Chapter 13
Game Theory

■ Chapter Outline

■ Teaching Tips

Chapter 13 is one that you may want to cover in detail, especially if you have a significant number of management majors or pre-MBA students. Some students view economic analysis, game theory in particular, and management strategy as separate entities. The value of this material is that it links pricing strategy, advertising profits, and entry decisions, and thus demonstrates the importance of understanding microeconomic theory for good managerial decision making.

The chapter begins with an overview of the game theory and then moves on to static games, where players must move simultaneously. It then reviews mixed strategies and dynamic games. There is a substantial amount of self-teaching that can occur by having the class work in groups on selected problems (see Additional Questions and Problems later in this chapter) designed to make the points from each section. If the class has to work out the rules for effective strategy on their own, they are more likely to understand why such rules for behavior are important.

When presenting game theory, consider dividing the class into small groups before making any formal presentation of types of equilibria and strategic rules. Give each group three or four games to solve, including a simple zero-sum game, a dominant strategy equilibrium where players follow a given strategy no matter what the other does, and a Nash equilibrium where the payoffs create a prisoners' dilemma. (The Additional Questions and Problems section includes sample payoff matrices.) You can ask the groups to simply play the games at first, under the following rules. First, assume that each player must move simultaneously, and that no cooperation is allowed. Second, allow collusion between players, and last, assume that one player gets to move first. Finally, you could ask the groups to try to write down general decision-making rules for players, and note whether they need to modify those rules when the game is played repeatedly. If you try this, you may find that students are quite good at identifying strategic decision-making rules once they understand the games. By asking students to play the games first, you can then go back through the various outcomes and identify them as Cournot, cartel, and so on when you reach Chapter 14. One of the great advantages of game theory is that by simply changing the rules, the same payoff matrix can be evaluated more than once, with different outcomes. You can return to this discussion when covering the final section of the chapter on the comparison of output, price, and welfare effects for the various models.

In the section that covers dynamic entry games, three points are worth emphasizing. The first is to be able to evaluate if the incumbent firm needs to deter entry, if they would want to deter entry, or if they should not act to do so. The second is to determine if the firm would be able to deter entry should they choose to. Finally, you may want to engage the class in a discussion of the normative aspects of this strategy. The VAA/British Airways and Coke/Pepsi incidents discussed in the chapter are two examples of a firm actively attempting to damage another firm's reputation or raise their cost in order to gain advantage. A similar but more widely accepted strategy is an advertising campaign where competing products are "shown" to be inferior in consumer tests. Without discussing how the relevant law reads in this area, you can ask students how they believe the law should read. What rules should firms have to abide by in a competition over a local market? Which behaviors should be allowed here and which should be illegal is likely to be a matter of debate within the class.

■ Additional Applications

Electric Utilities[1]

In 1996, the Federal Energy Regulatory Commission ordered electric utilities to open up their transmission systems to outside energy producers. The ruling allows for competition in the sale of wholesale power to firms.

[1] Agis Salpukas, "Electric Utilities Ordered to Open Distribution Systems to Rivals," *San Fransisco Chronicle,* April 25, 1996:A3; and "Utilities Unbound: Get Your Kilowatts Here!" *New York Times,* April 28, 1996:S3, 2.

Of the 166 large utilities under the commission's jurisdiction, 106 had already taken steps to open their transmission at the time of the order. The ruling, however, requires utilities to set the same conditions in transmitting power for others as they do for themselves, to reserve transmission capacity for outside power producers, to disclose prices for moving power, and to post information on the Internet about access to distribution systems.

This order should permit low-cost energy producers, like utilities in the Midwest, to use the national network to deliver electricity to high-cost areas, such as the Northeast. In 1995, the average price per kilowatt-hour of electricity in the Midwest was 6 cents compared to 10.4 cents in New England. According to some estimates, this ruling may lower electric bills by between $3.8 and $5.4 billion dollars per year.

1. Assuming the New England and Midwest power companies both produce using technologies that are characterized by extensive scale economies, what can you predict about the future of these firms?
2. What other strategies might New England power use to raise the costs of Midwest power companies in order to remain competitive?

