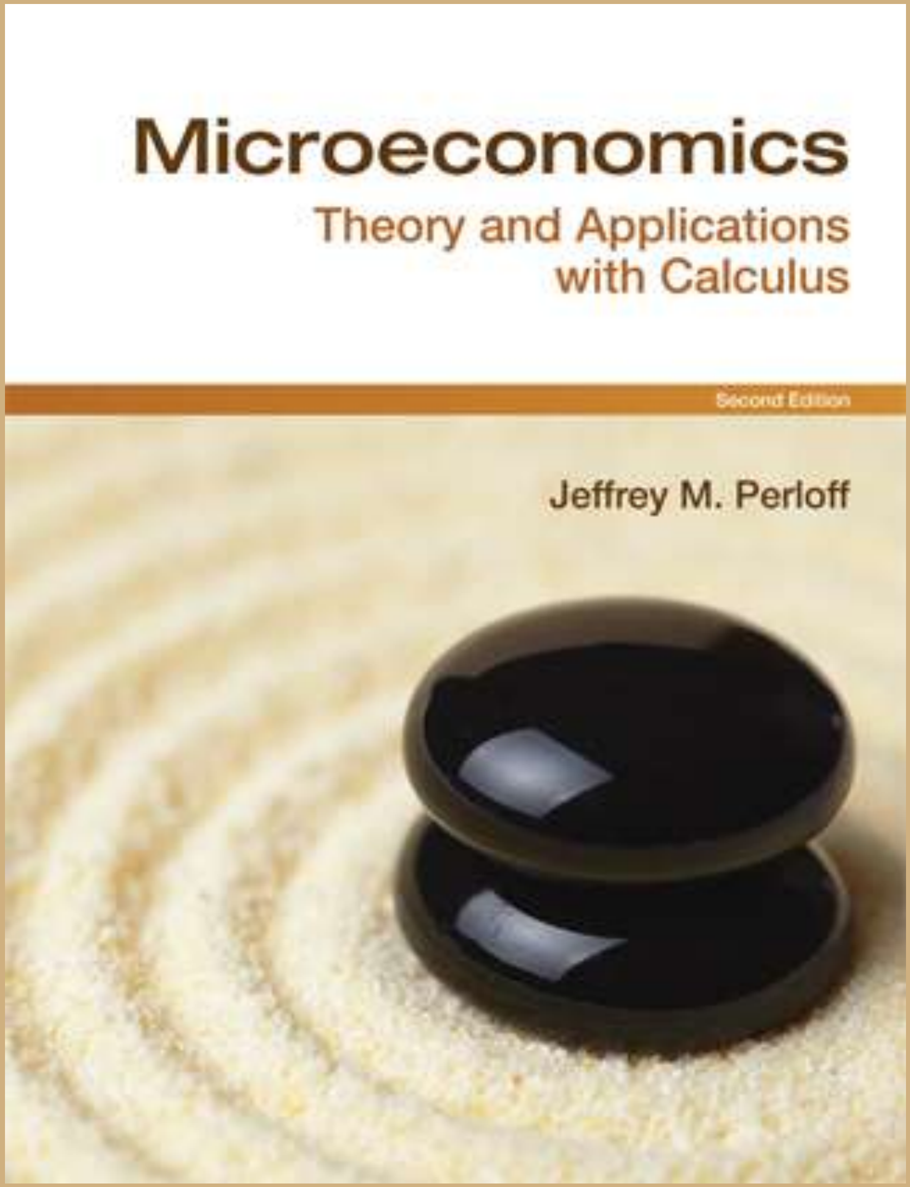
Chapter 3

A Consumer's Constrained Choice

If this is coffee, please bring me some tea; but if this is tea, please bring me some coffee.

Abraham Lincoln



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Chapter 3 Outline

Chapter 3: Model of Consumer Behavior

- Premises of the model:

1. Individual ***tastes*** or ***preferences*** determine the amount of pleasure people derive from the goods and services they consume.
2. Consumers face ***constraints***, or limits, on their choices.
3. Consumers ***maximize*** their well-being or pleasure from consumption subject to the budget and other constraints they face.

- Examples.

3.1 Preferences

- To explain consumer behavior, economists assume that consumers have a set of tastes or preferences that they use to guide them in choosing between goods.
- Goods are ranked according to how much pleasure a consumer gets from consuming each.
  - Preference relations summarize a consumer's ranking
  - $\succ$ is used to convey strict preference (e.g. $a \succ b$)
  - $\succsim$ is used to convey weak preference (e.g. $a \succsim b$)
  - $\sim$ is used to convey indifference (e.g. $a \sim b$)

3.1 Preferences

- Properties of preferences:

  1. Completeness

  - When facing a choice between two bundles of goods (e.g. *a* and *b*), a consumer can rank them so that either $a \succ b$, $b \succ a$, or $a \sim b$.

