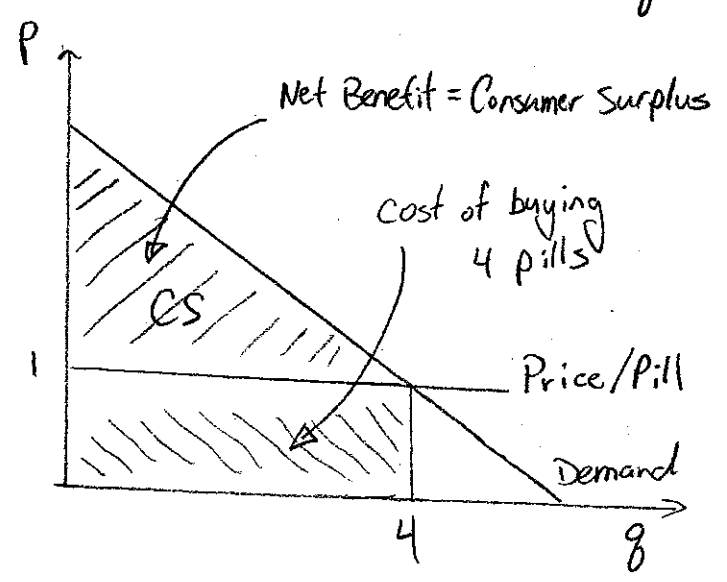
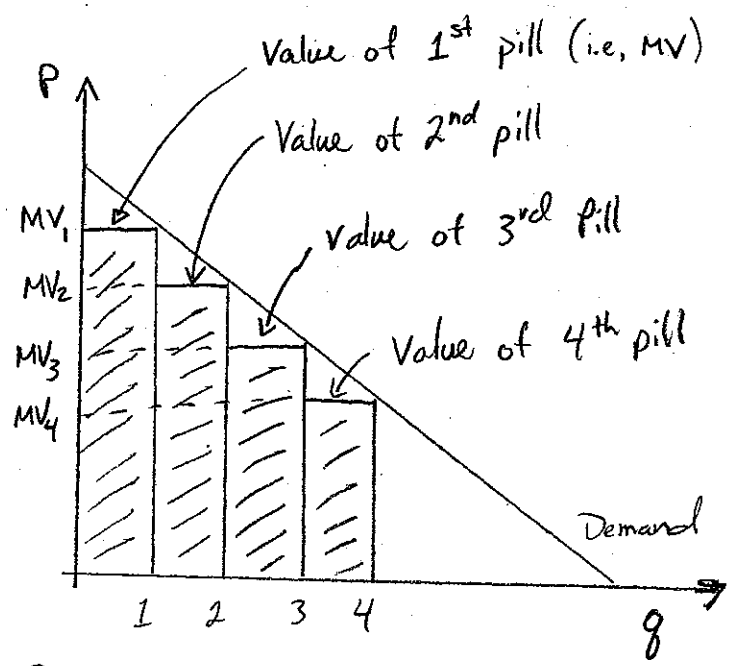
↳ Sum of Marginal Values (MV)

\*\* Demand refers to Marshallian Demand

Total Value minus Total cost of buying 4 units is Net Benefit or Consumer Surplus.

When price increases from \$1 to \$2, Qty demanded falls from 4 units to 3 units.

↳ Change in CS is shaded area.