Horizontal Mergers Create Entry Deterrence in the Airline Industry[2]

The following excerpts from *U.S. v. Northwest Airlines and Continental Airlines* provides a glimpse into the U.S. Department of Justice's (DOJ) view on mergers as an entry deterrent. The ability of new and existing carriers to enter new markets (routes) was a fundamental assumption of the initial deregulation of the airline industry in 1978. The DOJ now appears quite concerned about strategic mergers that might deter entry and thus limit competition.

"Under the "hub-and-spoke" system, an airline concentrates passengers from many points at the "hub" location, then provides nonstop service from the hub airport to a large number of destinations (the "spokes"). The hub-and-spoke system allows a carrier to serve more city pairs with more frequencies than would be profitable on a stand-alone basis.

In seven hub-to-hub city pair markets, Northwest and Continental together dominate the market for nonstop service and for all scheduled airline passenger service. These markets are Detroit-Cleveland, Detroit-New York City, Detroit-Houston, Cleveland-Minneapolis, Minneapolis-New York City, Houston-Minneapolis, and Houston-Memphis. Northwest and Continental's market shares for nonstop flights in each of the seven hub-to-hub city pairs are:

Northwest/Continental Hub-to-Hub Nonstop Shares

Route	NW Share of Nonstop Fights	CO Share of Nonstop Flights	Combined NW & CO Share of Nonstop Flights
Detroit-Cleveland	54%	40%	94%
Detroit-New York	70%	17%	87%
Detroit-Houston	36%	64%	100%
Cleveland-Minneapolis	53%	47%	100%
Minneapolis-New York	80%	20%	100%
Houston-Minneapolis	42%	58%	100%
Houston-Memphis	39%	61%	100%

[2] http://www usdoj.gov/atr/cases/f2000/2023.htm.

In two other hub-to-hub markets, Memphis-Newark and Cleveland-Memphis, Northwest currently has a nonstop monopoly. As the only airline with a hub at the other endpoint, Continental is the most likely potential entrant to challenge Northwest's nonstop monopoly.

In total, nearly four million passengers travel in the nine hub-to-hub city pairs annually, generating revenues of nearly $400 million per year.

Effective new entry for the provision of nonstop service in the hub-to-hub markets is unlikely by any carrier without a hub at one of the endpoints of the city pair. A hub carrier, such as Northwest or Continental, has significant cost advantages over a nonhub carrier attempting to offer service originating at the hub airport. Building a competing hub in the same city would require considerable time and investment, and is not likely to occur in response to fare increases in the hub-to-hub markets at issue here.

New entry also is impeded by other factors, including difficulty in obtaining access to gate facilities, the effects of travel agent incentive programs offered by dominant incumbents, frequent flyer programs, and the risk of aggressive responses to new entry by the dominant incumbent carrier serving a particular market.

In addition to the hub-to-hub routes where Northwest and Continental share a virtual duopoly, Northwest and Continental have a large share of the passengers traveling on connecting flights in numerous city pair markets. Because of the light traffic on these routes and the short flights to the Northwest or Continental hubs, carriers with more distant hubs are unlikely to initiate or expand competitive service to these destinations from their hubs in response to significant fare increases.

1. Why do you suppose that having a hub and gate facilities in one of the city pairs is so important?
2. Log on to the DOJ website (http://www.usdoj.gov/atr), locate this case under "Antitrust Filings," and determine how it was finally resolved.

■ Discussion Questions

1. Give as many examples as you can of situations where an incumbent firm may have a strategic advantage over a potential entrant.
2. Give as many examples as you can of situations where an entrant may have a strategic advantage over an incumbent firm. (Is it ever an advantage to be small, or to not have sunk fixed costs already?)
3. Can we distinguish between cooperative and noncooperative strategic behavior?
4. How can governments behave strategically toward other governments or firms? Explain.
5. What can firms do to make credible commitments?
6. List as many ways as you can in which a firm can raise its rivals' costs.
7. Can it ever pay for a firm to take an action that raises its own costs more than those of its rivals?
8. If a prisoner's dilemma game is played repeatedly with almost perfect information (see section on Repeated Games) by several players, would a cooperative or noncooperative strategy be a winning strategy? (*Note:* This is a reference to the Axelrod Tournament.)
9. Does advertising help or hurt consumers? Does it depend on the nature of the ads?
10. Does an advertisement that contains information about product characteristics and prices help consumers or harm them? Why?

