Econ 6500 Spring 2013
Sample Questions

Part I: True/False/Explain. ***Answer all 4 problems*** **[5 marks each, 20 marks total]. Answers without explanations will not receive any credit.**

1. Consumer surplus provides an exact measure of a change in consumer welfare if and only if the substitution effect is zero.

   Solution *FALSE. Consumer surplus CS is an exact measure of a change in consumer welfare if and only if it equals both equivalent and compensating variation, that is, $CS = EV = CV$. CS is measured as the area beneath the Marshallian demand curve between any two prices. EV and CV are measured as the area beneath the Hicksian demand curves between those same two prices. For a given price change, these areas are the same if and only if the Marshallian and Hicksian demand curves are the same. It is enough to show that these two demand curves are not the same when the substitution effect is zero. There are two ways to do this.*

   First Approach *Illustrate your argument with a diagram. Doing so requires you to make appropriate assumptions about consumer preferences, i.e., assumptions about the shape of the indifference curves. We want SE=0 so assume the consumer must consume goods in fixed proportion, i.e., perfect complements. Then, for a given price change, construct both demand curves to show that they are not the same, and thus the area beneath these two curves cannot be the same either.*

   Second Approach *Use the Slutsky equation, which provides a precise statement of our graphical analysis. The Slutsky equation is given as:*

$$\frac{\partial d_i}{\partial p_i} = \frac{\partial h_i}{\partial p_i} - \frac{\partial d_i}{\partial M} q_i$$

   where d_i is the Marshallian demand for good i, h_i is the Hicksian demand for good i, M is money income, and q_i is the initial quantity of good i. It says that the difference between the Marshallian demand response to a price change ($\frac{\partial d_i}{\partial p_i}$ - which measures the change in CS) and the Hicksian demand response to a price change ($\frac{\partial h_i}{\partial p_i}$ - which measures CV and EV) is equal to the income effect scaled by the effective change in income due to the price change (recalling that $q_i = \frac{\partial E}{\partial p_i}$). We can therefore conclude that $CS = CV = EV$ if and only if the income effect is zero, i.e., $\frac{\partial d_i}{\partial M} q_i$ - which in economic terms can happen if the good in question is neither inferior or normal (i.e., $\frac{\partial d_i}{\partial M} = 0$) or if the consumer is consuming $q_i = 0$ initially.

2. Suppose an individual consumes only two goods, q_1 and q_2. All else equal, if the price of q_1 rises and the price elasticity of demand for q_1 is 1.5, then the quantity of q_2 consumed will increase.

Solution *TRUE. We have* $\eta = \frac{\%\Delta q}{\%\Delta p} = 1.5$ *in absolute value. Thus, if the the price of good 1 rises by, say, 10%, then the quantity demanded of good 1 will decline by 15%. This means that the expenditure on good 1, i.e.* $p_1 q_1$*, must decline. But given the consumer's income* M*, a decline in expenditure on good 1 must imply an increase in expediture on good 2, i.e., since* $M = p_1 q_1 + p_2 q_2$*, and since* $(p_1 q_1) \downarrow$ *it follows that* $(p_2 q_2) \uparrow$.

3. Consider two different welfare policies: (i) a per unit subsidy s on good 1 or (ii) a lump sum cash grant S to all consumers. If both programs leave the consumer equally well off, the first program is always less expensive to taxpayers than the second.

Solution *False. Other way around. Let the initial quantity demanded of each good in the absence of a subsidy be given by the bundle* $A = (q_1^*, q_2^*)$ *on indifference curve* U_0*, where* $MRS(q_1^*, q_2^*) = \frac{p_1}{p_2}$ *and the budget constraint holds.*

- *A subsidy* s *applied to* p_1 *decreases the relative price of good 1 to* $\frac{p_1 - s}{p_2}$*. This gives the consumer an incentive to substitute toward good 1. Let the new quantities (with the subsidy) be given by bundle* $B = (q_1^{**}, q_2^{**})$ *on indifference curve* U_1*, where* $q_1^{**} > q_1^*$ *and* $U_1 > U_0$*. At this solution, we have* $MRS(q_1^{**}, q_2^{**}) = \frac{p_1 - s}{p_2}$ *and the government incurs a cost equal to* sq_1^{**} *to administer the subsidy program.*
- *A lump-sum cash grant* S *that is large enough to make bundle* B *affordable (i.e., shifts the budget constraint out such that is passes through bundle* B*) will cost the government the same amount of money as a subsidy, i.e.,* $S = sq_1^{**}$*, but will make the consumer better off (strictly so, assuming convex preferences). This is because, at bundle* B*, we now have* $MRS(q_1^{**}, q_2^{**}) < \frac{p_1}{p_2}$*; i.e., the slope of the indifference curve at bundle* B *is smaller than the slope of the original budget constraint shifted out by an amount equal to* S*. In words, this means that the relative price of good 1 exceeds the marginal value of good 1 and thus a consumer with carte blanche as to how to spend subsidy* S *will substitute away from good 1 (toward good 2) in order to make themselves better off (i.e., to reach indifference curve* $U_2 > U_1$*).*
- *This implies that a lump-sum cash grant need not be as large as* S *in order to help the consumer reach indifference curve* U_1*. Thus, lump-sum cash grants are less expensive to taxpayers.*
- *For a diagram, see Figure 1.*

4. In a two good world, the compensated (i.e., Hicksian) demand curve is always steeper than the ordinary (i.e., Marshallian) demand curve.

Solution *FALSE. Again we could use the Slutsky equation to answer this question:*

$$\frac{\partial d_i}{\partial p_i} = \frac{\partial h_i}{\partial p_i} - \frac{\partial d_i}{\partial M} q_i$$

where d_i *is the Marshallian demand for good* i*,* h_i *is the Hicksian demand for good* i*,* M *is money income, and* q_i *is the initial quantity of good* i*. The first two terms are the slopes of the Marshallian and Hicksian demand curves, respectively.*

As noted above, it is sufficient to show that these two slopes are the same if the income effect is zero, i.e. if the product $\frac{\partial d_i}{\partial M} q_i = 0$. *Of course, one could also argue that the Marshallian demand curve will be steeper than the Hicksian demand curve as long as good i is inferior, i.e.,* $\frac{\partial d_i}{\partial M} < 0$. *In other words, the Total Price Effect from a given price change* $\frac{\partial d_i}{\partial p_i}$ *is smaller than the pure SE* $\frac{\partial h_i}{\partial p_i}$ *if the IE* $\frac{\partial d_i}{\partial M}$ *is negative and countervailing.*

Part II: Longer Problems

1. Let $u(q_1, q_2) = \ln q_1 + q_2$ be the (direct) utility function, where q_1 and q_2 are two goods. Denote p_1 and p_2 as the prices of those two goods and let M be per period money income. Derive each of the following:

   (a) the ordinary or Marshallian demand functions $q_i^* = d_i(p_1, p_2, M)$ for $i = 1, 2$;

   (b) the compensated or Hicksian demand functions $q_i^* = h_i(p_1, p_2, M)$ for $i = 1, 2$;

   (c) the Indirect Utility Function $U^0 = V(p_1, p_2, M)$;

   (d) the Expenditure Function $E(p_1, p_2, U^0)$.

   (e) Draw a diagram of the solution. There should be two graphs, one above the other; the first containing the indifference curves and budget constraint that characterize the solution to the consumer's choice problem; the second characterizing the demand functions.

Solution:

- *For a derivation of the primal (i.e., utility max) problem, see my notes.*
- *The Marshallian demand functions are given by:*

$$\left.\begin{aligned} q_1^* = d_1(p_1, p_2, M) &= \tfrac{p_2}{p_1} \\ q_2^* = d_2(p_1, p_2, M) &= \tfrac{M - p_2}{p_2} \end{aligned}\right\} \quad if \quad M > p_2$$

$$\left.\begin{aligned} q_1^* = d_1(p_1, p_2, M) &= \tfrac{M}{p_1} \\ q_2^* = d_2(p_1, p_2, M) &= 0 \end{aligned}\right\} \quad if \quad M \leq p_2$$

- *Substituting these demand functions into the objective function gives us the following indirect utility function (with conditions):*

$$\begin{aligned} U^0 &= V(p_1, p_2, M) = \ln\left(\frac{p_2}{p_1}\right) + \frac{M - p_2}{p_2} \qquad \text{if } M > p_2 \\ U^0 &= V(p_1, p_2, M) = \ln\left(\frac{M}{p_1}\right) + 0 \qquad \text{if } M \leq p_2. \end{aligned}$$

- *To derive the expenditure function we can either (i) invert* $V(\cdot)$ *and solve for* M, *or (ii) set up the dual of the consumer's choice problem, solve for the Hicksian demand functions and substitute them into the objective function. Using approach (i), and assuming that the consumer has enough income to afford good 2 (i.e.,* $M > p_2$*), we can derive the Expenditure function as follows:*
  - *Start by rewriting the indirect utility function as:* $U^0 = \ln\left(\frac{p_2}{p_1}\right) + \frac{M}{p_2} - 1$.