  2. Transitivity

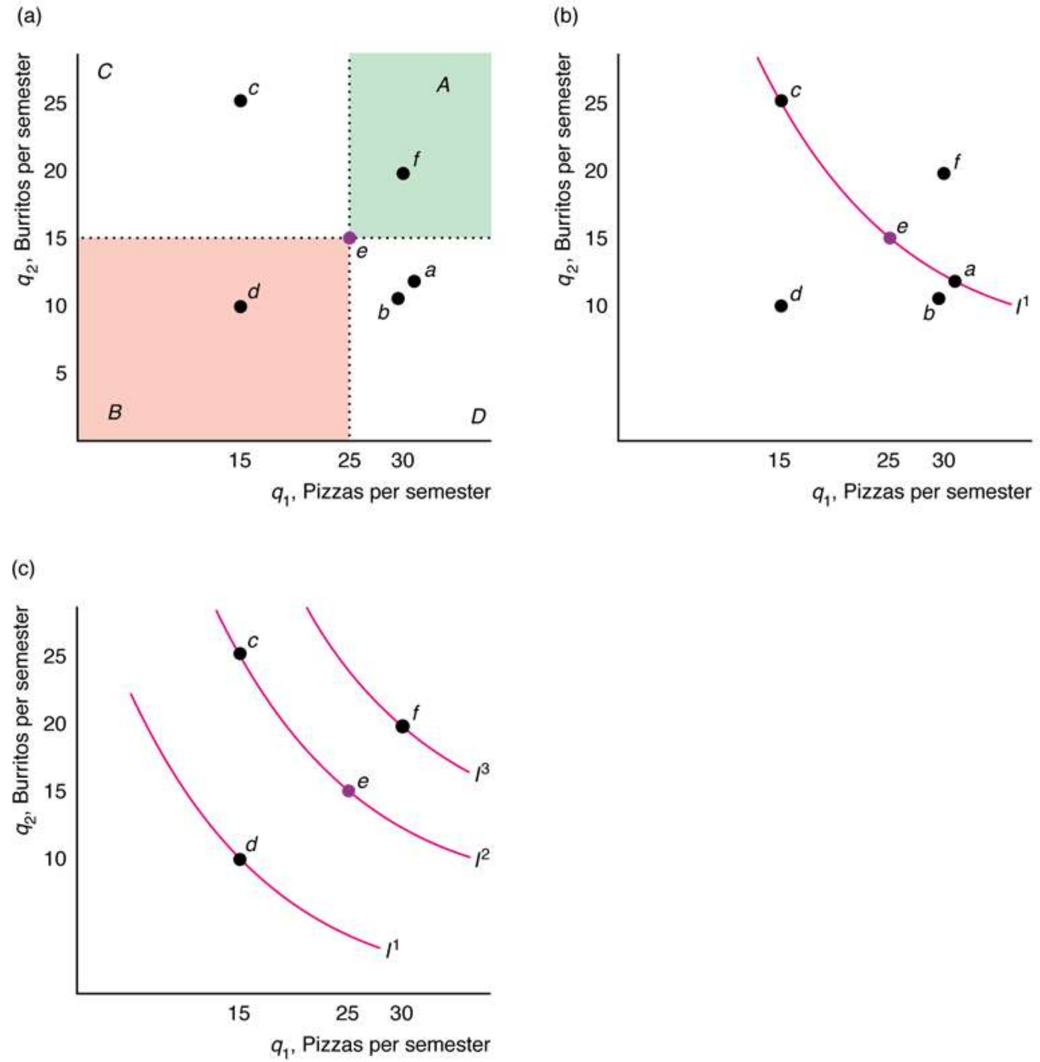
  - Consumers' rankings are logically consistent in the sense that if $a \succ b$ and $b \succ c$, then $a \succ c$.

  3. More is Better

  - All else the same, more of a commodity is better than less.
  - In this regard, a "good" is different than a "bad."

3.1 Preference Maps

- Graphical interpretation of consumer preferences over two goods:

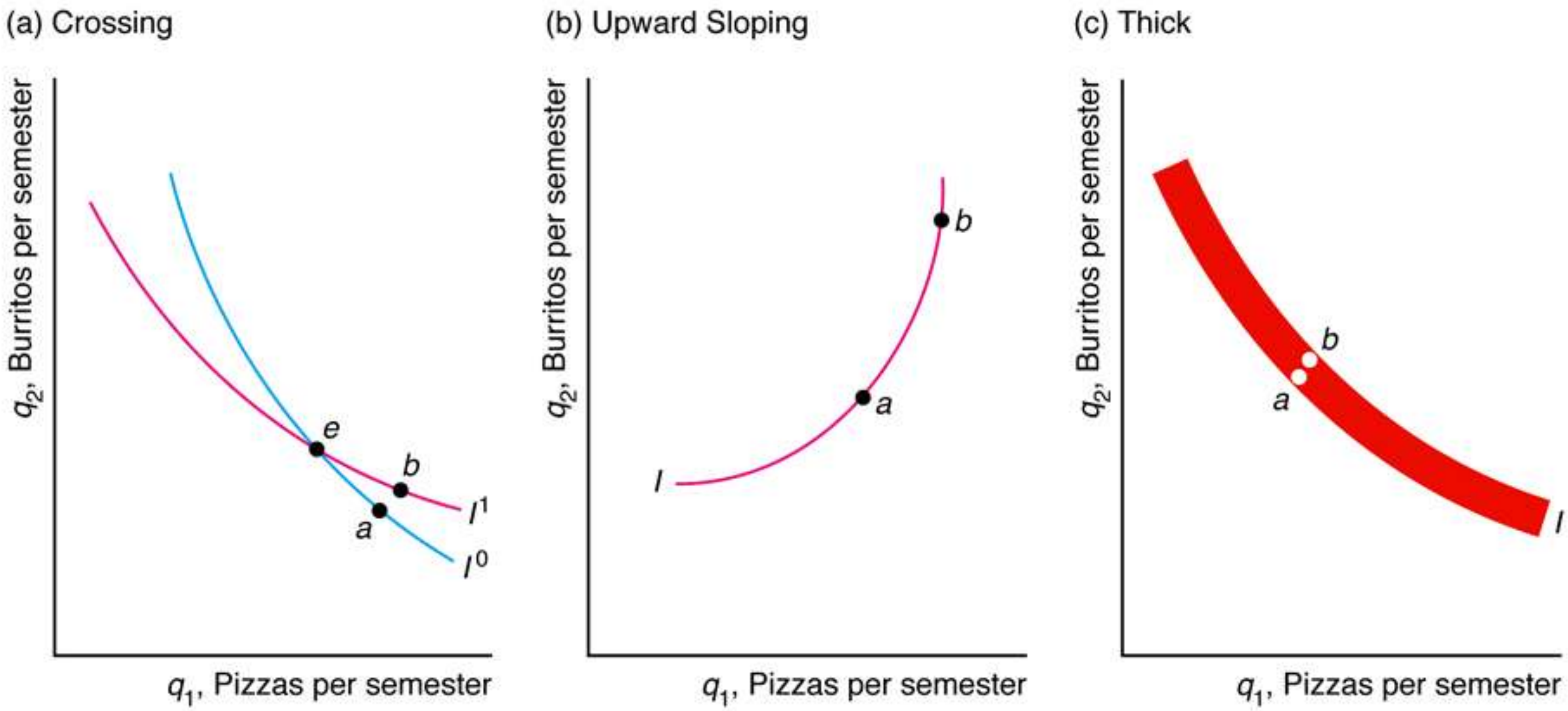


3.1 Indifference Curves

- The set of all bundles of goods that a consumer views as being equally desirable can be traced out as an ***indifference curve***.
- Five important properties of indifference curves:
  1. Bundles of goods on indifference curves further from the origin are preferred to those on indifference curves closer to the origin.
  2. There is an indifference curve through every possible bundle.
  3. Indifference curves cannot cross.
  4. Indifference curves slope downward.
  5. Indifference curves cannot be thick.

3.1 Indifference Curves

- Impossible indifference curves:



3.2 Utility

- ***Utility*** refers to a set of numerical values that reflect the relative rankings of various bundles of goods.
- The ***utility function*** is the relationship between utility measures and every possible bundle of goods.
  - Given a specific utility function, you can graph a specific indifference curve and determine exactly how much utility is gained from specific consumption choices.
  - Example: q_1 = pizza and q_2 = burritos

$$U = \sqrt{q_1 q_2}$$

    - Bundle x contains 16 pizzas and 9 burritos: $U(x) = 12$
    - Bundle y contains 13 pizzas and 13 burritos: $U(y) = 13$
    - Thus, $y \succ x$

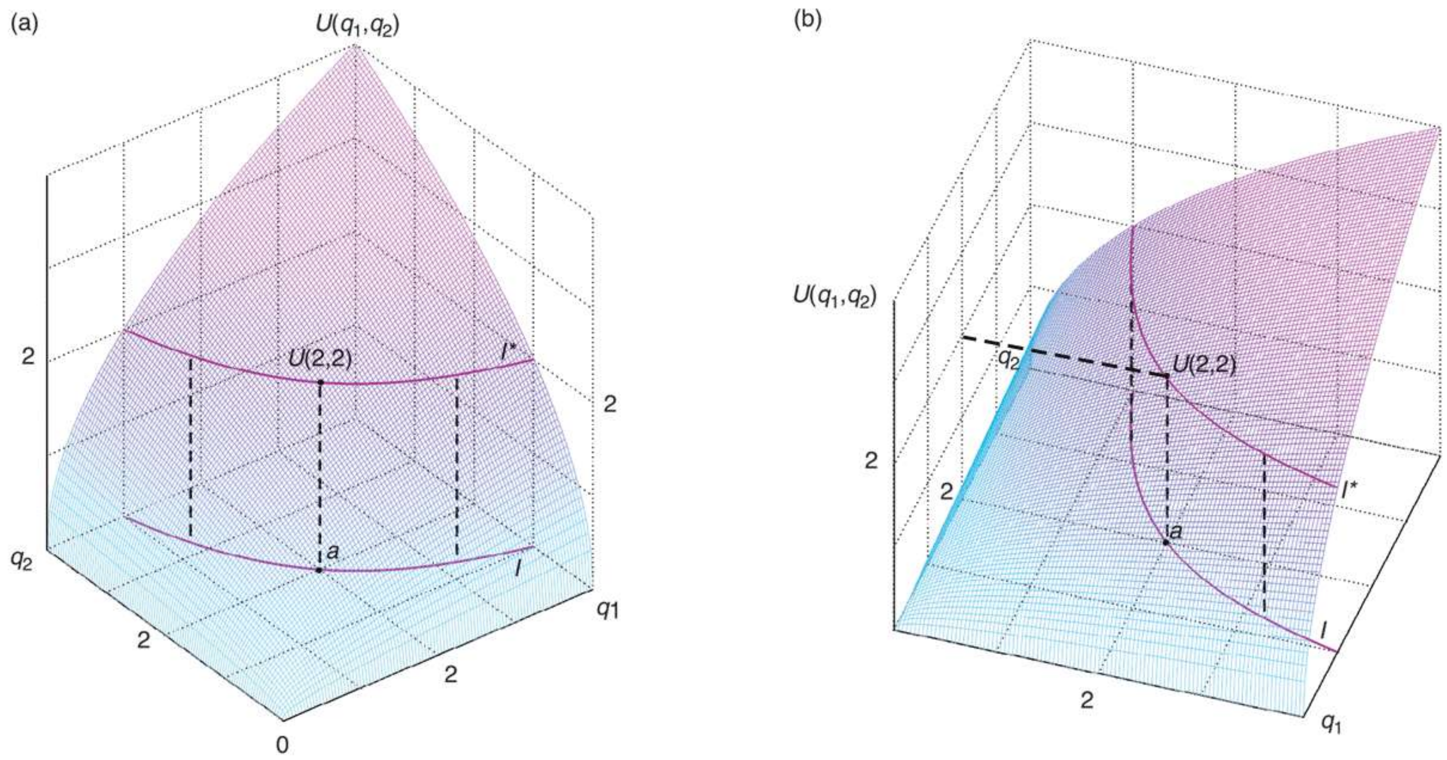
3.2 Utility

- Utility is an *ordinal* measure rather than a *cardinal* one.
  - Utility tells us the relative ranking of two things but not how much more one rank is valued than another.
  - We don't really care that $U(x) = 12$ and $U(y) = 13$ in the previous example; we care that $y \succ x$.
  - Any utility function that generated $y \succ x$ would be consistent with these preferences.

- A utility function can be transformed into another utility function in such a way that preferences are maintained.
  - ***Positive monotonic transformation***

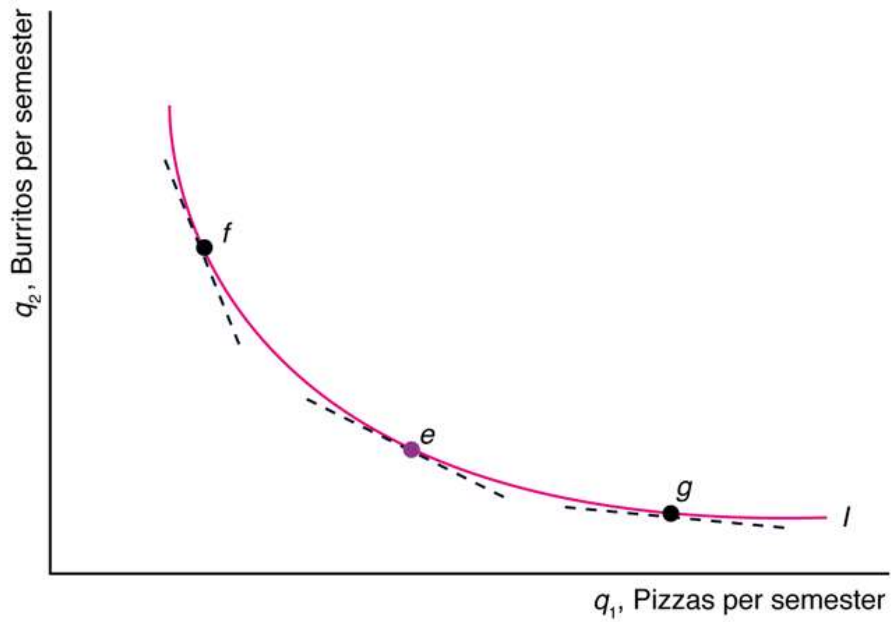
3.2 Utility and Indifference Curves

- The general utility function (for q_1 = pizza and q_2 = burritos) is $\overline{U} = U(q_1, q_2)$



3.2 Willingness to Substitute Between Goods

- ***Marginal Rate of Substitution* (MRS)** is the maximum amount of one good that a consumer will sacrifice (trade) to obtain one more unit of another good.
  - It is the slope at a particular point on the indifference curve
  - MRS = dq_2 / dq_1



3.2 Marginal Utility and MRS

- The MRS depends on how much extra utility a consumer gets from a little more of each good.
  - ***Marginal utility*** is the extra utility that a consumer gets from consuming the last unit of a good, holding the consumption of other goods constant.

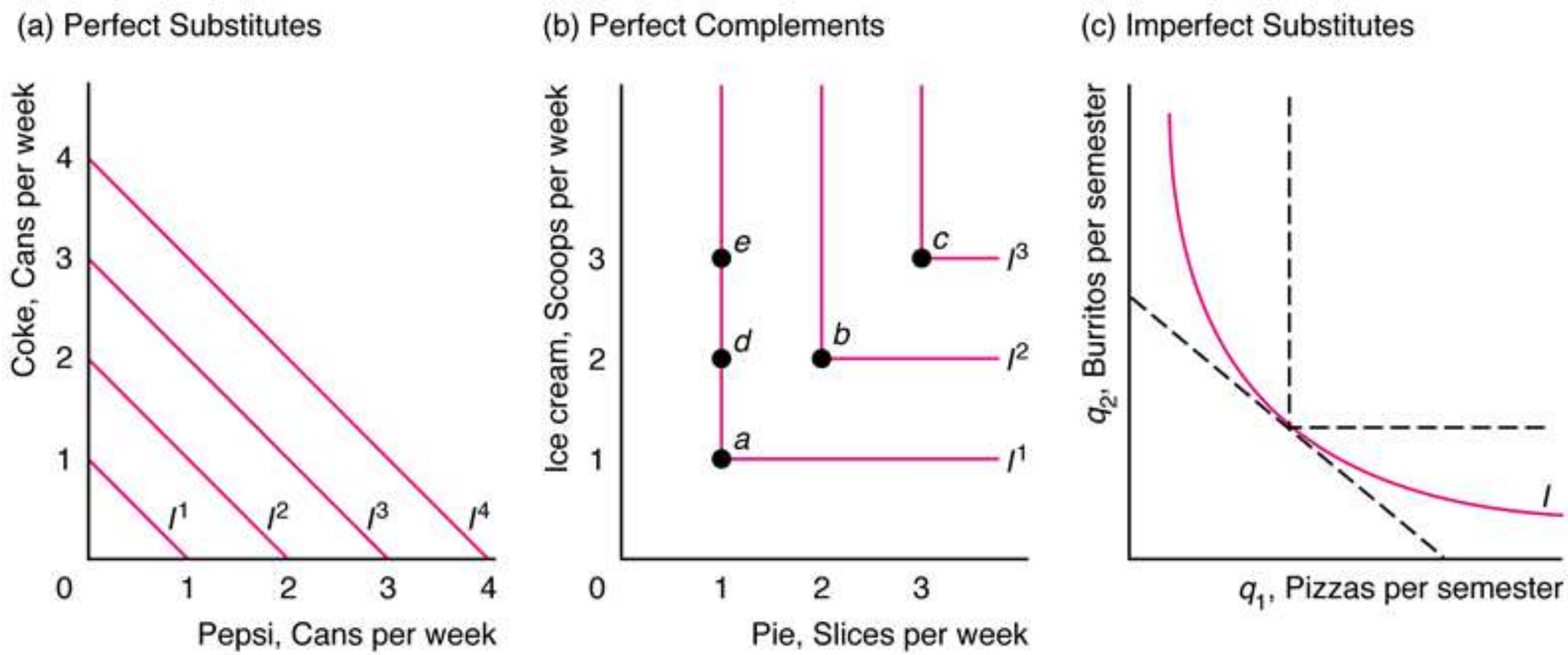
$$\text{marginal utility of pizza} = \frac{\partial U}{\partial q_1} = U_1$$

- Using calculus to calculate the MRS:

$$MRS = \frac{\mathrm{d}q_2}{\mathrm{d}q_1} = -\frac{\partial U / \partial q_1}{\partial U / \partial q_2} = -\frac{U_1}{U_2}$$

3.2 Curvature of Indifference Curves

- MRS (willingness to trade) diminishes along many typical indifference curves that are concave to the origin.
- Different utility functions generate different indifference curves:



3.2 Curvature of Indifference Curves

- Perfect Substitutes
  - Goods that a consumer is completely indifferent between
  - Example: Clorox (C) and Generic Bleach (G)

$$U(C, G) = iC + jG$$

  - MRS = -2 (constant)

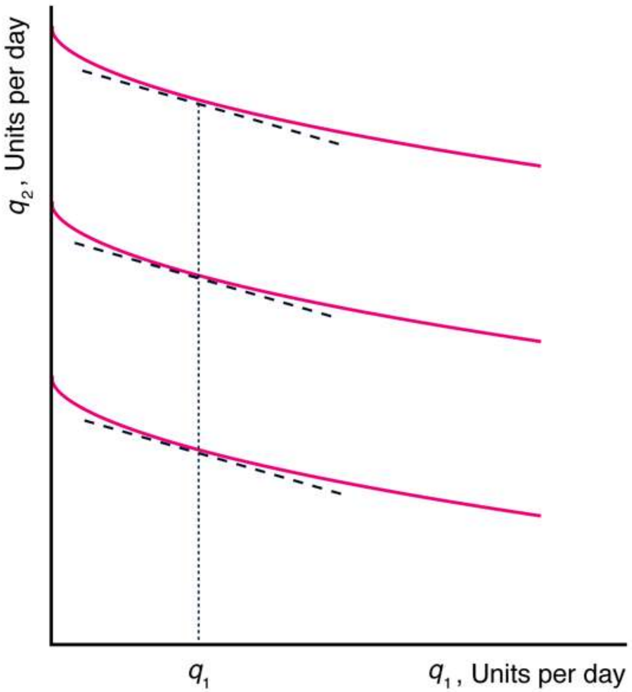
- Perfect Complements
  - Goods that are consumed in fixed proportions
  - Example: Apple pie (A) and Ice cream (I)