11. Are there any limitations to where auctions can be used as a price-discovery mechanism?

12. Can an auction be designed poorly? What is a "poorly designed auction," in your opinion?

■ Additional Questions and Problems

1. In the following payoff matrix, each firm has two possible strategies, and must move simultaneously. Assuming that each knows only its own payoff structure, what decision would each firm make? Is this a Nash equilibrium? Suppose each player can see the entire payoff structure, instead of only its own. How would this affect Firm 2? Payoffs shown are (Firm 1, Firm 2):

		Firm 2 *A*	Firm 2 *B*
Firm 1	*A*	3, 1	2, 0
	B	2, 4	1, 5

2. In the following game, players must move simultaneously. How many Nash equilibria are there? Which will occur without collusion? Which will occur if collusion is allowed?

		Firm 2 *A*	Firm 2 *B*
Firm 1	*A*	3, 1	7, 0
	B	2, 4	5, 3

3. In the following game, assume that you are an interested bystander (such as the local government in the town where Firm 1 is located). Can either firm make a credible commitment to enter? How could you alter the incentives (payoffs) with a non firm-specific prize to ensure that Firm 1 enters the market? Under what circumstances would it be worth it to do so?

		Firm 2 Enter	Firm 2 Don't enter
Firm 1	Enter	−1, −2	4, 0
	Don't enter	0, 4	0, 0

4. Two firms are considering entering a new market. Entrance requires construction of a highly specialized plant. Demand is sufficient for either one to be profitable, but not both. A newspaper writer, observing the posturing of the two firms, each stating that they are planning to go ahead with plans for the new facility, noted, "sunk costs make for credible threats." What does she mean by this statement?

5. Suppose an industry has one incumbent and three potential new entrants. Any firm can produce as the incumbent does, with no fixed costs, and marginal cost $MC = bq$. Is entry blockaded? Can/should it be deterred? What type of equilibrium will result?

6. A monopolist faces demand $p = 20 - Q + 0.5A^{0.5}$. Cost is $C = 4Q + A$, where A is the quantity of advertisement, measured in \$1 units. What are the profit-maximizing output and advertising levels? What are the profits?

7. What are the merits and disadvantages to developing word processing software that has a very different command structure than others on the market?

8. Show that if learning by doing results in the incumbent firm having the cost function $C = 100 + 5Q^{.5}$, the firm does not need any further action to deter entry.

9. Suppose the french fry market is a duopoly. Tests show that 95% of consumers prefer Brand *X*. Could it ever be shown that Brand *Z* is preferred in taste tests? What does this imply about such tests?

10. Jerry has the only ice cream store in town. There are only two methods of ice cream advertising that work: billboards and radio ads. Billboards are cheaper but less effective than radio. Upon learning that Ben will soon open a rival store, Jerry immediately rents all available billboard space. Why might he choose this strategy? Under what circumstances will it be effective?

■ Answers to Additional Questions and Problems

1. Firm 1 has a dominant strategy of *A*. However, without knowledge of the other player's possible outcomes, Firm 2 must guess, as they do not have a dominant strategy. The solution (*A*, *A*) is a Nash equilibrium, but (*A*, *B*) is not, since Firm 2 would rather switch, given Firm 1's choice of strategy *A*. If they can each see the entire matrix before play, Firm 2 will select strategy *A* rather than having to guess Firm 1's choice.

2. Each player has the dominant solution of strategy *A*, which is the only Nash equilibrium, and will occur if collusion is not allowed. However, if collusion is allowed, (*B*, *B*) will be the outcome, even though it is not a Nash equilibrium.

3. As the matrix is written, neither player has a dominant strategy, nor can they make a credible threat of entry, since each loses if both enter. However, if the local government offers a $1 prize to any firm that enters the market, Firm 1 can make a credible threat of entry.

   Payoffs with a $1 Prize for All Entering Firms:

		Firm 2	
		Enter	Don't enter
Firm 1	Enter	0, −1	5, 0
	Don't enter	0, 5	0, 0

4. The fact that a firm must invest in fixed costs does not ensure a credible commitment because the cost may be partially or totally recoverable (as in the case of an airplane placed on a particular route). Sunk costs are nonrecoverable. Thus a firm willing to begin the building process by spending funds that cannot be recovered is making a very credible threat that it plans to proceed.