- *Invert by solving for M to get the expenditure function:*

$$M = E(p_1, p_2, U^0) = p_2 + p_2\left[\overline{u} - \ln\left(\frac{p_2}{p_1}\right)\right].$$

- *To derive the Hicksian demand functions, apply Shephard's Lemma.*
  - *Start by rewriting the expenditure function as follows:* $E(p_1, p_2, U^0) = p_2 + p_2\overline{u} - p_2 \ln\left(\frac{p_2}{p_1}\right)$
  - *Which can rewritten again as:* $E(p_1, p_2, U^0) = p_2 + p_2\overline{u} - p_2\left[\ln p_2 - \ln p_1\right]$
  - *Now take derivatives with respect to p_1 and p_2 to get the demand functions:*

$$\begin{aligned} q_1^* &= h_1(p_1, p_2, U^0) = \frac{\partial E}{\partial p_1} = \frac{p_2}{p_1} \\ q_2^* &= h_2(p_1, p_2, U^0) = \frac{\partial E}{\partial p_2} = U^0 - \ln\left[\frac{p_2}{p_1}\right] \end{aligned}$$

Remark: *If $M \leq p_2$ the consumer cannot afford good 2. Thus all the consumer's per period income is spent on good 1. But if $M > p_2$, then the consumer spends enough income to buy the desired quantity of good 1 (which equals $q_1^* = p_2/p_1$) and then spends the rest on good 2. With quasi-linear preferences, indifference curves are vertically parallel, which implies that the MRS is constant along any vertical line, including the vertical line at $q_1^* = p_2/p_1$. This means there is no income effect - the total price effect equals the substitution effect. Finally, if the income effect is zero, this implies that the two demand curves are the same, i.e., $d_i(p_1, p_2, Y) = h_i(p_1, p_2, U)$ for $i = 1, 2$ and therefore the area beneath the demand curves between any two prices are the same, i.e., $EV = CV = CS$.*

Example *Suppose $M = \$10$, $p_1 = \$2$, and $p_2 = \$2$. Note that good 2 is affordable so the consumer will purchase a positive quantity of both goods. The Marshallian or ordinary demands are:*

$$\begin{aligned} d_1(2, 2, 10) &= \tfrac{p_2}{p_1} = \tfrac{2}{2} = 1 \\ d_2(2, 2, 10) &= \tfrac{M - p_2}{p_2} = \tfrac{10-2}{2} = 4 \end{aligned}$$

The indirect utility associated with these demands is given by:

$$U^0 = V(2, 2, 10) = \ln\left(\frac{p_2}{p_1}\right) + \frac{M - p_2}{p_2} = \ln\left(\frac{2}{2}\right) + \frac{10 - 2}{2} = 4$$

The Hicksian or compensated demand for each good is:

$$\begin{aligned} h_1(2, 2, 4) &= \frac{p_2}{p_1} = \frac{2}{2} = 1 \\ h_2(2, 2, 4) &= U^0 - \ln\left(\frac{p_2}{p_1}\right) = 4 - \ln\left(\frac{2}{2}\right) = 4 \end{aligned}$$

Now suppose $p_1 \downarrow$ to \$1. New Marshallian demand is:

$$d_1(1,2,10) = \tfrac{p_2}{p_1} = \tfrac{2}{1} = 2$$
$$d_2(1,2,10) = \tfrac{M-p_2}{p_2} = \tfrac{10-2}{2} = 4$$

New indirect utility is:

$$U^1 = V(1,2,10) = \ln\left(\frac{2}{1}\right) + \frac{10-2}{2} = 4.7$$

New Hicksian demand is:

$$\begin{aligned} h_1(1,2,4) &= \frac{p_2}{p_1} = \frac{2}{1} = 2 \\ h_2(1,2,4) &= 4.7 - \ln\left(\frac{2}{1}\right) = 4 \end{aligned}$$

Thus, d_1 and h_1 respond to a change in p_1 in exactly the same way!

- *For a graph of the demand curves see Figure 2.*

2. Same as question 1, but let preferences be given by the Cobb-Douglas utility function $u(q_1, q_2) = q_1^{\frac{1}{3}} q_2^{\frac{2}{3}}$.

Solution:
- *As always with Cobb-Douglas utility where the exponents sum to one, we have (i) the exponent on each variable telling us the share of the consumer's expenditure devoted to that good, and (ii) the quantity demanded of each good is independent of the other good's price.*
- *The Marshallian demand functions are given as:*

$$\begin{aligned} q_1^* &= d_1(p_1, p_2, M) = \frac{1}{3}\frac{M}{p_1} \\ q_2^* &= d_2(p_1, p_2, M) = \frac{2}{3}\frac{M}{p_2}. \end{aligned}$$

- *Substituting these demand functions into the objective (i.e., utility) function gives us the indirect utility function:*

$$\begin{aligned} U^0 &= V(p_1, p_2, M) = (q_1^*)^{1/3} (q_2^*)^{2/3} \\ U^0 &= V(p_1, p_2, M) = \left(\frac{1}{3p_1}\right)^{1/3} \left(\frac{2}{3p_2}\right)^{2/3} M \end{aligned}$$

- *To derive the expenditure function we can either (i) invert $V(\cdot)$ and solve for M, or (ii) set up the dual of the consumer's choice problem, solve for Hicksian demand functions and substitute them into the objective (i.e., expenditure) function. I will do both, but only to make a point.*

- *Method (i): Isolate M from the indirect utility function:*

$$\begin{aligned} M &= \frac{U^0}{\left(\frac{1}{3p_1}\right)^{1/3}\left(\frac{2}{3p_2}\right)^{2/3}} \\ &= U^0\left(\frac{1}{3p_1}\right)^{-1/3}\left(\frac{2}{3p_2}\right)^{-2/3} \\ &= U^0\left(\frac{3p_1}{1}\right)^{1/3}\left(\frac{3p_2}{2}\right)^{2/3} \\ &= U^0\left(\frac{p_1}{1/3}\right)^{1/3}\left(\frac{p_2}{2/3}\right)^{2/3} \\ E(p_1,p_2,U^0) &= U^0\left(\frac{p_1}{1/3}\right)^{1/3}\left(\frac{p_2}{2/3}\right)^{2/3} \end{aligned}$$

  I have written the expenditure function this way for reasons I explain below!

- *Method (ii): solve the dual of the problem. This will get messy and will produce lots of algebra and very little economics. The main lesson here is to understand method (i) in order to reduce the math and focus on what the results are telling us.*
- *Set up the Dual problem:*

$$Min \quad p_1q_1 + p_2q_2 \quad s.t \ \ q_1^{1/3}q_2^{2/3} = U^0$$

- *Form the Lagrange function and solve the system of first order equations for q_1, q_2 and λ. Solving yields the Hicksian demand functions:*

$$\begin{aligned} q_1^* &= h_1(p_1,p_2,U^0) = U^0\left(\frac{p_2}{2p_1}\right)^{2/3} \\ q_2^* &= h_2(p_1,p_2,U^0) = U^0\left(\frac{2p_1}{p_2}\right)^{1/3} \end{aligned}$$

- *Now substitute the Hicksian demand functions back into the objective function to get the expenditure function:*

$$E(p_1,p_2,U^0) = p_1\underbrace{\left[U^0\left(\frac{p_2}{2p_1}\right)^{2/3}\right]}_{q_1^*} + p_2\underbrace{\left[U^0\left(\frac{2p_1}{p_2}\right)^{1/3}\right]}_{q_2^*}$$

- *Basically we are done. If we had data on prices and income (which would allow us to compute U^0), we could compute expenditures for purposes of doing policy analysis (computing EV, CV, etc.).*
- *But ordinarily we collect like-terms and simplify the expression. For this problem (and most expenditure min problems with Cobb-Douglas utility), it will take quite a bit of work and the potential for error is high. I will do the algebra below, but I do it with a larger point to make. The point I wish to make is a mathematical one; one which allows you to save time in solving problems in order to focus on the economics.*

