Methodology and Modeling

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1 Methodology

Economics is a **science** concerned with *understanding, explaining, and predicting* the nature of individual and social behavior in terms of the *incentives* that motivate individual decision-making and the *constraints* that limit these choices.

It strikes most students as odd that economists regard their subject as a science. After all, economists don't go running around wearing white lab coats conducting experiments in carefully controlled lab environments. This caricature is no doubt the result of the process used to categorize subjects in education. Unfortunately, students in high school are taught to think that 'science' is a particular field of study like physics, chemistry or biology. But 'science' is not a particular field of study; nor is it a particular method to problem-solving involving 'real' experiments in controlled environments. Science is nothing more than a *process* of discovery, and there are many different approaches or methods considered to be 'scientific.'

1.1 The Scientific Process

All sciences (physics, chemistry, and biology included) are concerned with constructing arguments of the following form:

$$X \Longrightarrow Y,$$

where

Y is a set of phenomena to be explained,
X is a set of premises or assumptions (treated as 'given' or beyond explanation),
$\Longrightarrow$ is the operation of deductive logic.

We generally refer to the triplet $(X, \Longrightarrow, Y)$ as a theory of Y. The *theory* of Y *explains* phenomena Y in the context of what is assumed to be given (X) and what follows logically ($\Longrightarrow$) from these assumptions. Thus, an explanation is seen to consist of a *sequence of logically connected statements* beginning with a set of premises or assumptions (X) and concluding with phenomena (Y).

One thing to note here is the close relationship between the concepts of *theory* and *explanation*. As stated here, they are virtually equivalent. Hence we can conclude that explanation is not possible without theory! One important implication of this conclusion is that we cannot simply look at the world and 'let the facts speak for themselves.' Unfortunately, facts don't speak for themselves, and can only be interpreted in the context of some theory. Facts may be described without reference to theory, but description is not explanation ... and its important you don't confuse the two.

1.1.1 Criteria for Evaluating a 'Good' Theory

All theories require premises or assumptions in explaining some phenomena. Varying any one of these premises will typically alter the nature of a theory's conclusions *along some dimension*. Most business students have trouble with this element of scientific inquiry. The fact is most subjects in business school do not attempt to be scientific. The reason is that most authors of business textbook (excluding finance) are not explicit about the assumptions they make. Unfortunately, this leaves students with the impression that assumptions are unnecessary for making good arguments. This is false! There can be no explanation without theory; and no theory can explain everything. Thus, some elements of an argument must be taken as beyond explanation.

The most common criticism made about economic theories is that they contain assumptions that appear to be wildly inaccurate representations of reality. Yet, such criticism is often misguided. It turns out, physical scientists make simplifying assumptions that are known to be false all the time. Consider perhaps the most famous model of all time: Newton's universal law of gravitation. So famous is his model that everyone knows what would happen if you jumped off a ten story building without a parachute. Yet when physicists measure gravity they often do so by calculating how fast an object falls in a vacuum. By using a vacuum, the influence of air friction, which is present for all falling objects, is assumed away. Thus, physicists must be sensible about employing such an assumption and take into account the coefficient of air friction for different objects before proceeding.

Furthermore, although Newton's laws have been used successfully in predicting, among other things, the orbit of the planets, the law of gravitation - which describes the physical relationship that exists between any two objects on the basis of their mass - breaks down at the atomic level. Evidently, the law of gravitation does not apply to extremely 'small' objects. Yet no one seriously argues for discarding Newton's theories based on the fact that such laws do not provide a descriptively accurate representation of the behavior of sub-atomic particles.

It's important to remember that all theories will be false ... at least along some dimension! Rather than judge a theory based on the truth or falsity of its assumptions, most economists (and most other scientists) evaluate a theory on the basis of its **usefulness**. The main question is this: Is a theory more useful in providing us a plausible explanation for observed phenomena that in some sense is superior to competing explanations? If the answer is yes, then a theory is generally regarded as being superior to the alternatives regardless of the truth or falsity of its assumptions.

1.1.2 Three Characteristics of a 'Useful' Theory

Although 'usefulness' can be broadly interpreted, all useful theories share some basic characteristics. In economics, as with all sciences, the 'usefulness' of a theory has three main characteristics: formality, testability and simplicity. Laying out our assumptions so that they can be examined, and generating 'simple' propositions that can be tested and refuted, are all part of the scientific process.

Formality Economists express their theories using **models**. But models are nothing more than a formal representation of a theory. The use of graphs and mathematics force us to be quite formal about the substance of an argument. Thus, formality is useful because it forces us to be explicit about the assumptions being made. By being honest about our assumptions, we have a better idea of where our model needs fixing when it fails to explain some behavior.

Given the nature of business school training, students often have a difficult time with the formality of economic arguments. Compared with many other business school subjects, it seems as if there are too many steps in the sequence of logically connected statements. Such a comparison is misleading, however, since most 'models' or theories in business textbooks are not explicit about the assumptions being made. This is unfortunate because it gives students the impression that models with 'baggage' (i.e., assumptions) are somehow inferior to ones without – usually since one or more of the assumptions are descriptively false. It is important to note, however, that the models found in most business school textbooks have assumptions behind the scenes ... they just leave it you to uncover them. Equally important, providing a formal argument does not necessarily mean providing a mathematical one. Being formal means being explicit about all of the 'givens.' It is the formality of an argument that allows us to understand what follows logically from a set of assumptions.

Testability Models or theories which are not testable are impossible to refute. Such theories are not useful because they can never be wrong! Having a model that is testable means having a model that can be shown to be wrong. After all, if a model's assumptions are explicit, and the model fails to explain, we know that at least one of the assumptions is wrong and we know where to look to fix it.

The issue of testability raises some interesting problems for economists. Model building is complicated by the fact that we cannot test our theories in any direct way. As Robert Lucas points out, for example, we cannot build a model of the business cycle (the booms and recessions of the economy) and then test it by public policy. People would not look kindly at us if we deliberately engineered a recession (lower incomes, fewer jobs, etc.), even though this is the only convincing way to test the theory for real. Yet, as Lucas argues, this is actually a virtue of the democratic system – that we don't let people/kings experiment with our lives.

So how do we test such a theory when we cannot use real people as our laboratory? The short answer, as Lucas argues, is to create an abstract rendition of reality, i.e., a make-believe economic system which is similar in all important respects to the one in which we live. Thus, economists (like other scientists) are really story-tellers, operating most of the time in a world of make-believe. Story-telling is not only a useful activity but an essential one, since it's the *only* way we can think seriously about the world in which we live.

Of course, story-telling is not unique to economics. Indeed it is commonplace in the physical sciences as well. The subjects of cosmology, geology, and climatology are in similar straits. For example, climatologists may be keenly interested in the answer to the following question: suppose that the rate of world CO_2 emissions doubles during the next decade. What would happen to the average surface temperature of the Earth's atmosphere? As it turns out, climatologists have a pretty good idea of how to increase CO_2 emissions. The only really convincing way to test their theories would be to increase greenhouse gases and measure the impact on the earth's temperature. Of course, this is never going to happen – at least not in any deliberate way. Thus, in the absence of a make-believe atmospheric system, experimentation is simply not available.

In some respects, cosmologists and geologists face even greater obstacles. This is because the evolution of the physical universe is not influenced in any meaningful way by human behaviour. Hence, these scientists couldn't engineer a change in the physical environment even if space or time-travel was possible. Since it not possible (let alone practical) to alter the physical universe, cosmologists and geologists have only one meaningful alternative: tell stories using make-believe systems.

Simplicity Finally, as Einstein once said, a model must be "as simple as possible, but not simpler ...." The whole point of model-building is to make us better thinkers; the more simple the model, the more useful it is for thinking. Hence, there's no point in building models that are as complex as the world it describes. In order for a model to be useful, it must be a simplification of the real world.