5. In this case, the lack of fixed costs means that entry is neither blockaded nor should it be deterred. If the products are identical, a Bertrand equilibrium will likely occur, with firms pricing at marginal cost. If products are differentiated, a Cournot equilibrium is more likely. If the firms form a cartel, output and price will be identical to the monopoly result. (See Chapter 14.)

6. In this case, there is more than one choice variable. To solve, set up the profit function, differentiate with respect to Q and A, and solve two first-order conditions simultaneously to obtain Q^* and A^*.

$$\pi = (20 - Q + 0.5A^{0.5})Q - 4Q - A$$
$$\partial\pi/\partial Q = 16 - 2Q + 0.5A^{0.5} = 0$$
$$\partial\pi/\partial A = 0.25A^{-0.5}Q - 1 = 0$$
$$A^* = 4.55$$
$$Q^* = 8.53$$
$$p^* = \$12.54$$
$$\pi^* = \$68.30$$

7. Difference in command structure creates switching costs in the software market. If the product enjoys or is able to capture a high market share by some other means, such as advertising or quality, switching costs can create market power. If, however, a new product attempts to take market share from an established market leader, but users must pay switching costs, success is much less likely. In addition, with products such as word processing and spreadsheet programs such as Lotus 123 and Excel, users are reluctant to switch if the new software is unique and thus they would not be able to interact with other users of popular programs. This reluctance creates additional switching costs.

8. In this case, learning by doing results in a natural monopoly. Marginal cost ($MC = 2.5Q^{-0.5}$) declines continuously as output expands. Thus no further action is necessary by the firm to deter entry.

9. The company need only run the tests enough times, and eventually the firm will get a sample that prefers Brand *Z.* Recall from your statistics class that sample means are just that—based on samples. The implications for information conveyed by these tests is that they show only that preferences are not unanimous, and say very little about what most people prefer.

10. Jerry may choose to rent all available billboard space to raise Ben's cost of entry. In order to gain market share, Ben will have to advertise. By purchasing all available inexpensive ad space, Ben must use the more expensive radio. The strategy will be effective if the difference in ad effectiveness is not enough to justify the additional cost, given that Ben is a new firm new unknown to consumers.

■ Answers to Questions and Problems in the Text

1. In each panel, the dominant strategy for each firm is to advertise. This implies that pairs advertise-advertise are Nash equilibria in both panels of Table 14.4.

2. The payoff matrix in this prisoners' dilemma game is

		Duncan	
		Squeal	Stay Silent
Larry	Squeal	–2 –2	–5 0
	Stay Silent	0 –5	–1 –1

If Duncan stays silent, Larry gets 0 if he squeals and −1 (a year in jail) if he stays silent. If Duncan confesses, Larry gets −2 if he squeals and −5 if he does not. Thus Larry is better off squealing in either case, so squealing is his dominant strategy. By the same reasoning, squealing is also Duncan's dominant strategy. As a result, the Nash equilibrium is for both to confess.

3. A Nash equilibrium is a set of strategies such that, holding the strategies of all other firms constant, no firm can obtain a higher profit by choosing a different strategy. The Nash equilibria are for Firm 1 to pick low and Firm 2 to pick medium, and for Firm 1 to pick medium and Firm 2 to pick low. At those outcomes, neither firm can increase profit by changing their behavior given what the other firm selects.

   Conversely, it is not a Nash equilibrium for Firm 1 to pick low and Firm 2 to pick low. Firm 1 could increase profit from $1 to $21 by instead picking medium. Firm 2 could increase profit from $11 to $16 by instead picking medium.

   It is not a Nash equilibrium for Firm 1 to pick low and Firm 2 to pick high. Firm 1 could increase profit from $24 to $26 by instead picking medium. Firm 2 could increase profit from $5 to $16 by instead picking medium.

   It is not a Nash equilibrium for Firm 1 to pick medium and Firm 2 to pick medium. Firm 1 could increase profit from $2 to $25 by instead picking low. Firm 2 could increase profit from $3 to $19 by instead picking low.

   It is not a Nash equilibrium for Firm 1 to pick medium and Firm 2 to pick high. Firm 2 could increase profit from $14 to $19 by instead picking low.

   It is not a Nash equilibrium for Firm 1 to pick high and Firm 2 to pick low. Firm 1 could increase profit from $12 to $21 by instead picking medium. Firm 2 could increase profit from $6 to $23 by instead picking medium.

   It is not a Nash equilibrium for Firm 1 to pick high and Firm 2 to pick medium. Firm 1 could increase profit from $4 to $25 by instead picking low.

   It is not a Nash equilibrium for Firm 1 to pick high and Firm 2 to pick high. Firm 1 could increase profit from $11 to $26 by instead picking medium. Firm 2 could increase profit from $17 to $23 by instead picking medium.

4. Assume you're Lori. If Max works, your best strategy is to give no bonus (your payoff is 3), rather than give a bonus (your payoff is 1). If Max loafs, again, your best strategy is to give no bonus (your payoff is 0), rather than give a bonus (your payoff is −1). Hence "No Bonus" is Lori's dominant strategy. Now assume you're Max. If Lori offers you a bonus, your best strategy is to loaf because the payoff is 3, rather than 2 when you work. If Lori gives no bonus, again, your best strategy is to loaf (payoff of 0 vs. payoff of −1 if you work). This means that "Loaf" is Max's dominant strategy. Combining two dominant strategies together suggests that the pair No Bonus – Loaf is the Nash equilibrium outcome of this static game.

5. a. There are two Nash equilibria (the off diagonals). If either firm produces 20 while the other produces 10, neither player has an incentive to change strategies given the strategy of the other player.

   b. If Firm 1 can choose first, it will commit to selling 20 units, and Firm 2 sells 10 units. If Firm 1 were to choose 10 units, Firm 2 would choose to produce 20 units, reducing Firm 1's payoff by $10.

   c. If Firm 2 can choose first, it will sell 10 units, and Firm 1 will sell 20. If Firm 2 were to produce 20 units, Firm 1 would produce only 10, reducing Firm 2's payoff by $5.

6. a. If both must move simultaneously, neither has a dominant strategy because neither can credibly commit to producing the television.
   b. The two Nash equilibria are on the off diagonals where one firm enters and the other does not.
   c. With the subsidy, Zenith can credibly commit to entering the market, because the worst it can do is to gain 10 if Panasonic also enters. In this case, Zenith will enter and Panasonic will not.
   d. The equilibrium with the head start is the same as that with the subsidy. Once Zenith commits to producing the new product, there is no benefit to Panasonic if Panasonic produces it also.

7. There are no pure-strategy Nash equilibria in this game. In each cell, one of the players always would prefer to switch, given the move of the other.

8. The incumbent must compare the two-period profits under two scenarios. In the first scenario, the incumbent overproduces in the first period in order to reduce marginal cost in the second period, knowing that it will be a monopolist in the second period. In the second scenario, the incumbent produces the profit-maximizing output level in the first period, resulting in duopoly profits (with higher marginal cost) in the second period. In the first game tree below, the monopolist overproduces in the first period, resulting in total profits of $1000, which exceeds the profits with profit-maximizing production in the first period. In the second game tree, profits are greater if the incumbent does not overproduce in the first period.

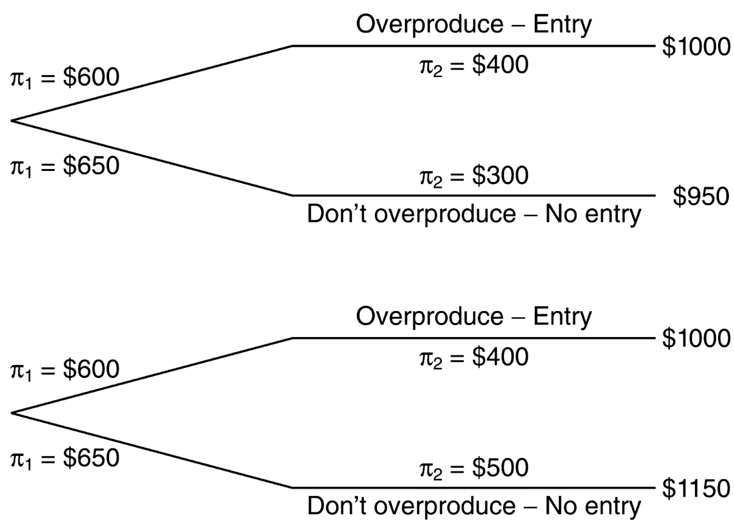


Figure 14.1

9. