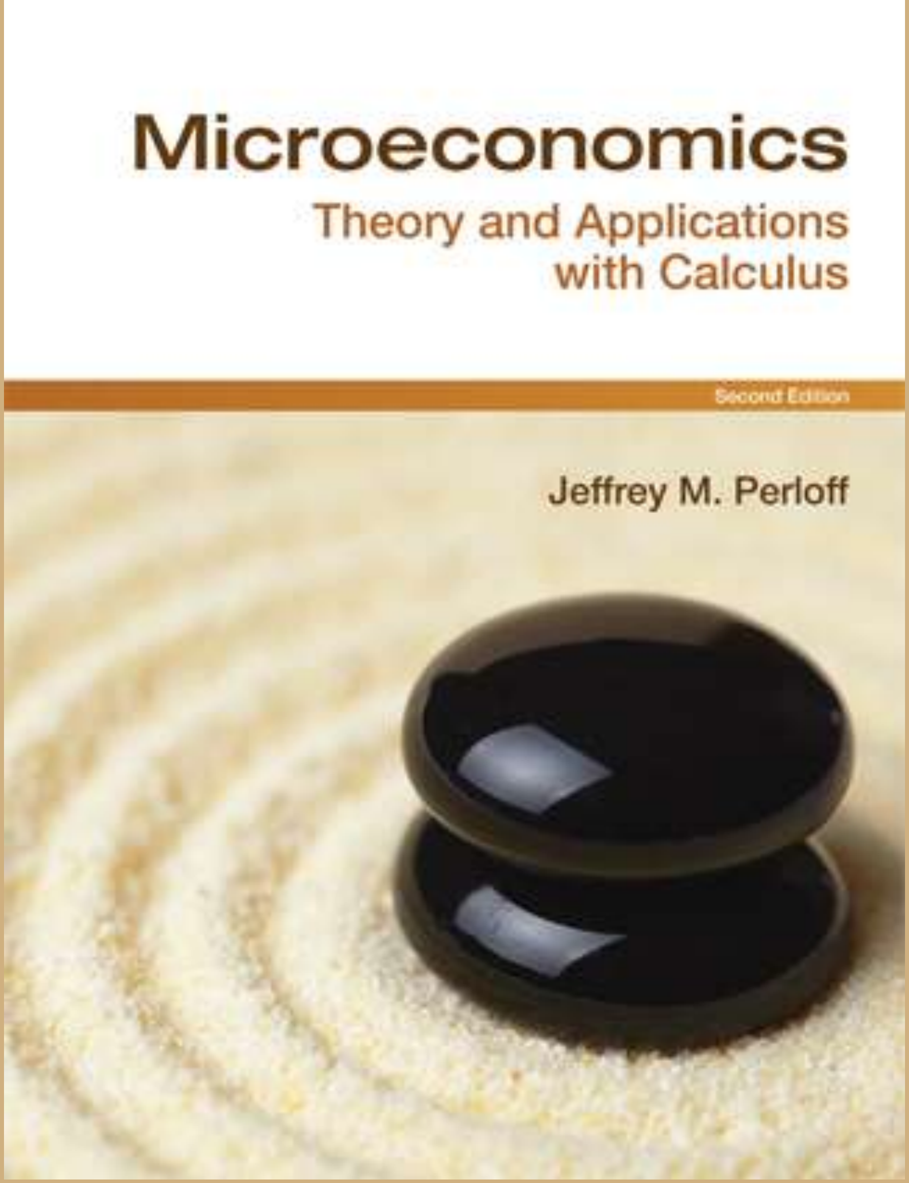
Chapter 13

Game Theory



A camper awakens to the growl of a hungry bear and sees his friend putting on a pair of running shoes. "You can't outrun a bear," scoffs the camper. His friend coolly replies, "I don't have to. I only have to outrun you!".

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Chapter 13 Outline

13.1 An Overview of Game Theory

- ***Game theory*** is a set of tools used by economists and many others to analyze players' strategic decision making.

- ***Games*** are competitions between players (individuals, firms, countries) in which each player is aware that the outcome depends on the actions of all players.