To illustrate, consider the use of a topological map for surveying, hiking, etc. A topological map describes the combinations of longitude and latitude that yield a given height above sea level. Move along a given contour line on a topological map and your height above sea level never changes. A contour map is extremely useful because it captures everything important to surveyors and hikers (and nothing more) in just 2 dimensions. Any 3 dimensional, real-world model would be worse (i.e., larger, heavier and more cumbersome) than a 2 dimensional piece of paper. Thus, all of the things that a contour map abstracts from are not essential to its main purpose.

Now if your not familiar with a contour map, consider a typical road map. Simplicity is the essence of a road map as well. In many ways, a road map is really a silly description of reality. Again, the reason is that a road map is 2 dimensional object and our world is not. Like a contour map, it also abstracts from really interesting features of the real world, such as liquor stores and ice-cream stands. But it is precisely this silliness that makes it useful, supplying good answers to questions like: where in the (expletive) am I?

The moral of the story here is that we should never attack a theory (or model) simply because it doesn't seem descriptive of the world in which we live. Often, simple, testable and formal models are the only practical way we can think sensibly (i.e., scientifically) about the world around us.

2 Modeling

As noted above, economists express their theories with models. A **model** is simply a formal representation of the theory. The mathematics used to express modern economic theory can get rather sophisticated. In some fields, the level of mathematical rigor is on par with that of theoretical physics. As it turns out, however, much of this rigor is directed toward convincing *other economists* that their theories are consistent, compelling and useful. This task generally requires formal proofs of propositions that comprise a particular theory.

But the use of such abstract mathematics is not necessary for introducing students to modern economic theory. Indeed, there are "diminishing marginal returns" to the degree of sophistication in mathematical technique. Students can quickly get bogged down in learning various mathematical techniques, leaving little or no time to understand what the math is telling them. For this reason, the fundamental constructs of modern economic theory will be conveyed and understood using nothing more than *high school algebra, analytic geometry, and basic differential calculus.* This section briefly reviews the first two of these.

2.1 Prerequisites

It is assumed that you can manipulate simple algebraic expressions. Consider the following expression:

$$ax + c - by = d,$$

where x and y are variables and a, b, c and d are constants. You should be able to verify that this expression can written in terms of y as follows:

$$y = \frac{a}{b}x + \frac{c-d}{b}.$$

Furthermore, you should recognize that the graph of this equation, drawn with x on the horizontal axis and y on the vertical axis, is a straight line with a slope equal to a/b and a vertical intercept equal to $(c-d)/b$. A slope of a/b means that if x increases by one unit, then y changes by a/b units. A vertical intercept of $(c-d)/b$ units means that y equals $(c-d)/b$ when x is equal zero.

It is also assumed you can solve a system of two equations with two unknowns. For example, the equation $y = 2x$ represents an upward sloping line through the origin with a slope equal to 2. Similarly, the line $y = 9-x$ represents a downward sloping line with a vertical intercept of 9 and a slope equal to -1. Since the two lines have different slopes, they must intersect at a single point. You must be able to solve these two equations for the point of intersection. You should verify that this point of intersection is given by $x = 3$ and $y = 6$.

2.1.1 Functions of One Variable

You should also be familiar with the concept of a **function**. It is assumed that you are familiar with the notation

$$y = f(x),$$

which, in words, says 'y is a function of x,' and the name of the function is f. The equation for the straight line given above is an example of a well behaved function. For the purpose of this course, we say a function is "well behaved" if, for every value of x, there is a unique corresponding value for y.

An example of a nonlinear function is given by

$$y = \sqrt{x},$$

or equivalently

$$y = x^{1/2},$$

where $x \geq 0$. You should be able to graph this function by evaluating it at various points in the domain ($x \geq 0$), and connecting the points in the x-y plane with a smooth curve.[1] You should also be equipped to do this quickly using a spreadsheet. Once drawn, verify that the function is positively sloped at each point in the domain, but that the slope of the function decreases as the value of x increases.

