Chapter 4

Microeconomics

Theory and Applications with Calculus

Demand

I have enough money to last me the rest of my life, unless I buy something. Jackie Mason



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Chapter 4 Outline

- 4.1 Deriving Demand Curves
- 4.2 Effects of an Increase in Income
- 4.3 Effects of a Price Increase
- 4.4 Cost-of-Living Adjustment
- 4.5 Revealed Preference

4.1 Deriving Demand Curves

- If we hold people's tastes, their incomes, and the prices of other goods constant, a change in the price of a good will cause a *movement along the demand curve*.
- We saw this in Chapter 2:



4.1 Deriving Demand Curves

- In Chapter 3, we used calculus to maximize consumer utility subject to a budget constraint.
 - This amounts to solving for the consumer's system of demand functions for the goods.
- Example: q_1 = pizza and q_2 = burritos
 - Demand functions express these quantities in terms of the prices of both goods and income:

$$q_1 = Z(p_1, p_2, Y)$$

 $q_2 = B(p_1, p_2, Y)$

 Given a specific utility function, we can find closedform solutions for the demand functions.

4.1 Example: Deriving Demand Curves

• Constant Elasticity of Substitution (CES) utility function:

$$U(q_1, q_2) = (q_1^{\rho} + q_2^{\rho})^{\frac{1}{\rho}}, \ 0 \neq \rho \leq 1$$

- Budget constraint:
 - Y= p1q1 + p2q2
- In Chapter 3, we learned that the demand functions that result from this constrained optimization problem are:

$$q_1 = \frac{Y p_1^{z-1}}{p_1^z + p_2^z} \qquad \qquad q_2 = \frac{Y p_2^{z-1}}{p_1^z + p_2^z}$$

• Quantity demanded of each good is a function of the prices of both goods and income.

4.1 Example: Deriving Demand Curves

- Cobb-Douglas utility function:
 - U(q1, q2) = q1a q2(1-a)
- Budget constraint:
 - Y= p1q1 + p2q2
- In Chapter 3, we learned that the demand functions that result from this constrained optimization problem are:

$$q_1 = a \frac{Y}{p_1} \qquad q_2 = (1-a) \frac{Y}{p_2}$$

• With Cobb-Douglas, quantity demanded of each good is a function of only the good's own-price and income.

4.1 Deriving Demand Curves

 Panel a below shows the demand curve for q₁, which we plot by holding Y fixed and varying p₁.



4.1 Deriving Demand Curves Graphically (a) Indifference Curves and Budget Constraints

- Allowing the price of the good on the x-axis to fall, the budget constraint rotates out and shows how the optimal quantity of the x-axis good purchased increases.
 - This traces out points along the demand curve.



4.2 Effects of an Increase in Income

- An increase in an individual's income, holding tastes and prices constant, causes a *shift of the demand curve*.
 - An increase in income causes an increase in demand (e.g. a parallel shift away from the origin) if the good is a *normal good* and a decrease in demand (e.g. parallel shift toward the origin) if the good is *inferior*.
- A change in income prompts the consumer to choose a new optimal bundle.
- The result of the change in income and the new utility maximizing choice can be depicted three different ways.

4.2 Effects of an Increase in Income

(a) Indifference Curves and Budget Constraints



4.2 Effects of an Increase in Income

- The result of the change in income and the new utility maximizing choice can be depicted three different ways.
 - **1.Income-consumption curve**: using the consumer utility maximization diagram, traces out a line connecting optimal consumption bundles.
 - **2.Shifts in demand curve**: using demand diagram, show how quantity demanded increases as the price of the good stays constant.
 - **3.Engle curve**: with income on the vertical axis, show the positive relationship between income and quantity demanded.

4.2 Consumer Theory and Income Elasticities

• Recall the formula for income elasticity of demand from Chapter 2:

 $\xi = \frac{\text{percentage change in quantity demanded}}{\text{percentage change in income}} = \frac{\Delta Q/Q}{\Delta Y/Y} = \frac{\partial Q}{\partial Y}\frac{Y}{Q}$

- **Normal goods**, those goods that we buy more of when our income increases, have a positive income elasticity.
 - Luxury goods are normal goods with an income elasticity greater than 1.
 - Necessity goods are normal goods with an income elasticity between 0 and 1.
- **Inferior goods**, those goods that we buy less of when our income increases, have a negative income elasticity.

4.2 Income-Consumption Curve and Income Elasticities

• The shape of the income-consumption curve for two goods tells us the sign of their income elasticities.