# EQUIVALENT AND COMPENSATING VARIATION

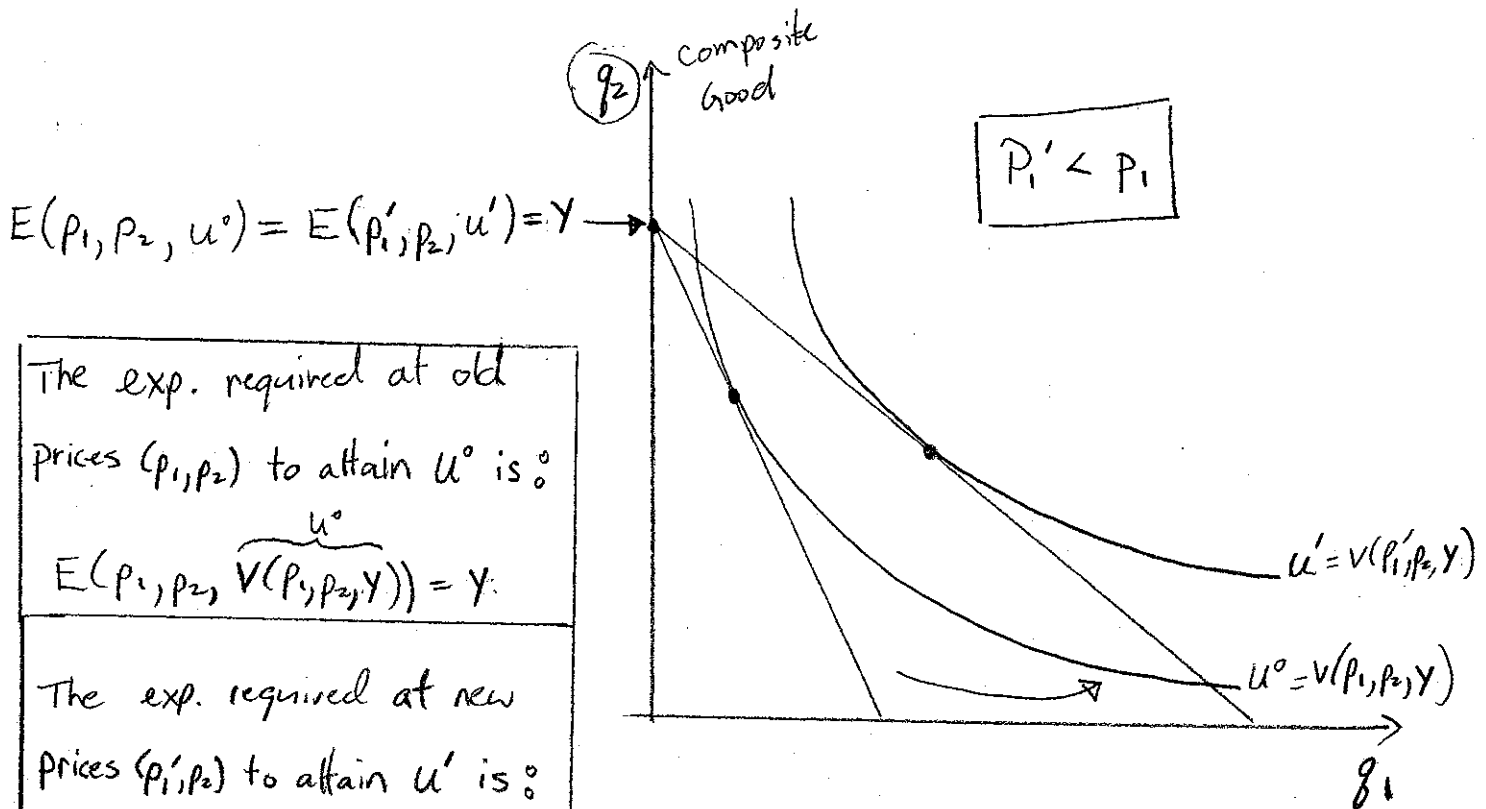
Change in CS is a crude approximation to the change in consumer welfare.

\* There are more precise measures called :

- (1) Equivalent Variation (EV)
- (2) Compensating Variation (CV)

Initial Prices :  $\{P_1, P_2, Y\} \Rightarrow U^0 = V(P_1, P_2, Y)$

New Prices :  $\{P_1', P_2, Y\} \Rightarrow U' = V(P_1', P_2, Y)$



The exp. required at old prices  $(p_1, p_2)$  to attain  $u'$  is :