$$U(A, V) = \min(iA, jV)$$

  - MRS is undefined

3.2 Curvature of Indifference Curves

- Imperfect Substitutes
  - Between extreme examples of perfect substitutes and perfect complements are standard-shaped, convex indifference curves.
  - Cobb-Douglas utility function (e.g. $U = q_1^a q_2^{1-a}$) indifference curves never hit the axes.
  - Quasilinear utility function (e.g. $U(q_1, q_2) = u(q_1) + q_2$) indifference curves hit one of the axes.



3.3 Budget Constraint

- Consumers maximize utility subject to constraints.
- If we assume consumers can't save and borrow, current period income determines a consumer's budget.
- Given prices of pizza (p_1) and burritos (p_2), and income Y, the ***budget line*** is

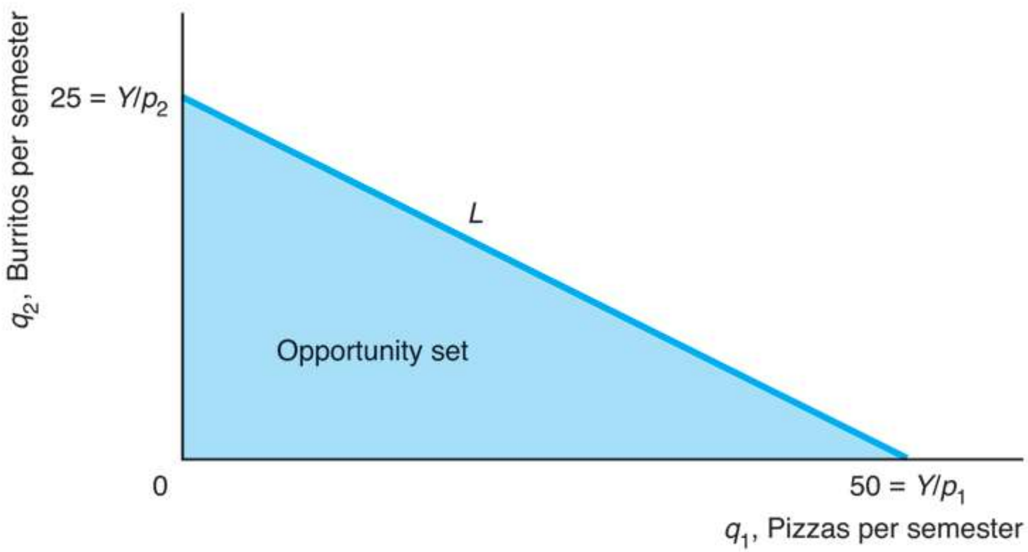
$$p_1 q_1 + p_2 q_2 = Y$$

- Example:
  - Assume p_1 = \$1, p_2 = \$2 and Y = \$50
  - Rewrite the budget line equation for easier graphing ($y=mx+b$ form):

$$q_2 = \frac{\$50 - (\$1 \times q_1)}{\$2} = 25 - \frac{1}{2} q_1$$

3.3 Budget Constraint

- ***Marginal Rate of Transformation*** (MRT) is how the market allows consumers to trade one good for another.
  - It is the slope of the budget line: $MRT = \frac{dq_2}{dq_1} = -\frac{p_1}{p_2}$