2.1.2 Functions of Two or More Variables

Your past experience with mathematics probably required you to memorize too many formulas (Hint: memorization in this course will get you in deep trouble!). For example, the area of a triangle equals one-half the base of the triangle times its height. This formula is typically expressed more compactly as

$$A = \frac{1}{2}bh,$$

where A is the area of the triangle, b is the length of its base, and h is the height of the triangle. We say A is a function of the two variables b and h.

Note that the letters in the above formula are chosen for the their mnemonic value. That is, they remind us of the geometric concepts that they represent,

[1] By convention, the "x-y plane" means the following. "x-y" means the plane is 2 dimensional. If it were "x-y-z," it would be a 3 dimensional plane. The order "x" then "y" means we are referring to horizontal axis (the x axis) first and vertical axis (the y axis) second.

i.e., b for base and h for height. We shall continue to employ this approach to modelling wherever possible. Of course, it is entirely possible, even likely, that this function (or formula) is useful or meaningful outside of Euclidean geometry. Thus, we may wish to rewrite it more generally as

$$z = \frac{1}{2}xy$$

with the variables renamed. In economic terms, this function could potentially describe a production relationship between quantities of inputs employed (x and y) and the maximum quantity of output produced (z) in a given period.

For many applications it is often unnecessary to specify the exact algebraic form of a function. The more general expression

$$z = f(x, y)$$

simply states that z is a function of the variables x and y. The name of this function is "f," but we could have called it "frank" or anything else we wanted; we use the notation "f" because it's more economical and less confusing. In this example, the function f determines a unique real number z (if it's well behaved) for any given pair of real numbers (x, y). In a sense, the value of z determined by the function is *dependent* on our choice of x and y. For this reason, z is sometimes called the **dependent variable** and x and y are called **independent variables**. Less intuitively, economists frequently refer to z as an **endogenous variable**, while x and y are called **exogenous variables**.

More formally, we say that the function f maps points in the **domain** of f into the set of real numbers. In the example given above, the domain is comprised of all pairs of real numbers such that $x \geq 0$ and $y \geq 0$. Although the function z is well defined over negative values of x and y, it doesn't make sense to use negative values if both x and y represent a unit of length. Often in economics, the variables of interest are naturally non-negative (e.g., quantities produced or consumed in a given period, prices of goods or services in a given period, etc.). Thus, the domain of functions will typically consist of combinations of non-negative values of the independent (exogenous) variables x and y.

Recall the "triangle area function," given by the following *specific* function form

$$z = \frac{1}{2}xy,$$

where $x \geq 0$ and $y \geq 0$. If we label this function "f" we can also write it as

$$f(x, y) = \frac{1}{2}xy.$$

One way to make sense of a function is to evaluate it at some of the points in its domain and plot these points on ordinary graph paper - or, better yet, plot the function using a spreadsheet. If the points are closely enough spaced, the graph of the function is easily inferred from the points. This procedure, tedious in the absence of a spreadsheet program, usually works fine for functions in one variable, but is more complicated with functions of two variables (you have to be good at drafting). Suppose that we decided to evaluate the area function for all integers between 0 and 9 for each x and y. That is, we compute z, i.e., $f(x,y)$, for each point such that

$$x \in \{0,1,2,3,4,5,6,7,8,9\} \text{ and } y \in \{0,1,2,3,4,5,6,7,8,9\}.$$

Thus we have

$$\begin{aligned} f(0,0) &= \frac{1}{2}(0)(0) = 0 \\ f(0,1) &= \frac{1}{2}(0)(1) = 0 \\ f(0,2) &= \frac{1}{2}(0)(2) = 0 \\ &\vdots \\ f(0,9) &= \frac{1}{2}(0)(9) = 0. \end{aligned}$$

And then we have

$$\begin{aligned} f(1,0) &= \frac{1}{2}(1)(0) = 0 \\ f(1,1) &= \frac{1}{2}(1)(1) = 1 \\ f(1,2) &= \frac{1}{2}(1)(2) = 2 \\ &\vdots \\ f(1,9) &= \frac{1}{2}(1)(9) = 9 \end{aligned}$$

and so on. Clearly, patience is required. For each x chosen there are 10 y's and vice versa. In all, there are $10^2 = 100$ separate points on which to evaluate the function.[2] Even if we do carry through with all of these computations, the graph of the function is a *surface* - a 3 dimensional object. Again, plenty of patience is required to depict the surface on a 2 dimensional piece of paper. Once drawn, the graph of the surface may be pretty but it is difficult to analyze at best. For functions in three or more exogenous variables, this technique fails entirely.

2.1.3 Holding Independent (Exogenous) Variables Constant

We need to simplify life. Suppose that we simply remove one variable from the analysis by holding its value constant. This converts the function in three variables (one dependent or endogenous variable, and two independent or exogenous

[2] Actually, the task is somewhat simpler because x and y enter the function symmetrically. That is, $f(a,b) = f(b,a)$ for all values of a and b. So really, the function only needs to be evaluated at 50 points.

variables) to a function in two variables (one dependent, and one independent variable). Suppose that we first hold y constant at 1. That is, let $y = 1$. Then we have the function

$$f(x, 1) = \frac{1}{2}x(1) \quad \text{or} \quad f(x, 1) = \frac{1}{2}x \quad \text{or} \quad z = \frac{1}{2}x,$$

where $x \geq 0$. The graph of this function is quickly recognized as a positively sloped straight line through the origin with a slope equal to $1/2$ (the coefficient on x). In terms of our geometry example, we have a triangle with a fixed height at 1 unit.[3] Holding the height constant at 1 unit while increasing the length of the triangle's base by 1 unit, increases the area of the triangle by one-half a square unit. That is, the slope of this new function tells us how fast the area of a 1 unit high triangle increases as the base increases. Also, since the line begins at the origin, a triangle with a base of zero length can only be a point with zero mass or area. Finally, note that the right endpoint of this line does not exist. That is, holding the height of the triangle constant, the area of the triangle increases without bound as the base becomes infinitely large.

Now let's consider the function generated by holding y constant at 2. That is, let $y = 2$. Then we have

$$f(x, 2) = \frac{1}{2}x(2) \quad \text{or} \quad f(x, 2) = x \quad \text{or} \quad z = x,$$

where $x \geq 0$. The graph of this function is a positively sloped straight line through the origin with a slope equal to 1. Returning once again to the geometry example, we have a triangle with a height now fixed at 2 units. Holding the height constant at 2 while increasing the length of the base by 1 unit, increases the area of the triangle by 1 square unit. You should graph this function too. We can continue this exercise by fixing $y = 3$, $y = 4$, and so on, and plot each equation in the same graph. It is apparent that increasing y has the effect of "rotating" the graph of the (partial) function relating z to x upward or counterclockwise.

This type of exercise reveals alot about the function f. First, it shows how the dependent variable z depends on independent variable x, *holding constant the value of y*. Each equation corresponding to different values of y are all positively sloped. Thus we say that "f is increasing in x" which means that an increase in x, *holding constant the value of y*, increases the value of z. In the language of introductory economics, the relationship between the dependent variable z and the independent variable x, holding independent variable y constant, is depicted by "movements along" one of these area curves (which one depends on the value of y).

Second, the exercise also illustrates how z depends on independent variable y, *holding constant the value of x*. Suppose that x is held constant at 3 units. In terms of a graph, in which independent variable x is plotted along the horizontal

[3] Here, a unit refers to a measurement of distance. A unit could be measured in centimeters, inches, feet, meters, etc.

axis, holding x constant is depicted by a vertical line at $x = 3$. The intersection of the vertical line at $x = 3$ with each of the individual curves indicates that z (the area) increases as y (the height) increases, holding x (the base) constant at 3. More precisely, the area of a triangle with a base fixed at 3 units increases from 1.5 square units to 3 square units to 6 square units as y increases from 1 unit to 2 units to 4 units. Thus, we say "f is increasing in y," which means that an increase in y, *holding constant the value of* x, increases the value of z. In the language of introductory economics, the relationship between the dependent variable z and the independent variable y, holding independent variable x constant, is depicted by "shifts" in the area curve.

Now, the choice of which independent variable to place on the horizontal axis of a diagram and which to hold constant depends on the particular application. In the above example, the choice is trivial because each independent variable enters the function symmetrically. That is, x and y have the exact same effect on z holding the other variable constant. Thus a diagram with either x or y on the horizontal axis will look identical as long as we evaluate the function with the same values for x or y. Note, however, this is generally not the case. In fact, we should expect to encounter examples of functions that are increasing in one independent variable and decreasing in another.

2.1.4 Functions of More than Three Variables

Although our models are designed to simplify the world, the world can be a complicated place. Understanding various aspects of the economy often requires models with more than three variables. For example, a model with four variables may include a function with all four:

$$z = f(w, x, y).$$

Fortunately, the addition of variables does not entail additional analytical difficulties. The techniques described above still apply. We can always graphically illustrate the relationship between the dependent variable and any one of the independent variables provided we *hold all other independent variable constant*. In addition, we can illustrate the effect on z caused by a change in one of the constant variables (the ones not measured on the horizontal axis) holding all other independent variables constant (including the one being measured on the horizontal axis) with the appropriate shift in the curve.

2.1.5 Holding the Dependent Variable Constant (Level Sets)

Have you ever gone hiking in the mountains with aid of a contour map? Although you are moving around on a 3 dimensional surface (the earth), the contour map occupies only 2 dimensions. Indeed, the map derives its usefulness from this property. The contours of the map represent combinations of longitude and latitude that have identical elevations (usually measured as the distance above sea level). Hike along a contour line and your elevation never changes. Ascend in an area where the contours are closely spaced and you can plan on climbing a steep hill. The contour map is an excellent example of how collapsing 3 dimensions into 2 can be useful. Another example is a weather map that displays combinations of longitude and latitude that yield identical surface temperatures (isotherms) or identical levels of atmospheric pressure (isobars). Contours, isotherms, and isobars are all examples of what we call "level sets."

To see how this works, let's return to the area function given by:

$$z = \frac{1}{2}xy.$$

This time we hold the dependent variable z (area) constant at some predetermined level. We can then use the algebraic description of the function to derive all the combinations of x (base) and y (height) that yield the predetermined area z. Suppose that we are interested in a graphic representation of all bases and heights that produce triangles of 2 square units in area. First, substitute $z = 2$ into the equation describing the area and simplify:

$$2 = \frac{1}{2}xy \quad \text{or} \quad 4 = xy.$$

In graphing this expression it may help to first rearrange it in terms of y:

$$y = \frac{4}{x}.$$

Then, in accordance with existing convention, we graph the resulting expression with x on the horizontal axis and y on the vertical. You should verify (with paper or spreadsheet) that the resulting graph yields an iso-area line that is downward sloping. It has a downward slope because an increase in x (the base of the triangle) requires a *compensating* reduction in y (the height of the triangle) to maintain the triangle's area of 2 square units.

Now set $z = 4$ and find all combinations of x and y whose product yields an area equal to 4 square units. Doing so yields another downward sloping iso-area curve which lies everywhere above the previous one, i.e., to the "northeast" of the iso-area curve of $z = 2$. This means that increases in x or y or both, increases z - the area of the triangle. This makes perfect sense: larger bases and heights produce larger triangles.