$$E(p_1, p_2, \overbrace{V(p_1, p_2, Y)}^{u'}) = Y$$

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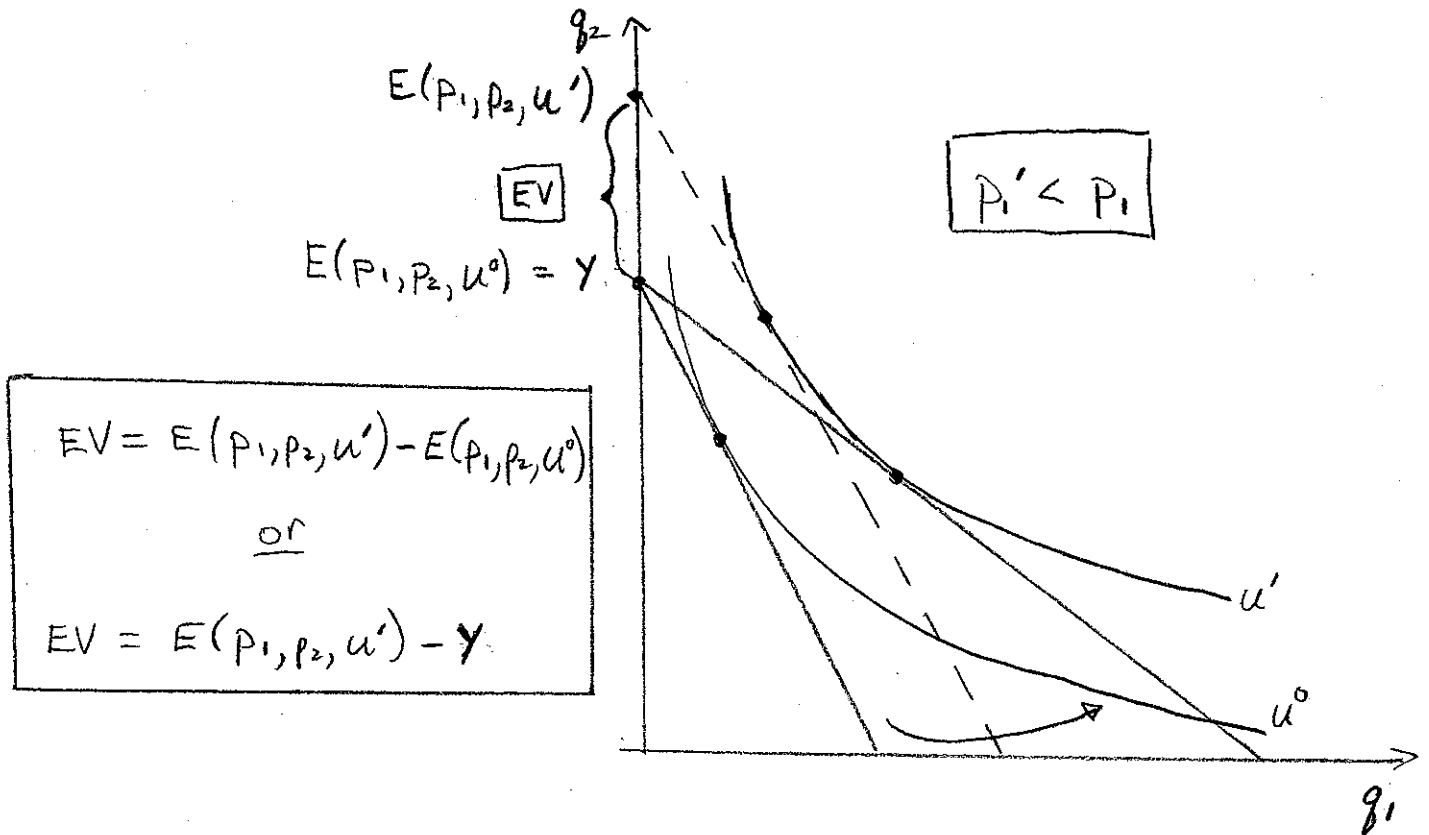
$$E(p_1', p_2, \overbrace{V(p_1', p_2, Y)}^{u'}) = Y$$

We now ask 2 questions.

(1) What is the variation in income that's equivalent to the price change in terms of its welfare impact?

↳ what amount of cash is consumer indifferent about accepting in lieu of the price change?

↳ This is the EV, and the reference price is the old prices b/c it's a change in income producing same change in utility, so it's as if the price change never occurred.



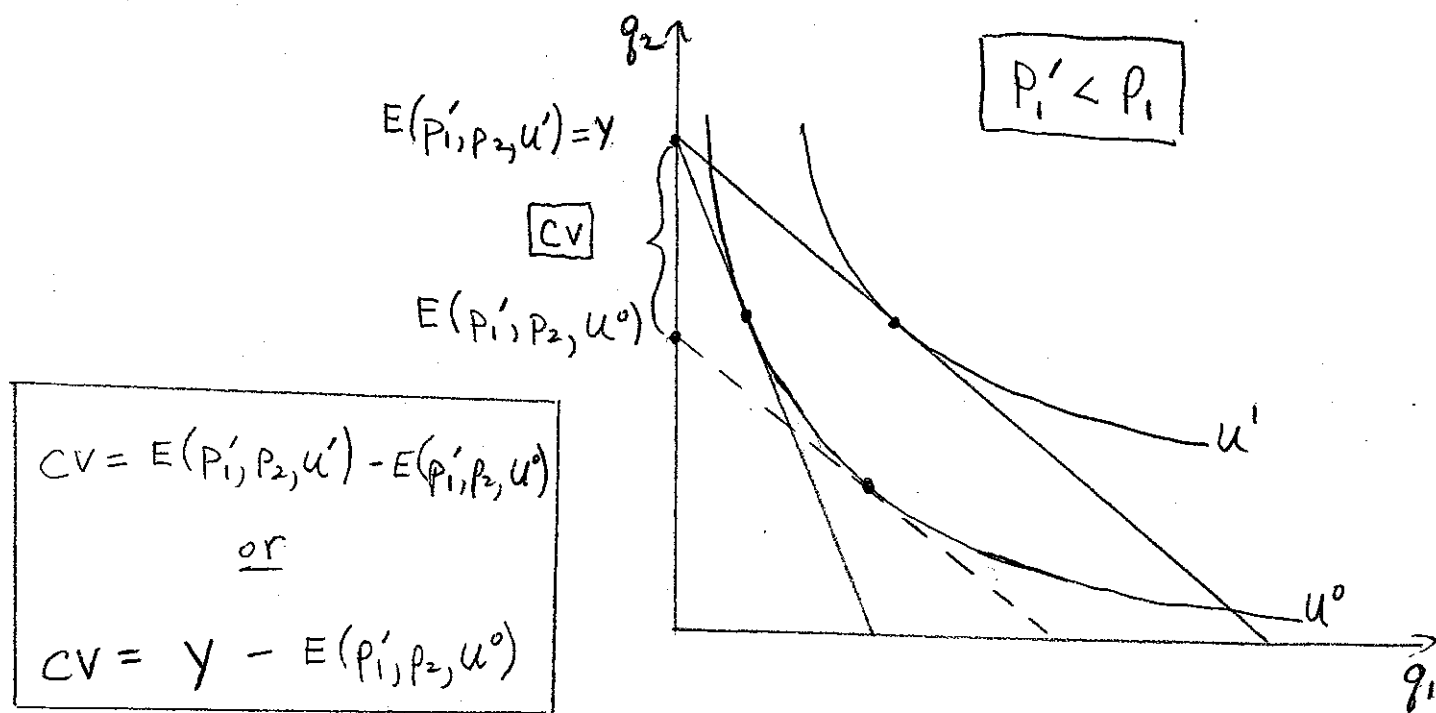
\* EV can be seen as a way to measure the "distance" btw indifference curves as if a price change had occurred.



(2) What is the variation in income the compensates the consumer for the price change after it occurs.