- *Here goes ....*
- *Rewriting the expenditure function above:*

$$E(p_1,p_2,U^0) = p_1\left[U^0\left(\frac{p_2}{2p_1}\right)^{2/3}\right] + p_2\left[U^0\left(\frac{2p_1}{p_2}\right)^{1/3}\right]$$

- *Factor out U^0 to get:*

$$E(p_1,p_2,U^0) = U^0\left[p_1\left(\frac{p_2}{2p_1}\right)^{2/3} + p_2\left(\frac{2p_1}{p_2}\right)^{1/3}\right]$$

- *Expand:*

$$E(p_1,p_2,U^0) = U^0\left[\frac{p_1p_2^{2/3}}{(2p_1)^{2/3}} + \frac{p_2(2p_1)^{1/3}}{p_2^{1/3}}\right]$$

- *Bring the terms downstairs to the upstairs:*

$$E(p_1,p_2,U^0) = U^0\left[p_1p_2^{2/3}(2p_1)^{-2/3} + p_2(2p_1)^{1/3}p_2^{-1/3}\right]$$

- *Collect like terms inside the brackets:*

$$E(p_1,p_2,U^0) = U^0\left[p_1^{1/3}p_2^{2/3}2^{-2/3} + p_1^{1/3}p_2^{2/3}2^{1/3}\right]$$

- *Factor out $p_1^{1/3}p_2^{2/3}$:*

$$E(p_1,p_2,U^0) = U^0\left[p_1^{1/3}p_2^{2/3}\left(2^{-2/3} + 2^{1/3}\right)\right]$$

- *Get a common denominator for the inner most brackets:*

$$\begin{aligned} E(p_1,p_2,U^0) &= U^0\left[p_1^{1/3}p_2^{2/3}\left(\frac{1}{2^{2/3}} + \frac{2^{1/3}}{1}\cdot\left(\frac{2^{2/3}}{2^{2/3}}\right)\right)\right] \\ E(p_1,p_2,U^0) &= U^0\left[p_1^{1/3}p_2^{2/3}\left(\frac{1}{2^{2/3}} + \frac{2}{2^{2/3}}\right)\right] \end{aligned}$$

- *Simplify:*

$$E(p_1,p_2,U^0) = U^0\left[p_1^{1/3}p_2^{2/3}\left(\frac{3}{2^{2/3}}\right)\right]$$

- *Distribute the 3 (i.e., $3^{2/3}\cdot 3^{1/3}$):*

$$E(p_1,p_2,U^0) = U^0\left[\frac{(3p_1)^{1/3}(3p_2)^{2/3}}{2^{2/3}}\right]$$

- *Rewrite the denominator, recognizing that $2^{2/3}\cdot 1^{1/3} = 2^{2/3}$ since $1^{1/3} = 1$:*

$$E(p_1,p_2,U^0) = U^0\left[\frac{(3p_1)^{1/3}(3p_2)^{2/3}}{1^{1/3}\cdot 2^{2/3}}\right]$$

- *Simplify:*

$$E(p_1, p_2, U^0) = U^0 \left(\frac{3p_1}{1}\right)^{1/3} \left(\frac{3p_2}{2}\right)^{2/3}$$

- *Finally, multiply both terms inside both brackets by 1/3 to get:*

$$E(p_1, p_2, U^0) = U^0 \left(\frac{p_1}{1/3}\right)^{1/3} \left(\frac{p_2}{2/3}\right)^{2/3}$$

THIS THE SAME ANSWER WE GOT USING THE INDIRECT UTILITY FUNCTION!

- *Main point: There is a reason I went through the algebra and reduced it in the manner I did. To see why, consider the utility function*

$$u = q_1^{\frac{\boxed{1}}{\boxed{3}}} q_2^{\frac{\boxed{2}}{\boxed{3}}}$$

The Marshallian demand functions were given as:

$$\begin{aligned} q_1^* &= d_1(p_1, p_2, M) = \frac{\boxed{1}}{\boxed{3}}\frac{M}{p_1} \\ q_2^* &= d_2(p_1, p_2, M) = \frac{\boxed{2}}{\boxed{3}}\frac{M}{p_2} \end{aligned}$$

The exponents tell us the share of expenditures on each good. The Expenditure function is given as:

$$E(p_1, p_2, U) = U \left(\frac{p_1}{\frac{\boxed{1}}{\boxed{3}}}\right)^{\frac{\boxed{1}}{\boxed{3}}} \left(\frac{p_2}{\frac{\boxed{2}}{\boxed{3}}}\right)^{\frac{\boxed{2}}{\boxed{3}}}$$

The exponents show up both as ***divisors*** *and* ***powers*** *of the price of each good!*

- *This worth memorizing!!! And it is the only time you will see me write this. Given the form of a Cobb-Douglas utility function I know the ordinary demand functions and the expenditure function without having to compute anything. From here, I apply Shephard's Lemma to get the Hicksian demand functions, and I can invert the expenditure function to get the indirect utility function.*
- *There you have it (at least with Cobb-Douglas). This will save time, not to mention errors in working through problems in order to get to the results so you can interpret them.*
- *Important caveat: This result works for exponents that sum to one!*
- *However, consider a couple of other examples:*

$$u = q_1 q_2^2$$

or

$$u = q_1 q_2$$

- *There is a more general result you can use, which I simply state without derivation.*
- *Recall for $u = q_1^a q_2^b$ where $a + b \neq 1$ we have:*

$$\begin{aligned} q_1^* &= d_1(p_1, p_2, M) = \left(\frac{a}{a+b}\right)\frac{M}{p_1} \\ q_2^* &= d_2(p_1, p_2, M) = \left(\frac{b}{a+b}\right)\frac{M}{p_2} \end{aligned}$$

Expenditure shares are given by good i's exponent divided by the sum of the exponents.

- *The expenditure function is given by:*

$$E(p_1, p_2, U) = (a+b)\, U^{\boxed{\frac{1}{a+b}}} \left(\frac{p_1}{a}\right)^{\boxed{\frac{a}{a+b}}} \left(\frac{p_2}{b}\right)^{\boxed{\frac{b}{a+b}}}$$

See the pattern? When $a + b = 1$, the $(a + b)$ terms disappear and we get our previous result above.

3. A consumer has the following utility over childcare c (measured in hours) and food f (measured in units):

$$u(c, f) = c^{\frac{1}{5}} f^{\frac{4}{5}}.$$

The price of childcare is $p_c = 2$, the price of food is $p_f = 4$ and income is $M = 20$.

(a) What is the consumer's ordinary or uncompensated demand for childcare and food?

Solution • *Using our short-cut (since preferences are given by Cobb-Douglas utility), the uncompensated or Marshallian demands are given by:*

$$\begin{aligned} c^* &= \frac{1}{5}\frac{20}{2} = 2 \\ f^* &= \frac{4}{5}\frac{20}{4} = 4. \end{aligned}$$

(b) Suppose the government gives the consumer an income subsidy of $S = 10$. How will the consumer allocate the subsidy in the consumption of goods c and f (i.e., what are c^* and f^* given the subsidy)? Draw a carefully labeled graph where you show the pre- and post subsidy solutions to the consumer's choice problem.

Solution *This is a cash subsidy of $S = \$10$, so the consumer has carte blanche as to how to allocate it. The new Marshallian demand functions are given by:*

$$\begin{aligned} c_{sub}^* &= \frac{1}{5}\frac{M+S}{p_c} = \frac{1}{5}\frac{20+10}{2} = 3 \\ f_{sub}^* &= \frac{4}{5}\frac{M+S}{p_f} = \frac{4}{5}\frac{20+10}{4} = 6. \end{aligned}$$

(c) Suppose now the government decides to give an in-kind transfer to the consumer. The in-kind transfer takes the form of 4 hours of childcare and 0.5 units of food. Assume that the transfer cannot be resold.

1. What are the new consumption levels after the in-kind transfer is given? What is the level of utility attained by the consumer at this consumption level? Carefully draw the in-kind budget contraint along with the solutions to this new choice problem in your graph from part (b). [You may need to use a different color to draw this new budget contraint.]