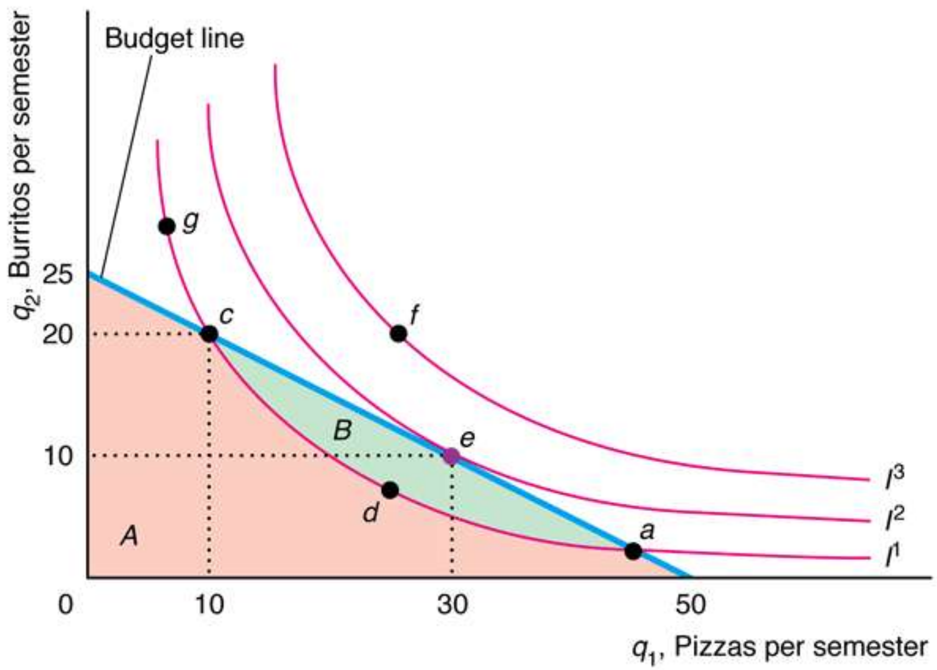
3.4 Constrained Consumer Choice

- Consumers maximize their well-being (utility) subject to their budget constraint.
- The highest indifference curve attainable given the budget is the consumer's ***optimal bundle***.
- When the optimal bundle occurs at a point of tangency between indifference curve and budget line, this is called an ***interior solution***.
  - Mathematically, $$MRS = -\frac{U_1}{U_2} = -\frac{p_1}{p_2} = MRT$$
  - Rearranging, we can see that the marginal utility per dollar is equated across goods at the optimum:

$$\frac{U_1}{p_1} = \frac{U_2}{p_2}$$

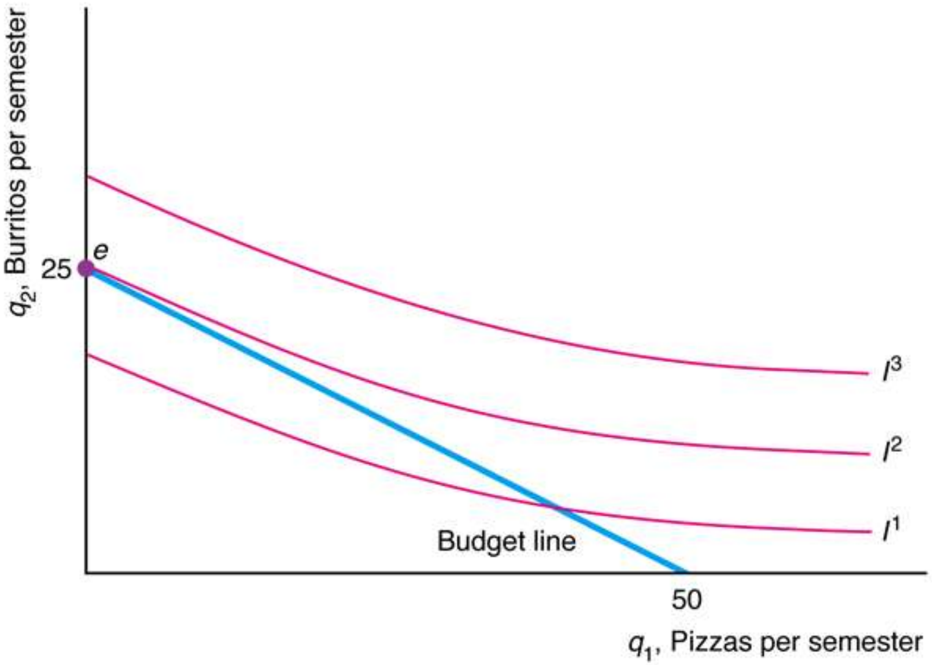
3.4 Constrained Consumer Choice

- The interior solution that maximizes utility without going beyond the budget constraint is Bundle *e*.
- The interior optimum is where $MRS = -\frac{U_1}{U_2} = -\frac{p_1}{p_2} = MRT$



3.4 Constrained Consumer Choice

- If the relative price of one good is too high and preferences are quasilinear, the indifference curve will not be tangent to the budget line and the consumer's optimal bundle occurs at a ***corner solution***.



3.4 Consumer Choice with Calculus

- Our graphical analysis of consumers' constrained choices can be stated mathematically:

$$\max_{q_1, q_2} U(q_1, q_2)$$

$$\text{s.t.} \quad Y = p_1 q_1 + p_2 q_2$$

- The optimum is still expressed as in the graphical analysis:

$$MRS = -\frac{U_1}{U_2} = -\frac{p_1}{p_2} = MRT$$

- These conditions hold if the utility function is quasi-concave, which implies indifference curves are convex to the origin.
- Solution reveals utility-maximizing values of q_1 and q_2 as functions of prices, p_1 and p_2, and income, Y.

3.4 Consumer Choice with Calculus

- Example (Solved Problem 3.5):

$$U(q_1, q_2) = \left(q_1^{\rho} + q_2^{\rho}\right)^{\frac{1}{\rho}}, \text{ where } 0 \neq \rho \leq 1.^{13}$$

$$\max_{q_1, q_2} U(q_1, q_2) = \left(q_1^{\rho} + q_2^{\rho}\right)^{\frac{1}{\rho}}$$

$$\text{s.t. } Y = p_1 q_1 + p_2 q_2$$

$$\max_{q_1} U\left(q_1, \frac{Y - p_1 q_1}{p_2}\right) = \left(q_1^{\rho} + \left[\frac{Y - p_1 q_1}{p_2}\right]^{\rho}\right)^{1/\rho}$$

$$\frac{1}{\rho}\left(q_1^{\rho} + \left[\frac{Y - p_1 q_1}{p_2}\right]^{\rho}\right)^{\frac{1-\rho}{\rho}}\left(\rho q_1^{\rho-1} + \rho\left[\frac{Y - p_1 q_1}{p_2}\right]^{\rho-1}\left[-\frac{p_1}{p_2}\right]\right) = 0$$

$$\frac{1}{\rho}\left(q_1^{\rho} + \left[\frac{Y - p_1 q_1}{p_2}\right]^{\rho}\right)^{\frac{1-\rho}{\rho}}\left(\rho q_1^{\rho-1} + \rho\left[\frac{Y - p_1 q_1}{p_2}\right]^{\rho-1}\left[-\frac{p_1}{p_2}\right]\right) = 0$$

$$q_1 = \frac{Y p_1^{z-1}}{p_1^{z} + p_2^{z}},$$

$$q_2 = \frac{Y p_2^{z-1}}{p_1^{z} + p_2^{z}}$$

3.4 Consumer Choice with Calculus

- A second approach to solving constrained utility maximization problems is the Lagrangian method:

$$\max_{q_1, q_2, \lambda} \mathcal{L} = U(q_1, q_2) + \lambda(Y - p_1 q_1 - p_2 q_2)$$

- The critical value of $\mathcal{L}$ is found through first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial q_1} = \frac{\partial U}{\partial q_1} - \lambda p_1 = U_1 - \lambda p_1 = 0 \qquad \frac{\partial \mathcal{L}}{\partial q_2} = U_2 - \lambda p_2 = 0$$

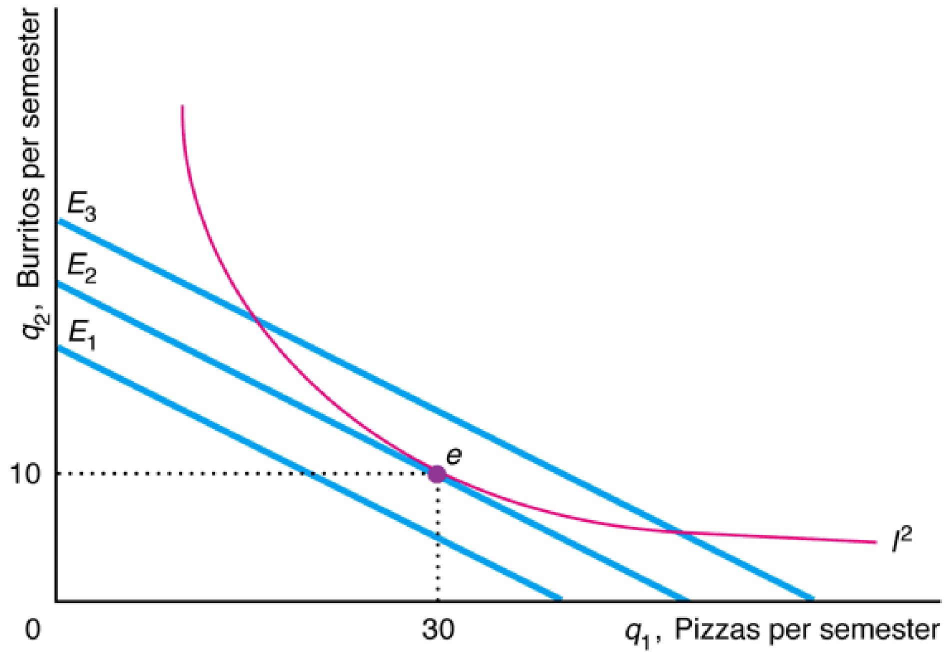
$$\frac{\partial \mathcal{L}}{\partial \lambda} = Y - p_1 q_1 - p_2 q_2 = 0$$

- Equating the first two of these equations yields:

$$\lambda = \frac{U_1}{p_1} = \frac{U_2}{p_2}$$

3.4 Minimizing Expenditure

- Utility maximization has a dual problem in which the consumer seeks the combination of goods that achieves a particular level of utility for the least expenditure.



3.4 Expenditure Minimization with Calculus

- Minimize expenditure, *E*, subject to the constraint of holding utility constant:

$$\min_{q_1, q_2} E = p_1 q_1 + p_2 q_2$$

$$\text{s.t. } \overline{U} = U(q_1, q_2)$$

- The solution of this problem, the ***expenditure function***, shows the minimum expenditure necessary to achieve a specified utility level for a given set of prices:

$$E = E\left(p_1, p_2, \overline{U}\right)$$

3.5 Behavioral Economics

- What if consumers are not rational, maximizing individuals?
  - ***Behavioral economics*** adds insights from psychology and empirical research on cognition and emotional biases to the rational economic model.
    - **Tests of transitivity**: evidence supports transitivity assumption for adults, but not necessarily for children.
    - **Endowment effect:** some evidence that endowments of goods influence indifference maps, which is not the assumption of economic models.
    - **Salience:** evidence that consumers are more sensitive to increases in pre-tax prices than post-tax price increases from higher ad valorem taxes.
      - **Bounded rationality** suggests that calculating post-tax prices is “costly” so some people don’t bother to do it, but they would use the information if it were provided.

Figure 3.10 Optimal Bundles on Convex Sections of Indifference Curves