4.2 Income-Consumption Curve and Income Elasticities

• The shape of the incomeconsumption and Engle curves can change in ways that indicate goods can be both inferior and normal depending on an individual's income level.

(a) Indifference Curves and Budget Constraints



4.3 Effects of a Price Increase

- Holding tastes, other prices, and income constant, an increase in the price of a good has two effects on an individual's demand:
 - **1.Substitution effect**: the change in quantity demanded when the good's price increases, holding other prices and consumer utility constant.
 - **2.Income effect**: the change in quantity demanded when income changes, holding prices constant.
- When the price of a good increases, the total change in quantity demanded is the sum of the substitution and income effects.

4.3 Income and Substitution Effects

- The direction of the substitution effect is always unambiguous.
 - When price increases, individuals consume less of it because they are substituting away from the now more expensive good.
- The direction of the income effect depends upon whether the good is normal or inferior; it depends upon the income elasticity.
 - When price increases and the good is normal, the income effect is negative.
 - When price increases and the good is inferior, the income effect is positive.

4.3 Income and Substitution Effects with a Normal Good

- Beginning from budget constraint L¹, an increase in the price of music tracks rotates budget constraint into L².
 - The total effect of this price change, a decrease in quantity of 12 tracks per quarter, can be decomposed into income and substitution effects.



4.3 Compensated Demand Curve

- The demand curves shown thus far have all been uncompensated, or Marshallian, demand curves.
 - Consumer utility is allowed to vary with the price of the good.
 - In the figure from the previous slide, utility fell when the price of music tracks rose.
- Alternatively, a *compensated*, or *Hicksian*, *demand curve* shows how quantity demanded changes when price increases, holding utility constant.
 - Only the pure substitution effect of the price change is represented in this case.
 - An individual must be compensated with extra income as the price rises in order to hold utility constant.

4.3 Compensated Demand Curve



4.3 Compensated Demand Curve

- Deriving the compensated, or Hicksian, demand curve is straight-forward with the expenditure function:
 - E is the smallest expenditure that allows the consumer to achieve a given level of utility based on given market prices:

$$E = E(p_1, p_2, \overline{U})$$

• Differentiating with respect to the price of the first good yields the compensated demand function for the first good: ∂E

$$\frac{\partial E}{\partial p_1} = H(p_1, p_2, \overline{U}) = q_1$$

- A \$1 increase in p₁ on each of the q₁ units purchased requires the consumer increases spending by \$q₁ to keep utility constant.
- This result is called Shephard's lemma.

4.3 Slutsky Equation

- We graphically decomposed the total effect of a price change on quantity demanded into income and substitution effects.
- Deriving this same relationship mathematically utilizes elasticities and is called the *Slutsky equation*.

total effect = substitution effect + income effect

 $\varepsilon = \varepsilon^* + (-\theta\xi)$

- *E* is elasticity of uncompensated demand and the total effect
- *E*[^] is elasticity of compensated demand and the substitution effect
- heta is the share of the budget spent on the good
- ξ is the income elasticity
- $\check{ heta}\xi$ is the income effect

4.4 Cost-of-Living Adjustment

- **Consumer Price Index (CPI)**: measure of the cost of a standard bundle of goods (market basket) to compare prices over time.
 - Example: In 2010 dollars, what is the cost of a McDonald's hamburger in 1955? $\frac{\text{CPI for 2010}}{\text{CPI for 1955}} \times \text{price of a burger} = \frac{216.7}{26.8} \times 15 \notin = \1.21
- Knowledge of substitution and income effects allows us to analyze how accurately the government measures inflation.
- Consumer theory can be used to show that the costof-living measure used by governments overstates inflation.

4.4 Cost-of-Living Adjustment (COLA)

• CPI in first year is the cost of buying the market basket of food (F) and clothing (C) that was actually purchased that year:

$$Y_1 = p_C^1 C_1 + p_F^1 F_1$$

• CPI in the second year is the cost of buying the first year's bundle in the second year:

$$Y_2 = p_C^2 C_1 + p_F^2 F_1$$

• The rate of inflation determines how much additional income it took to buy the first year's bundle in the second year: $Y_2 = p_C^2 C_1 + p_F^2 F_1$

4.4 Cost-of-Living Adjustment (COLA)

- If a person's income increases automatically with the CPI, he can afford to buy the first year's bundle in the second year, but chooses not to.
 - Better off in the second year because the CPI-based COLA overcompensates in the sense that utility increases.



4.5 Revealed Preference

- Preferences \rightarrow predict consumer's purchasing behavior
- Purchasing behavior \rightarrow infer consumer's preferences



(b) Four budget constraints

(a) Two budget constraints