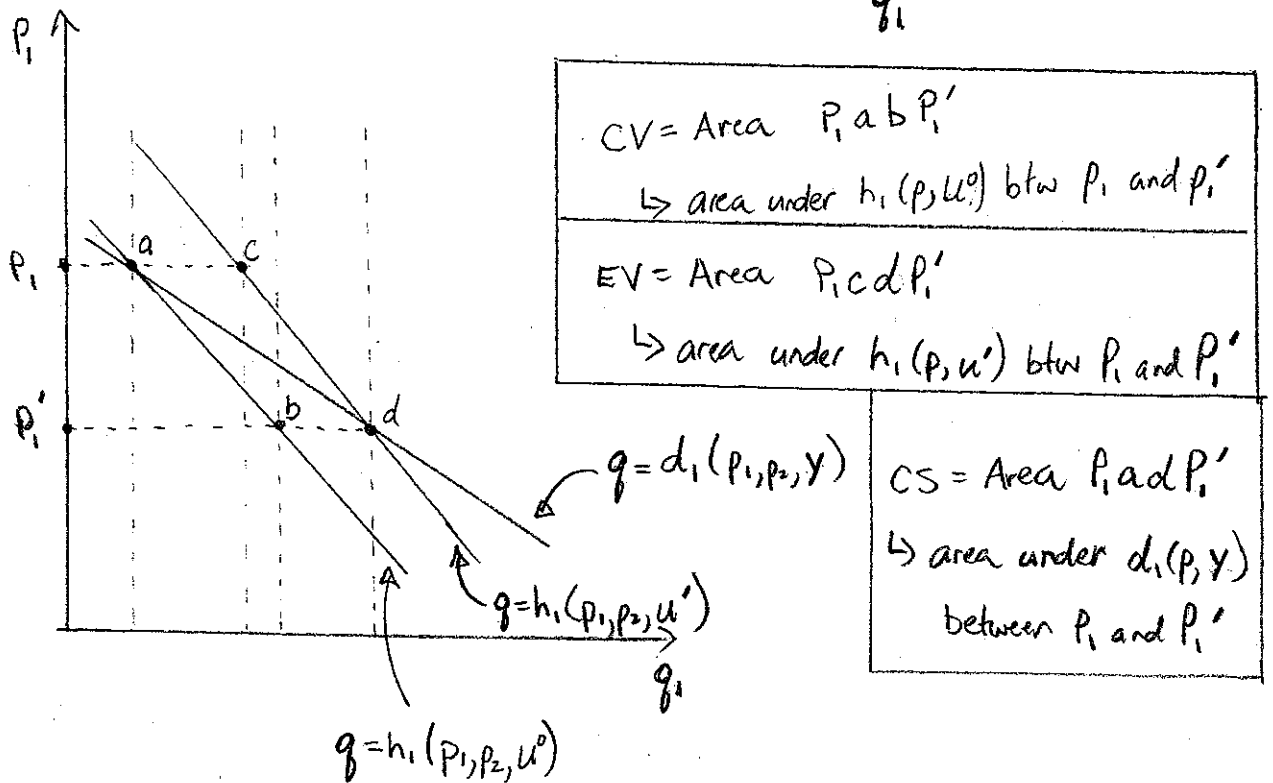
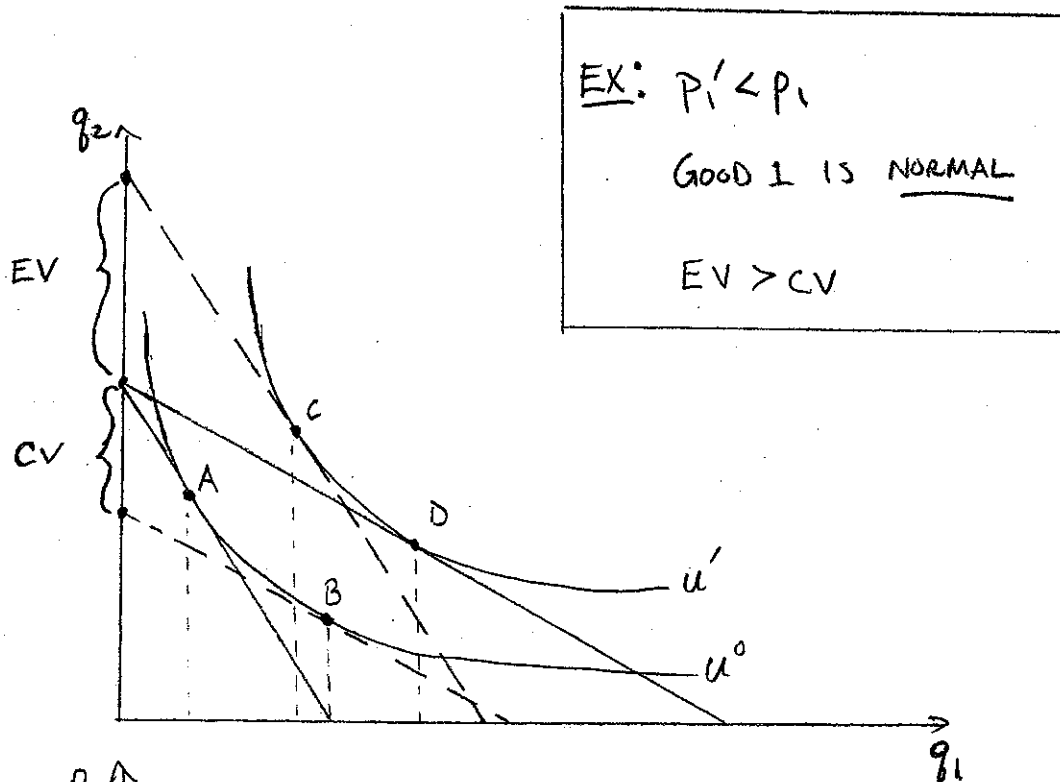
↳ what change in income brings the consumer back to original indifference curve  $u^0$  after the price change occurs.

↳ This is CV and the reference prices are new prices b/c it measures the welfare impact after a price change occurs.



\* CV can also be seen as a way to measure the "distance" btw indifference curves once a price change occurs.

\* EV and CV have important representations in terms of the Hicksian or Compensated Demand curves.



- NOTES: (1) THIS relation btw EV and CV reverses when good 1 is inferior  
 (2) IF IE=0  $\Rightarrow$   $EV = CV = CS$  !