We could, of course, continue this exercise by finding all combinations of x and y that produce an area (z) equal to 6, 8, 100, 576, etc. Again, doing so would, in each case, yield a downward sloping iso-area curve that lies to the "northeast" (i.e., above) the previous one. Just as we called the entire family of contour lines a *contour map,* the entire family of iso-area curves is called an *iso-area map.*

Utility Functions: An Economic Application Understanding utility functions is critical to understanding both intermediate microeconomics and the modern approach to macroeconomics. The reason is that modern macroeconomics is based entirely on microfoundations. We will have much more to say about utility functions and microfoundations as we proceed in the course, but it is instructive to introduce the concept of utility functions here because they nicely illustrate the concept of the *level set* we developed above.

Economists represent an individual's preferences over bundles of goods or services using a **utility function**. Although a typical person will have many goods and services they consume over a given period of time, we frequently simplify the utility function by assuming the typical person consumes just two goods per unit of time. As illustrated above, this simplifying assumption is without loss of generality, as it easily extends to utility functions defined over many goods per unit of time. If we define two distinct commodities as good x and y, we can write the utility function with a lower case u as follows:

$$u(x, y),$$

where the pair (x, y) represents the quantities of good x and y. While we haven't written down a specific functional form of u, it seems reasonable to require that

$$u(2,1) > u(1,1) \text{ and } u(1,2) > u(1,1).$$

That is, the bundle comprised of 2 units of x and 1 unit of y is preferred to the bundle comprised of 1 unit of x and 1 unit of y. Similarly, the bundle comprised of 1 unit of x and 2 units of y is preferred to the bundle comprised of 1 unit of x and 1 unit of y. In the language developed above, we say that u is increasing in each of its arguments (i.e., the 2 independent variables x and y).

We now give two different interpretations of goods x and y we'll use to model different economic problems. In either case, however, the technique for deriving level sets will be the same as developed above.

We are frequently interested in an individual's allocation of time between work and 'leisure' (more on this later). Since time cannot simultaneously be spent working and engaging in leisure, and since time spent working generates income to buy various goods and services, every person faces a fundamental choice in life: the choice between the consumption of 'goods' and the consumption of 'leisure,' both of which are things people want more of. Thus, if we denote the consumption of 'goods' by c and the consumption of 'leisure' by l, we can describe an individual's preferences as follows:

$$u(c, l),$$

where c and l represent the quantities of consumption and leisure at a *specific point in time.*

Consider now a second interpretation for variables x and y. In economics, the dynamic behavior of various prices and quantities is of particular importance. For instance, how does aggregate consumption in the economy evolve over time. To sort this out, it is first necessary to understand how individuals choose among quantities of consumption goods at different points in time. In this respect, commodities are not only characterized by their physical properties, but also by when these commodities are available for consumption. In other words, a hamburger today is not the same good as a hamburger tomorrow. We will find it convenient to use a date subscript to denote the time period (such as a year) in which a good is available to consume. Thus, if the economic problem at hand is explaining individual consumption *over time*, rather than explaining the choice between consumption and leisure at a *specific point in time*, individual preferences can be described as follows:

$$u(c_1, c_2),$$

where u is a function of c_1 and c_2, the quantity of goods consumed in periods 1 and 2, respectively. We refer to this function as a **time-dated utility function**.

Now, it doesn't matter which example we use to illustrate the application of a level set, as the same technique applies to both. Choosing somewhat arbitrarily, we'll illustrate the economic application of level sets using the utility function defined over consumption and leisure at a specific point in time. Consider an individual whose preferences are defined as follows:

$$U = u(c, l),$$

where c and l represent the quantities of consumption and leisure at a specific date. U is the total utility or satisfaction (i.e., a number) received by an individual from consuming a specific quantity of c and l, while u is just the name of the function which we have yet to specify.

As it turns out, the specific functional form (or formula) that describes the area of a triangle is also a reasonable description of an individual's preferences (though the coefficient or scaling factor of 1/2 has no particular significance for this application). More on this later. Thus, we could write the utility function as

$$U = \frac{1}{2}(c)(l).$$

We can now repeat the exercise above (for level sets) by re-labeling the axes. Place l on the horizontal axis and c on the vertical (as is customary). Set the total level of utility U at some predetermined level, and identify all possible combinations of consumption and leisure that yield that amount. Since these points represents combinations of consumption and leisure that yield the same level of satisfaction or happiness, an individual is said to be *indifferent* to having one combination (or commodity bundle) over another. Thus, economists have a name for these iso-utility curves: we call them **indifference curves**. You should verify that the indifference curve is downward sloping and that the slope becomes flatter as you move down the curve (just as it was with the iso-area curves of a triangle whose mass or area was fixed as some predetermined level). Now set U at a higher predetermined level and identify all combinations of c and l that yield this new amount. When graphed, the new iso-utility curve, or indifference curve, will lie everywhere above (i.e., to the northeast) of the old one. Each new indifference curve (associated with different predetermined levels of U) gives all the combinations of consumption and leisure that yield a specific amount of total utility at a specific point in time. And just as we called the entire set or family of contour lines a *contour map* (and the entire set or family of iso-area curves for a triangle an *iso-area map)*, economists call the entire set or family of indifference curves an *indifference map*.

2.2 Exercises

1. Consider the function $z = 2 + x - 2y$, where $x \geq 0$ and $y \geq 0$.

   (a) Is z increasing or decreasing in x? Explain. Is z increasing or decreasing in y? Explain.

   (b) Using graph paper or a spreadsheet, draw the graph of this function in the $x - z$ plane assuming that $y = 1$. What is the slope of the resulting line? What is the vertical intercept?

   (c) In the same diagram for part (b), draw the graph of this function assuming that $y = 2$. What is the slope of the resulting line? What is the vertical intercept?

   (d) Using the diagram drawn for parts (b) and (c), indicate the effect of a 1 unit increase in y on z assuming that $x = 2$. Does your answer change if we assume that x is fixed at some value other than 2?

2. Consider the function $z = x\sqrt{y}$, or equivalently, $z = xy^{1/2}$ where $x \geq 0$ and $y \geq 0$.

   (a) Using graph paper or a spreadsheet, draw the graph of this function in the $y-z$ plane assuming that $x = 1$. Is the slope of the resulting curve positive or negative? In other words, is z increasing or decreasing in y? What happens to the slope as y increases?

   (b) In the same diagram for part (a), draw the graph of this function assuming that $x = 2$.

   (c) Use your diagram for parts (a) and (b) to answer this part. Suppose that y is constant at 4, i.e., $y = 4$. What happens to the slope of the function as x increases from 1 to 2? Is this true for all values of y?

   (d) Using the diagrams for parts (a) and (b), indicate the effect of a 1 unit increase in x on z assuming initially that $x = 4$. Does your answer change if we assume that x is initially fixed at some value other than 4?

   (e) In the $x - y$ plane, draw the level sets that correspond to $z = 2$ and $z = 4$. Are the slopes of the resulting curves positive or negative? Explain.

3. Consider the function $z = f(w, x, y)$ where $w \geq 0$, $x \geq 0$, and $y \geq 0$. Suppose that we only know the following about the specific form of f: (i) it does not take on negative values, i.e., $z \geq 0$, and (ii) it is increasing in w and y, but decreasing in x.

(a) Draw the graph of this function in the $x - z$ plane that is consistent with its assumed properties. By doing this, you are implicitly assuming that independent variables w and y are constant.

(b) In the same diagram as part (a), illustrate the effect of an increase in w holding y and x constant.

(c) Now draw a graph of this function in the $x-w$ plane that is consistent with its assumed properties. By doing so, you are implicitly assuming that independent variables x and y are constant. Now suppose that you wish to illustrate the effect of a *simultaneous* increase in x and y. Is this possible? Explain.