- Game theory is particularly useful for examining how a small group of firms in a market with substantial barriers to entry, an ***oligopoly***, interact.
  - Examples: soft drink industry, chain hotel industry, smart phones

13.1 An Overview of Game Theory

- Useful definitions:
  - The ***payoffs*** of a game are the players' valuation of the outcome of the game (e.g. profits for firms, utilities for individuals).
  - The ***rules of the game*** determine the timing of players' moves and the actions players can make at each move.
  - An ***action*** is a move that a player makes at a specified stage of a game.
  - A ***strategy*** is a battle plan that specifies the action that a player will make condition on the information available at each move and for any possible contingency.
  - ***Strategic interdependence*** occurs when a player's optimal strategy depends on the actions of others.

13.1 An Overview of Game Theory

- Assumptions:
  - All players are interested in maximizing their payoffs.
  - All players have common knowledge about the rules of the game
  - Each player's payoff depends on actions taken by all players
  - Complete information (payoff function is common knowledge among all players) is different from perfect information (player knows full history of game up to the point he is about to move)

- We will examine both static and dynamic games in this chapter.

13.2 Static Games

- In a ***static game*** each player acts simultaneously, only once and has complete information about the payoff functions but imperfect information about rivals' moves.
  - Examples: employer negotiations with a potential new employee, teenagers playing "chicken" in cars, street vendors' choice of locations and prices

- Consider a ***normal-form*** static game of complete information which specifies the players, their strategies, and the payoffs for each combination of strategies.
  - Competition between United and American Airlines on the LA-Chicago route.

13.2 Quantity-Setting Game

- Quantities, *q*, are in thousands of passengers per quarter; profits are in millions of dollars per quarter

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	United: \$4.1; American: \$4.1	United: \$5.1; American: \$3.8
	$q_U = 48$	United: \$3.8; American: \$5.1	United: \$4.6; American: \$4.6

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

13.2 Predicting a Game's Outcome

- Rational players will avoid strategies that are *dominated* by other strategies.
- In fact, we can precisely predict the outcome of any game in which every player has a ***dominant strategy***.
  - A strategy that produces a higher payoff than any other strategy for every possible combination of its rivals' strategies
- Airline Game:
  - If United chooses *high-output*, American's *high-output* strategy maximizes its profits.
  - If United chooses *low-output*, American's *high-output* strategy still maximizes its profits.
  - For American, *high-output* is a dominant strategy.

13.2 Quantity-Setting Game

- The *high-output* strategy is dominant for American and for United. This is a dominant strategy equilibrium.

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	\$4.1 \$4.1	\$3.8 \$5.1
	$q_U = 48$	\$5.1 \$3.8	\$4.6 \$4.6

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

- Players choose strategies that don’t maximize joint profits.
  - Called a ***prisoners’ dilemma*** game; all players have dominant strategies that lead to a profit that is less than if they cooperated.

13.2 Iterated Elimination of Strictly Dominated Strategies

- In games where not all players have a dominant strategy, we need a different means of predicting the outcome.

		American Airlines		
		$q_A = 96$	$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 96$	$0 $0	$2.0 $3.1	$2.3 $4.6
	$q_U = 64$	$3.1 $2.0	$4.1 $4 1	$3.8 $5.1
	$q_U = 48$	$4.6 $2.3	$5.1 $3.8	$4.6 $4.6

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

13.2 Static Games

- When iterative elimination fails to predict a unique outcome, we can use a related approach.
- The ***best response*** is a strategy that maximizes a player's payoff given its beliefs about its rivals' strategies.
- A set of strategies is a ***Nash equilibrium*** if, when all other players use these strategies, no player can obtain a higher playoff by choosing a different strategy.
  - No player has an incentive to deviate from a Nash equilibrium.

13.2 Nash Equilibrium

- Every game has at least one Nash equilibrium and every dominant strategy equilibrium is a Nash equilibrium, too.

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	\$4.1 / \$4.1	\$3.8 / \$5.1
	$q_U = 48$	\$5.1 / \$3.8	\$4.6 / \$4.6

(In each cell, the upper-right value is American's profit and the lower-left value is United's profit.)

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

13.2 Mixed Strategies

- So far, the firms have used ***pure strategies***, which means that each player chooses a single action.
- A ***mixed strategy*** is when a player chooses among possible actions according to probabilities the player assigns.
  - A pure strategy assigns a probability of 1 to a single action.
  - A mixed strategy is a probability distribution over actions.
- When a game has multiple pure-strategy Nash equilibria, a mixed-strategy Nash equilibrium can help to predict the outcome of the game.

13.2 Simultaneous Entry Game

- This game has two Nash equilibria in pure strategies and one mixed-strategy Nash equilibrium.

Firm 1

Firm 2	*Do Not Enter*	*Enter*
Do Not Enter	$0 (Firm 1); $0 (Firm 2)	$1 (Firm 1); $0 (Firm 2)
Enter	$0 (Firm 1); $1 (Firm 2)	–$1 (Firm 1); –$1 (Firm 2)

13.2 Advertising Game

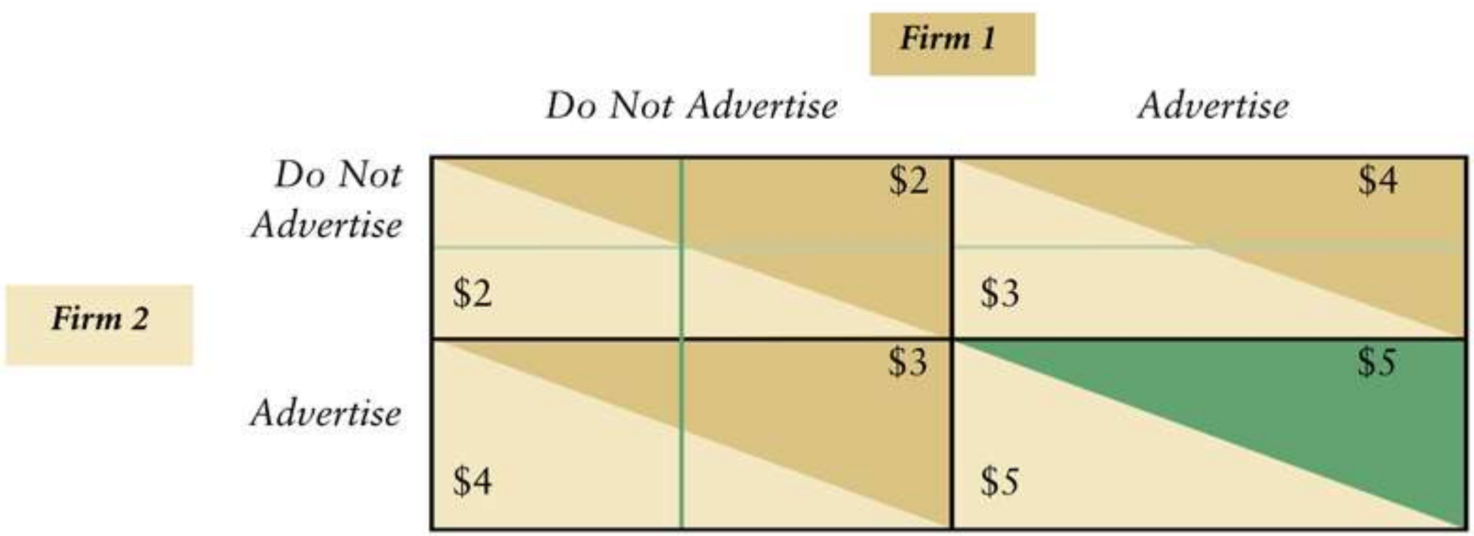
- Firms don’t cooperate in this game and the sum of firms’ profits is not maximized in the Nash equilibrium

		Firm 1	
		Do Not Advertise	Advertise
Firm 2	Do Not Advertise	$2 (Firm 1) $2 (Firm 2)	$3 (Firm 1) $0 (Firm 2)
	Advertise	$0 (Firm 1) $3 (Firm 2)	$1 (Firm 1) $1 (Firm 2)

13.2 Advertising Game

- If advertising by either firm attracts new customers to the market, then Nash equilibrium does maximize joint profit.