   Solution *For the geometric solution, see Figure 3.*

   - *The consumer's choice problem is:*

$$Max \quad u(c,f) = c^{1/5} f^{4/5} \quad s.t. \quad \begin{cases} 2c + 4f \leq 20 \\ c \geq 4 \\ f \geq 0.5 \end{cases}$$

   The proper way to solve this is to use inequality constraints (i.e., Kuhn-Tucker conditions). But we don't need to do this. What we have to do is to look at the optimal choices in the case of a cash transfer and compare it to the constraints to see if the constraints are binding.

   - *Recall that the consumer's choice of child care with a cash transfer is $c^*_{sub} = 3$, but that one of the constraints with an in-kind transfer is $c \geq 4$. This means the in-kind transfer forces the consumer to consume more childcare than she would have liked if she had been cash instead. Since she prefers 3 hours of childcare to 4 hours, she would never choose more than 4 hours. Thus, we know that the solution is to only consume as much child care as required to get the transfer .... and no more. Since 4 hours are "free" to her, all of her budget is spent on food plus whatever food is given in-kind. The solution with an in-kind transfer is therefore given by:*

$$\begin{aligned} c^*_{ik} &= 4 \\ f^*_{ik} &= 5.5 \end{aligned}$$

   And the level of utility to the consumer is equal to:

$$U_{ik} = V_{ik}(p_c, p_f, M) = (4)^{1/5} (5.5)^{4/5} = 5.16$$

2. What is the minimum expenditure level required to attain the same utility if the consumer were buying all goods on the market instead of being given goods in-kind?

   Solution

   - *To solve for the expenditure function, we invert the indirect utility function for M.*
   - *To do this we need to write V in more general terms (i.e., without numbers). Of course, we could use our trick, since preferences are Cobb-Douglas. Doing so yields:*

$$E(p_1, p_2, U_{ik}) = U_{ik} \left(\frac{p_c}{1/5}\right)^{1/5} \left(\frac{p_f}{4/5}\right)^{4/5}$$

Now plug in prices (which are given) and utility derived from part (i) to get:

$$E(p_1, p_2, U_{ik}) = 5.16\,(10)^{1/5}\,(5)^{4/5} = \$29.64.$$

3. What is the cash equivalent of the in-kind transfer? This cash is the variation in income that is equivalent (in utility terms) to being given the specified quantity of goods for "free."

   Solution

   - *The government could have provided the same level of utility by giving a cash transfer of*

     $$E(p_1, p_2, U_{ik}) - M = \$29.64 - \$20 = \$9.64$$

     instead of the in-kind transfer, which cost the government \$10 on the open market.
   - *Note: we are computing the equivalent variation (EV) here. Given original prices, how much money would it take to make her just as well off as if she had been an in-kind transfer instead.*
   - *Also note: this amount of cash is less than the \$10 subsidy in part (b). Implication: cash is more efficient at enhancing consumer welfare than in-kind transfers; it also requires much lower overhead (i.e., administration costs).*

4. Hillary spends her money on two goods q_1 and q_2, with prices p_1 and p_2. Her ordinary demand for good 1 is estimated to be:

   $$q_1^* = d_1(p_1, p_2, M) = \frac{M}{p_1} - 1.$$

   [You need the Slutsky decomposition (i.e., equation) for part of this question.]

   (a) What is her ordinary or Marshallian demand for good 2?

   - *Notice we are not given a utility function. But we don't need one to answer the question.*
   - *At the solution to every consumer choice problem we have (i) the slope of a line tangent to the indifference curve equal to the slope of the budget constraint and (ii) the solution lies on the budget constraint.*
   - *Since we have one of the solutions to the choice problem given to us, all we have to do is evaluate either the indifference curve at q_1^* or the budget constraint at q_1^* to get q_2^*. Since we don't know the utility function (and can't compute the MRS at q_1^*), we use the budget constraint.*
   - *The budget constraint is given by:*

     $$p_1 q_1^* + p_2 q_2^* = M$$

   - *Evaluate the budget constraint at q_1^* to get:*

     $$p_1\left(\frac{M}{p_1} - 1\right) + p_2 q_2^* = M$$

- *Now solve for* q_2^* :

$$q_2^* = d_2(p_1, p_2, M) = \frac{p_1}{p_2}$$

(b) What is the slope of her Hicksian or compensated demand for good 1?

- *The question asks us to compute* $\partial h_1 / \partial p_1$.
- *Recall the Slutsky equation:*

$$\frac{\partial h_1}{\partial p_1} = \frac{\partial d_1}{\partial p_1} + \frac{\partial d_1}{\partial M} q_1$$

- *Taking the first partial derivative of the Marshallian demand function for good 1 (which is given) first with respect to* p_1 *and then with respect to* M *we get:*

$$\begin{aligned} \frac{\partial d_1}{\partial p_1} &= -\frac{M}{p_1^2} \\ \frac{\partial d_1}{\partial M} &= \frac{1}{p_1} \end{aligned}$$

Substituting back into the Slutsky equation yields:

$$\begin{aligned} \frac{\partial h_1}{\partial p_1} &= -\frac{M}{p_1^2} + \left(\frac{1}{p_1}\right) q_1 \\ &= -\frac{M}{p_1^2} + \left(\frac{q_1}{p_1}\frac{p_1}{p_1}\right) \\ &= \frac{1}{p_1^2}(p_1 q_1 - M) \leq 0 \end{aligned}$$

- *The slope of the compensated demand curve must be non-positive. It could, of course, be zero if the consumer cannot afford to buy good 2 (i.e.,* $M \leq p_2$*), which means the consumer's expenditures on good 1 will equal per period income (i.e.,* $p_1 q_1 = M$*). Otherwise, the consumer can afford good 2 and the consumer's expenditures on good 1 will be less than per period income, making the slope negative.*

(c) Is good 2 a substitute or complement for good 1?

- *Solution is asking you to compute* $\partial q_1^* / \partial p_2$. *But* q_1^* *is independent of* p_2, *so* $\partial q_1^* / \partial p_2 = 0$.
- *The goods are neither substitutes or complements; they are unrelated.*

(d) Is good 1 a substitute or complement for good 2?

- *Compute* $\partial q_2^* / \partial p_1$. *From part (a), we know* $q_2^* = \frac{p_1}{p_2}$. *Thus we have*

$$\frac{\partial q_2^*}{\partial p_1} = \frac{1}{p_2} > 0.$$

- *So good 2 is a substitute for good 1.*

(e) Is good 1 a normal good?

- *Check $\partial q_1^*/\partial M$. We have already done this in part (b). We found:*

$$\frac{\partial q_1^*}{\partial M} = \frac{1}{p_1} > 0$$

- *So good 1 is a normal good.*

(f) Is good 1 a Giffen good?

- *Check $\partial q_1^*/\partial p_1$. We have already done this in part (b). We found:*

$$\frac{\partial q_1^*}{\partial p_1} = -\frac{M}{p_1^2} < 0$$

- *So good 1 is an ordinary good and the law of demand holds, i.e., the Marshallian demand curves is downward sloping (as it should be).*

5. You are going to spend \$100 on either new CD's (good q_1) which cost \$10 or old CD's (good q_2) which cost only \$5. You like a bit of old and new music, so your utility function has the Cobb-Douglas form:

$$u = q_1^{0.5} q_2^{0.5}$$

Given this utility function, you initially purchase 5 new CD's and 10 old CD's. [You may want to verify this.]

(a) If the price of old CD's falls to \$2, how many do you purchase of each?

Solution *With Cobb-Douglas preferences, our original solution is given by:*

$$\begin{aligned} q_1^* &= \frac{1}{2}\frac{M}{p_1} = \frac{1}{2}\frac{100}{10} = 5 \\ q_2^* &= \frac{1}{2}\frac{M}{p_2} = \frac{1}{2}\frac{100}{5} = 10 \end{aligned}$$

If the price of good 2 falls to \$2, we now have:

$$\begin{aligned} q_1^{**} &= \frac{1}{2}\frac{M}{p_1} = \frac{1}{2}\frac{100}{10} = 5 \\ q_2^{**} &= \frac{1}{2}\frac{M}{p_2} = \frac{1}{2}\frac{100}{2} = 25. \end{aligned}$$

Thus the total price effect is to increase the amount of good 2 purchased by $q_2^{**} - q_2^* = 25 - 10 = 15$ *old CD's.*

(b) How much of the increase in your demand for old CD's (good q_2) is due to the fact that they have become relatively cheaper (the SE effect) and how much is due to the fact that your overall purchasing power, or real income, has increased (the IE effect)?

- *To find the SE, we must shift the new budget constraint back so that it is just tangent to the original indifference curve, i.e., we must remove the IE.*
- *We call this new budget constraint a compensated budget constraint.*
- *The solution will amount to solving two equations in two unknowns.*
- *We know two things. First, at the original solution we know that utility is given by:*

$$\begin{aligned} U^0 &= (q_1^*)^{1/2} (q_2^*)^{1/2} \\ &= (5)^{1/2} (10)^{1/2} \\ &= (50)^{1/2} \end{aligned}$$

  Thus, the amount of good 1 and good 2 the consumer buys resulting from a pure SE must still satisfy the following equation:

$$U^0 = (q_1)^{1/2} (q_2)^{1/2} = (50)^{1/2}$$

  which is the same thing as

$$q_1 q_2 = 50.$$

- *The second thing we know is that any tangency between the compensated budget constraint and the original indifference curve means that the marginal rate of substitution of good 1 for good 2 must equal the relative price of good 1, i.e.* $MRS = p_1/p_2$.
- *Deriving the MRS and setting it equal to the new price ratio yields the following:*

$$\begin{aligned} MRS &= \frac{q_2}{q_1} = \frac{10}{2} = \frac{p_1}{p_2} \\ \frac{q_2}{q_1} &= \frac{10}{2} \\ q_2 &= 5q_1 \end{aligned}$$

  Thus, whatever our new solution is, we know that q_2 *must be 5 times as large as* q_1. *This is the second equation that must hold.*
- *Now we just solve the two equations for their two unknowns. Our equations are:*

$$\begin{aligned} (i) \quad q_1 q_2 &= 50 \\ (ii) \quad q_2 &= 5q_1 \end{aligned}$$

  The first says the product of q_1 *and* q_2 *(whatever they are) must equal 50. The second says that, at the solution,* q_2 *must be 5 times larger than* q_1.
- *Define our new solutions as* $\widehat{q}_1$ *and* $\widehat{q}_2$. *Solve yields:*

$$\begin{aligned} \widehat{q}_1 &= (10)^{1/2} \\ \widehat{q}_2 &= 5\,(10)^{1/2} \end{aligned}$$

- *So the SE for good 2 is given by the difference between* $\widehat{q}_2$ *and* q_2^**:*

$$\begin{aligned} SE &= \widehat{q}_2 - q_2^* \\ &= 5\,(10)^{1/2} - 10 \\ &\cong 5.8 \end{aligned}$$

And the IE for good 2 is what's left of the total price effect.

- *Recall the total price effect was 15 old CD's. Thus the IE is given by:*

$$\begin{aligned} IE &= q_2^{**} - \widehat{q}_2 \\ &= 25 - 5\,(10)^{1/2} \\ &\cong 9.2 \end{aligned}$$

(c) Draw a carefully labelled graph that shows the substitution and income effects.

Solution *See Figure 4*

6. The government wants to raise a specific amount of revenue with either (1) an excise tax t or (2) a lump-sum tax τ. Assuming that preferences for the taxed good and all other goods are strictly convex, and assuming that the lump-sum tax must raise the same exact amount of revenue as the per unit excise tax, which tax do people prefer? Draw a carefully labelled diagram to illustrate your answer.

Solution *Let the initial quantity demanded of each good in the absence of a tax be given by the bundle* $A = (q_1^*, q_2^*)$*, where* $MRS(q_1^*, q_2^*) = \frac{p_1}{p_2}$ *and the budget constraint holds.*

- *An excise tax* t *imposed on* p_1 *increases the relative price of good 1 to* $\frac{p_1+t}{p_2}$*. This gives the consumer an incentive to substitute out of good 1. Let the new quantities (with the excise tax) be given by bundle* $B = (q_1^{**}, q_2^{**})$*, where* $q_1^{**} < q_1^*$*. At this solution, we have* $MRS(q_1^{**}, q_2^{**}) = \frac{p_1+t}{p_2}$ *and the government raises an amount of tax revenue equal to* tq_1^{**}*.*
- *A lump-sum tax* τ *must raise the same amount of tax revenue, i.e.,* $\tau = tq_1^{**}$*. This means bundle* B *must still be affordable with the lump-sum tax, i.e., the budget constraint shifts inward through bundle* B*. But, at bundle* B*, we now have* $MRS(q_1^{**}, q_2^{**}) > \frac{p_1}{p_2}$*; that is, the slope of the indifference curve at bundle* B *is larger than the slope of the original budget constraint shifted in by an amount equal to* τ*. In words, this means that the marginal value of good 1 exceeds its relative price and thus the consumer will substitute toward good 1 to make themselves better off.*
- *Thus, lump-sum taxes are always (weakly) preferred to excise taxes of the same size because consumers have carte blanche as to how to spend any remaining income and can therefore make any substitutions necessary to make themselves better off (i.e., reach a higher indifference curve).*
- *For a diagram, see Figure 5.*

7. Nesli has \$1000 income per month which she spends on fitness classes and all other goods (i.e., a composite good whose price is equal to \$1). Her preferences are given by $U(f,c) = f^{1/4}c^{3/4}$, where f is the amount of fitness classes and c is the amount (of money) she buys of the composite good (the price of the composite good is \$1). The fitness centre she attends has two plans. Non-members pay \$10 per class and members pay \$5 per class. Membership costs \$100 per month.

   (a) Find her optimal bundle if she does *not* become a member.

   Solution *Using our short-cut, the optimal bundle for a non-member is given by:*

$$\begin{aligned} q_f^{non} &= \frac{1}{4}\frac{M}{p_f} = \frac{1}{4}\frac{1000}{10} = 25 \\ q_c^{non} &= \frac{3}{4}\frac{M}{p_c} = \frac{3}{4}\frac{1000}{1} = 750 \end{aligned}$$

   (b) Find her optimal bundle if she becomes a member.

   Solution *Again, using our short-cut, the optimal bundle for a member is given by:*

$$\begin{aligned} q_f^{mem} &= \frac{1}{4}\frac{M - fee}{p_f'} = \frac{1}{4}\frac{1000-100}{5} = 45 \\ q_c^{mem} &= \frac{3}{4}\frac{M - fee}{p_c} = \frac{3}{4}\frac{1000-100}{1} = 675 \end{aligned}$$

   (c) Which plan should Nesli choose? Explain your answer.

   Solution *Note we cannot say by inspection which bundle is more preferred. In comparing the two plans, one bundle has more of one good but less of the other. Since utility is increasing in both goods, it is not obvious which of these two plans is more preferred. The proper way to analyze this is to use the indirect utility function $V(p_f, p_c, M)$. Given any two (or more) price vectors, the indirect utility function tells us which set of prices is most preferred. Substituting our Marshallian demand functions into the (direct) utility function yields:*

$$\begin{aligned} U^{non} &= V(p_f, p_c, M) = \left(q_f^{non}\right)^{\frac{1}{4}}\left(q_c^{non}\right)^{\frac{3}{4}} \\ U^{mem} &= V(p_f', p_c, M - fee) = \left(q_f^{mem}\right)^{\frac{1}{4}}\left(q_c^{mem}\right)^{\frac{3}{4}} \end{aligned}$$

   which gives us:

$$\begin{aligned} U^{non} &= V(10, 1, 1000) = (25)^{\frac{1}{4}}(750)^{\frac{3}{4}} \cong 320 \\ U^{mem} &= V(5, 1, 900) = (45)^{\frac{1}{4}}(675)^{\frac{3}{4}} \cong 344. \end{aligned}$$

   The membership plan yields higher utility and is therefore the more preferred plan.

Remark *An alternative approach would be to show that the optimal bundle with no membership* $(25, 750)$ *is still budget feasible with new prices and new income yet is no longer regarded as optimal. The expenditure associated with no membership at new prices is:*

$$25 \cdot \$5 + 750 \cdot \$1 = \$875$$

which is less than the \$900 *worth of monthly income available to her after purchasing the* \$100 *membership. She therefore has* \$25 *of income to spend on either 5 more fitness classes, 25 units of all other goods, or some combination of the two. This approach simply tells us that it is easy to find another bundle which is strictly preferred. But note that it does not tell us which bundle. It is therefore a less satisfactory approach.*

8. There are continuing efforts to open up the Alaskan wilderness for further energy exploration. Suppose there are only 2 goods, natural gas q_1, and other goods, q_2. Suppose further that the current income of the typical individual in the U.S. is \$25 thousand dollars, that the current price of natural gas is $p_1 = \$5$ per unit and that the price of other goods is $p_2 = \$1$ per unit. If further exploration is allowed, the price of natural gas is projected to fall to \$1 per unit. However, there is a group of environmentalists who will be outraged by any damage to this natural habitat. Assume that the typical, non-environmentalist American has the utility function:

$$u(q_1, q_2) = 30q_1^{1/2}q_2^{1/2}$$

   (a) At initial prices, what is the demand for q_1, for q_2, and what is the initial utility of the typical American?

   Remark *Note that the utility function is given in the form of* $u(q_1, q_2) = Aq_1^{1/2}q_2^{1/2}$, *where A is a positive constant and thus a monotone transformation of the utility function* $u(q_1, q_2) = q_1^{1/2}q_2^{1/2}$. *Hence, our solutions in no way depend on the parameter A. If you were to ignore it, you get the same answers either way. We could show this by computing the MRS at any bundle and show that it is the same,* $MRS = q_2/q_1$. *Where you must be careful is in using our "trick" to write the expenditure function. If you have multiplied by A to obtain indirect utility* $U(\cdot)$, *you must divide* $U(\cdot)$ *by A in your short-cut to derive the expenditure function. In the solutions that follow, I use the utility function given above, where* $A = 30$.

   Solution *Ordinary demand functions are as follows:*

$$\begin{aligned} q_1^* &= \frac{1}{2}\frac{M}{p_1} = \frac{1}{2}\frac{25,000}{5} = 2,500 \\ q_2^* &= \frac{1}{2}\frac{M}{p_2} = \frac{1}{2}\frac{25,000}{1} = 12,500 \end{aligned}$$

   The indirect utility associated with orginal prices and income is:

$$\begin{aligned} U^o &= 30 \cdot (q_1^*)^{1/2}(q_2^*)^{1/2} \\ &= 30 \cdot (2,500)^{1/2}(12,500)^{1/2} \\ &= 167,705 \end{aligned}$$

(b) Supposing that gas exploration proceeds and the price of natural gas falls, what would be the new demand for q_1, for q_2, and the new utility level of the typical non-environmentalist?

Solution *New demand is as follows:*

$$\begin{aligned} q_1^{**} &= \frac{1}{2}\frac{M}{\widehat{p}_1} = \frac{1}{2}\frac{25,000}{1} = 12,500 \\ q_2^{**} &= \frac{1}{2}\frac{M}{p_2} = \frac{1}{2}\frac{25,000}{1} = 12,500 \end{aligned}$$

The indirect utility associated with new prices is:

$$\begin{aligned} U^1 &= 30 \cdot (q_1^{**})^{1/2}(q_2^{**})^{1/2} \\ &= 30 \cdot (12,500)^{1/2}(12,500)^{1/2} \\ &= 375,000 \end{aligned}$$

(c) How much of the increase in demand for q_1 resulting from the price decrease is due to the substitution effect? To the income effect?

Solution *First, we identify the new quantity of good 1 resulting from a change in the relative price of good 1 holding the typical consumer's purchasing power constant, i.e. the change in the quantity of good 1 purchased is the pure SE. To do this we first need to find the tangency between indifference curve u^0 and the compensated budget constraint with new prices. The budget constraint has slope of $\frac{p_1}{p_2} = -1$. The $MRS = -\frac{q_2}{q_1}$. Setting the two equal implies*

$$q_2 = q_1$$

The second thing we know is that

$$30 \cdot (q_1)^{1/2}(q_2)^{1/2} = 167,705$$

Solving these two equations for the quantity demanded of good 1, we get $\widehat{q}_1 =$ 5590. Thus, the SE and IE are as follows:

$$\begin{aligned} SE &= \widehat{q}_1 - q_1^* = 5590 - 2500 = 3090 \\ IE &= q_1^{**} - \widehat{q}_1 = 12,500 - 5590 = 6910 \end{aligned}$$

(d) Suppose that the exploration has been approved and is about to proceed but has not yet started. How much would the environmentalists need to offer to the typical non-environmentalist (those who do not care about the environment and like low natural gas prices) to compensate them if the environmentalists were successful in getting companies to abandon their exploration efforts.

Solution *Calculate EV, which is the difference between the expenditure function with original prices and new utility, i.e., $E(p_1, p_2, U^1)$ and original income $M = \$25,000$. To find the expenditure function, invert the indirect utility function for M :*

$$U^1 = 30\left(\frac{1}{2}\frac{M}{p_1}\right)^{1/2}\left(\frac{1}{2}\frac{M}{p_2}\right)^{1/2} = 375,000$$

Isolating M yields the following expenditure function:

$$M = 12,500\,(2p_1)^{1/2}\,(2p_2)^{1/2}$$

Plug in original prices to get the level of expenditures required to achieve new utility:

$$\begin{aligned} M &= 12,500(20)^{1/2} \\ M &= \$55,901 \end{aligned}$$

Now subtract original income to get EV :

$$\begin{aligned} EV &= \$55,901 - \$25,000 \\ EV &= \$30,901 \end{aligned}$$

(e) Suppose that the environmentalists were not successful in putting a stop to exploration and that the new lower price of natural gas has materialized. Because of the "green" media campaign, there is debate about closing the Alaskan wilderness down again and preventing further gas removal. How much income would the typical non-environmentalist be willing to pay in order to preserve this source of natural gas.

Solution *Calculate CV, the difference between original income,* \$25,000, *and the expenditure function with new prices and original utility,* $E(\widehat{p_1}, p_2, U^0)$. *To find the expenditure function, invert the indirect utility function given above, but with original utility and new prices:*

$$U^0 = 30\left(\frac{1}{2}\frac{M}{\widehat{p_1}}\right)^{1/2}\left(\frac{1}{2}\frac{M}{p_2}\right)^{1/2} = 167,750$$

which yields

$$M = 5591(2\widehat{p_1})^{1/2}(2p_2)^{1/2}$$

Now plug in new prices to get:

$$\begin{aligned} M &= 5591.67(2) \\ M &= \$11,183 \end{aligned}$$

Thus, CV is given as:

$$\begin{aligned} CV &= \$25,000 - \$11,183 \\ CV &= \$13,817 \end{aligned}$$

FIGURE 1

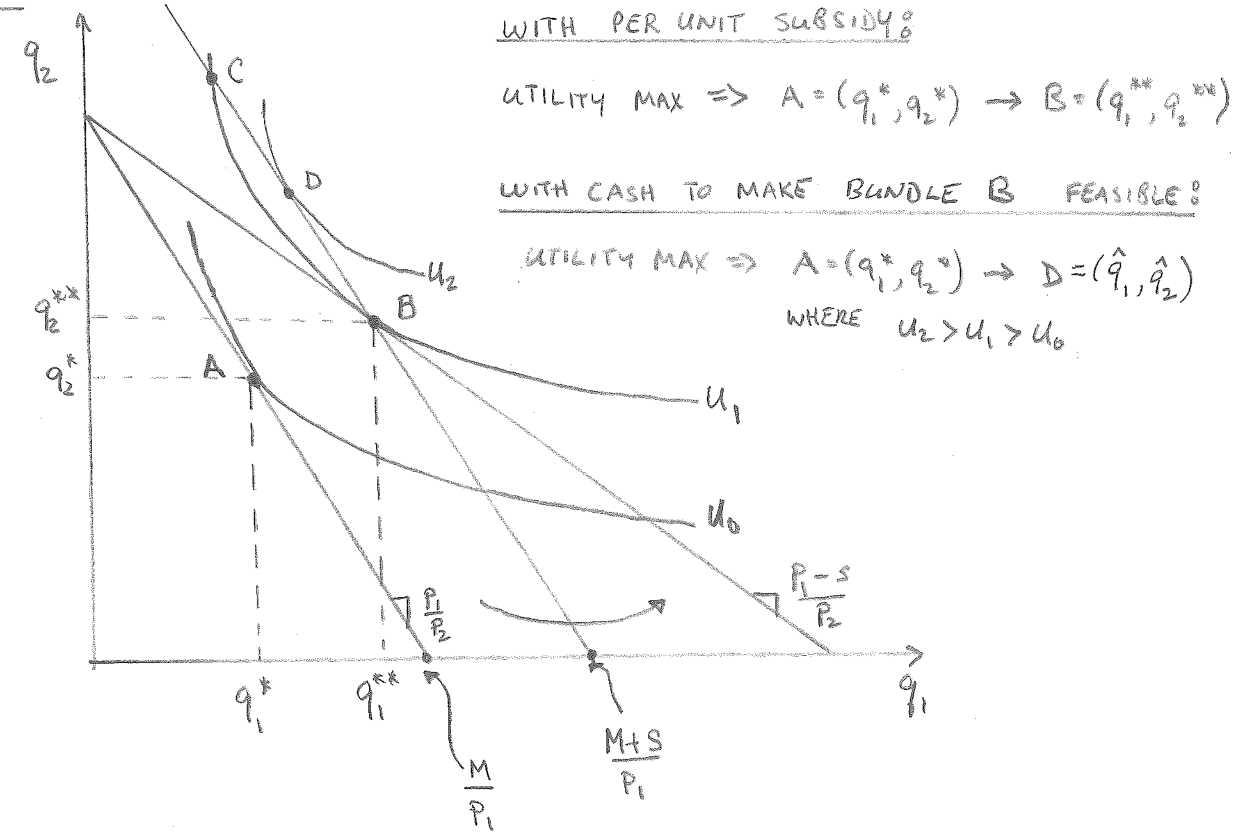


FIGURE 2

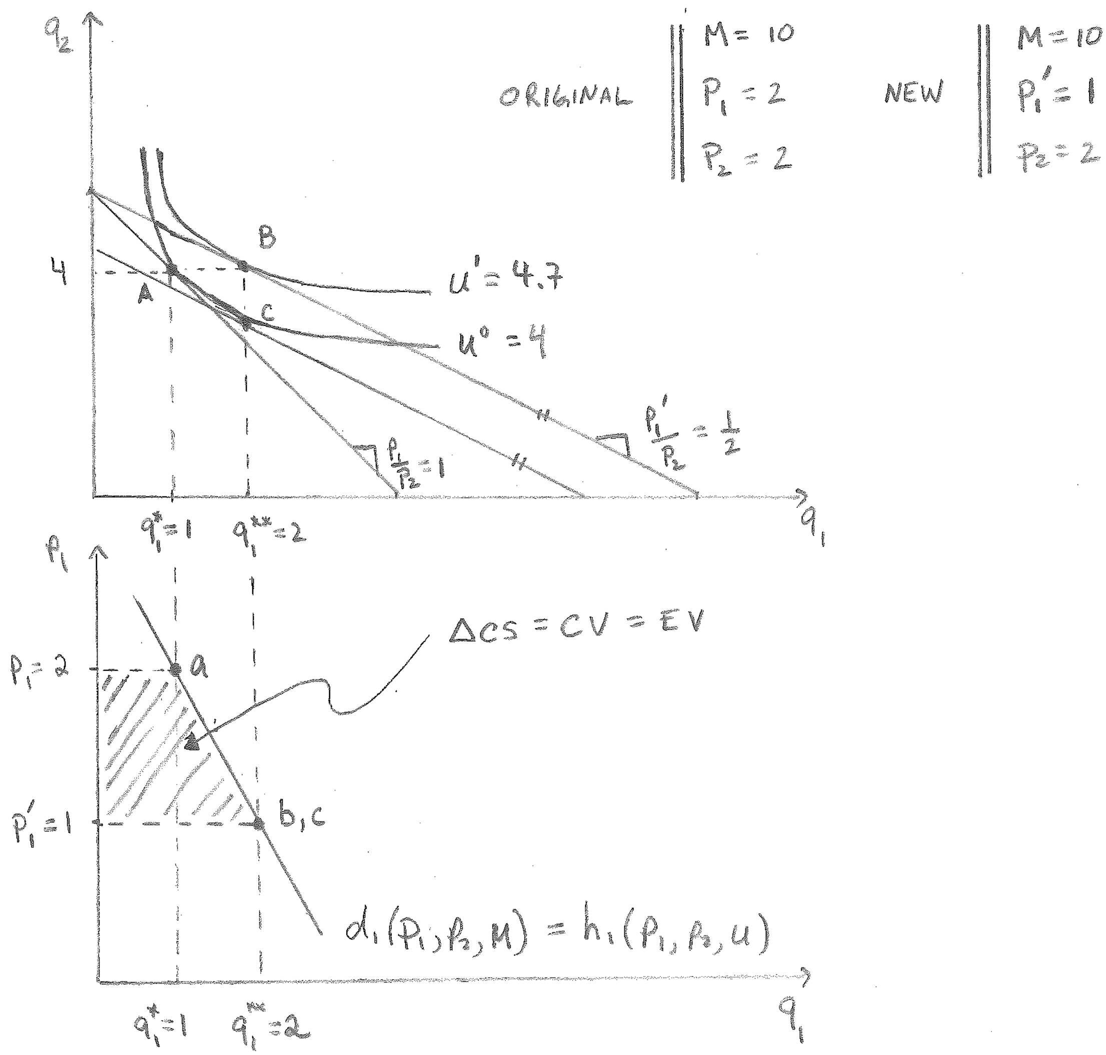
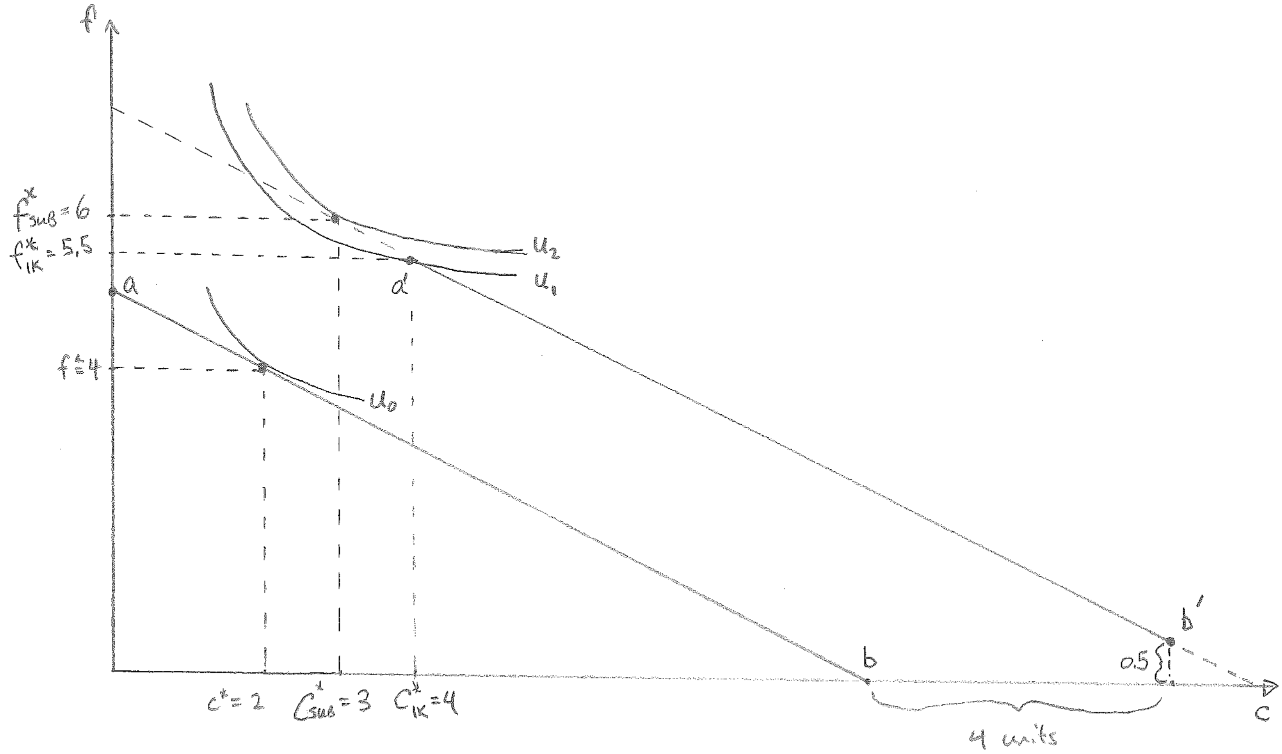


FIGURE 3



- original Budget line: ab

  New budget line (with in-kind transfer): a'b'

  ↳ point a is moved 0.5 units up and 4 units over to get a'

  ↳ same for point b

- ORIGINAL SOLUTION: $c^* = 2$, $f^* = 4$

  SOLUTION WITH CASH TRASFER: $c^*_{SUB} = 3$, $f^*_{SUB} = 6$

  ↳ with cash, utility ↑ from u_0 to u_2

  ↳ But this choice and resulting utility level is not feasible with an in-kind transfer, since $c \geq 4$ and $f \geq 0.5$

- Since the preferred solution is $c^*_{SUB} = 3$ but is forced to consume $c \geq 4$ the consumer will choose the minimum c (and no more)

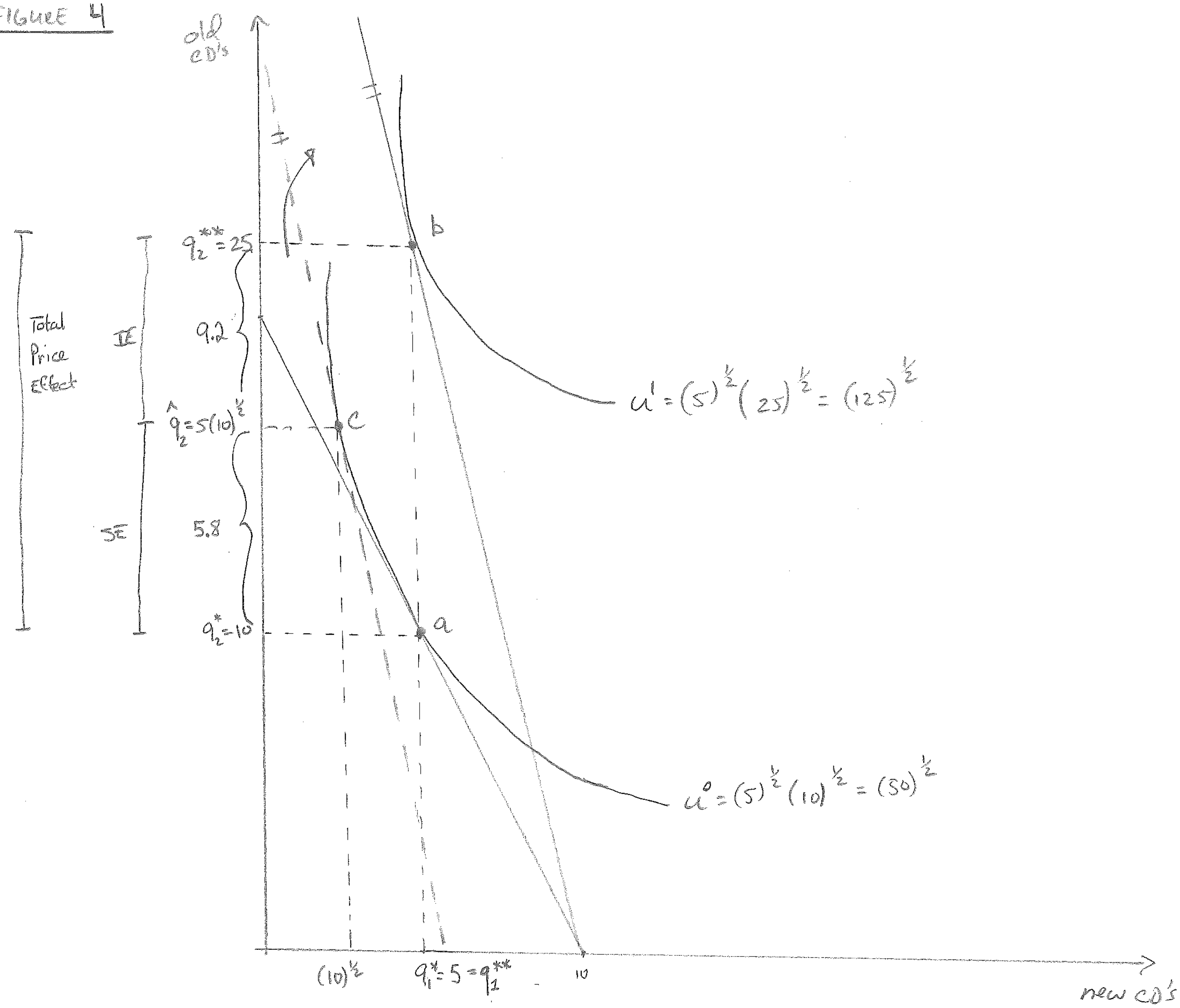
- SOLUTION WITH IN-KIND TRASFER: $c^*_{IK} = 4$, $f^*_{IK} = 5.5$

  ↳ utility ↑ from u_0 to only u_1

  ↳ solution is at the kink

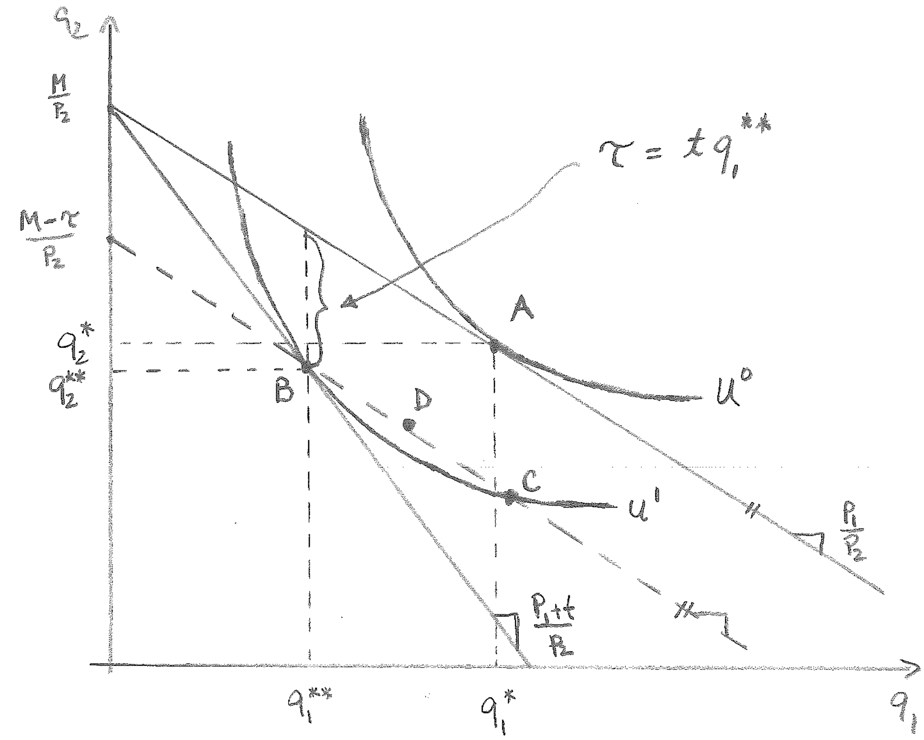
  ↳ Here the constraints are binding.

FIGURE 4



old CD's
b
c
a
$q_2^{**} = 25$
9.2
$\hat{q}_2 = 5(10)^{\frac{1}{2}}$
5.8
$q_2^* = 10$
Total Price Effect
IE
SE
$u^1 = (5)^{\frac{1}{2}}(25)^{\frac{1}{2}} = (125)^{\frac{1}{2}}$
$u^0 = (5)^{\frac{1}{2}}(10)^{\frac{1}{2}} = (50)^{\frac{1}{2}}$
$(10)^{\frac{1}{2}}$
$q_1^* = 5 = q_1^{**}$
10
new CD's

FIGURE 5



(i) - START AT BUNDLE $A = (q_1^*, q_2^*)$

- EXCISE TAX => NEW OPTIMUM AT BUNDLE $B = (q_1^{**}, q_2^{**})$
- TAX REVENUE RAISED EQUALS PER UNIT TAX, t, MULTIPLIED BY NEW QUANTITY OF GOOD 1, I.E, tq_1^{**}.

(ii) LUMP-SUM TAXES MUST RAISE SAME AMOUNT SO THAT $tq_1^{**} = \tau$

- MEANS BUNDLE $B = (q_1^{**}, q_2^{**})$ MUST STILL BE AFFORDABLE WITH LUMP-SUM TAX

(iii) GIVEN $\tau = tq_1^{**}$, SUBSTITUTION TO ANY BUNDLE (SUCH AS D) ON LINE SEGMENT $\overline{BC}$ IS POSSIBLE.

- SINCE BUNDLE $D \succ B$ => LUMP-SUM TAXES ARE PREFERABLE!