(b) Advertising Attracts New Customers to the Market



13.3 Dynamic Games

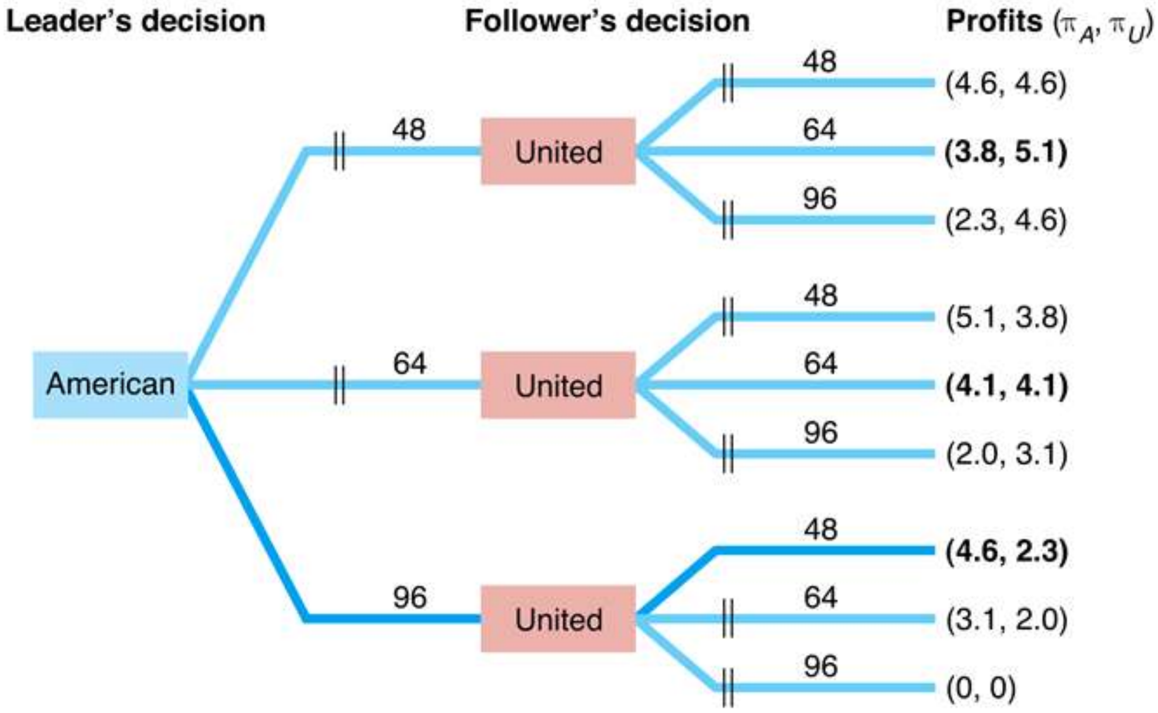
- In ***dynamic games***:
  - players move either sequentially or repeatedly
  - players have complete information about payoff functions
  - at each move, players have perfect information about previous moves of all players
- Dynamic games are analyzed in their ***extensive form***, which specifies
  - the *n* players
  - the sequence of their moves
  - the actions they can take at each move
  - the information each player has about players' previous moves
  - the payoff function over all possible strategies.

13.3 Dynamic Games

- Consider a single period *two-stage* game:
  - First stage: player 1 moves
  - Second stage: player 2 moves
- In games where players move sequentially, we distinguish between an action and a strategy.
  - An action is a move that a player makes a specified point.
  - A strategy is a battle plan that specifies the action a player will make condition on information available at each move.
- Return to the Airline Game to demonstrate these concepts.
  - Assume American chooses its output before United does.

13.3 Dynamic Games

- This Stackelberg ***game tree*** shows
  - *decision nodes*: indicates which player's turn it is
  - *branches:* indicates all possible actions available
  - *subgames*: subsequent decisions available given previous actions



13.3 Dynamic Games

- To predict the outcome of the Stackelberg game, we use a strong version of Nash equilibrium.

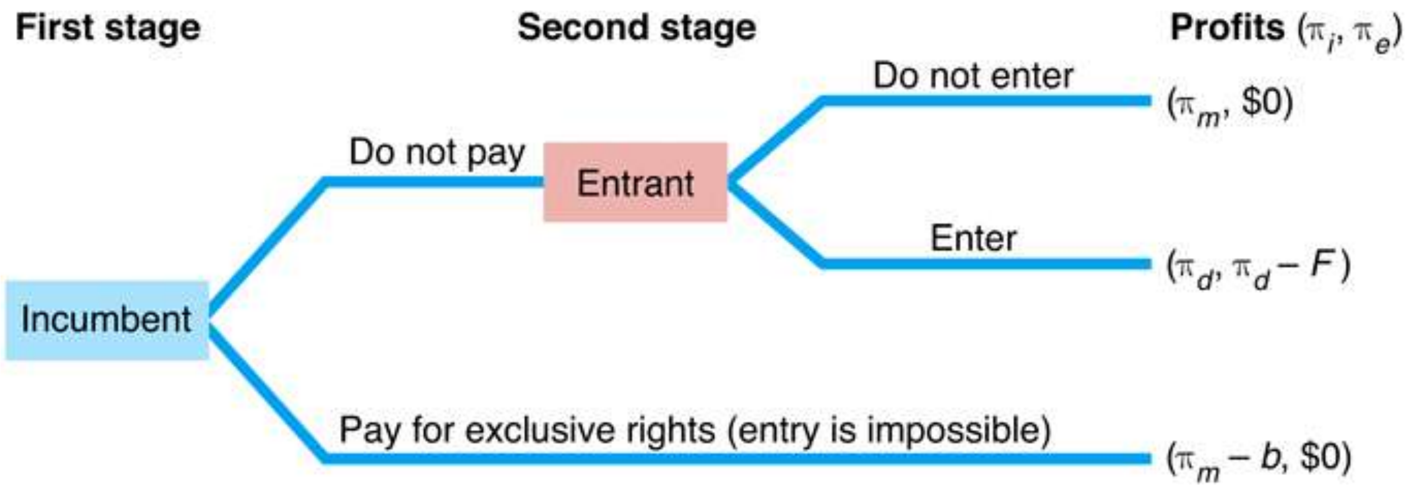
- A set of strategies forms a ***subgame perfect Nash equilibrium*** if the players' strategies are a Nash equilibrium in every subgame.
  - This game has four subgames; three subgames at second stage where United makes a decision and an additional subgame at the time of the first-stage decision.
  - We can solve for the subgame perfect Nash equilibrium using ***backward induction***.

13.3 Dynamic Games

- ***Backward induction*** is where we determine:
  - the best response by the last player to move
  - the best response for the player who made the next-to-last move
  - repeat the process until we reach the beginning of the game
- Airline Game
  - If American chooses 48, United selects 64, American's profit=3.8
  - If American chooses 64, United selects 64, American's profit=4.1
  - If American chooses 96, United selects 48, American's profit=4.6
  - Thus, American chooses 96 in the first stage.

13.3 Dynamic Entry Games

- Entry occurs unless the incumbent acts to deter entry by paying for exclusive rights to be the only firm in the market.



13.4 Auctions

- What if the players in a game don't have complete information about payoffs?
  - Players have to devise bidding strategies without this knowledge.
- An ***auction*** is a sale in which a good or service is sold to the highest bidder.
- Examples of things that are exchanged via auction:
  - Airwaves for radio stations, mobile phones, and wireless internet access
  - Houses, cars, horses, antiques, art

13.4 Elements of Auctions

- Rules of the Game:

1. Number of units
   - Focus on auctions of a single, indivisible item
2. Format
   - English auction: ascending-bid auction; last bid wins
   - Dutch auction: descending-bid auction; first bid wins
   - Sealed-bid auction: private, simultaneous bids submitted
3. Value
   - Private value: each potential bidder values item differently
   - Common value: good has same fundamental value to all

13.4 Bidding Strategies in Private-Value Auctions

- In a ***first-price*** sealed-bid auction, the winner pays his/her own, highest bid.
- In a ***second-price*** sealed-bid auction, the winner pays the amount bid by the second-highest bidder.
- In a second-price auction, should you bid the maximum amount you are willing to spend?
  - If you bid more, you may receive negative consumer surplus.
  - If you bid less, you only lower the odds of winning without affecting the price that you pay if you do win.
  - So, yes, you should bid your true maximum amount.

13.4 Bidding Strategies in Private-Value Auctions

- English Auction Strategy
  - Strategy is to raise your bid by smallest permitted amount until you reach the value you place on the good being auctioned.
  - The winner pays slightly more than the value of the second-highest bidder.

- Dutch Auction Strategy
  - Strategy is to bid an amount that is equal to or slightly greater than what you expect will be the second-highest bid.
  - Reducing your bid reduces probability of winning but increases consumer surplus if you win.

13.4 Auctions

- The ***winner's curse*** is that the auction winner's bid exceeds the common-value item's value.
  - Overbidding occurs when there is uncertainty about the true value of the good
  - Occurs in common-value but not private-value auctions
- Example:
  - Government auctions of timber on a plot of land
  - Bidders may differ on their estimates of how many board feet of lumber are on the plot
  - If average bid is accurate, then high bid is probably excessive
  - Winner's curse is paying too much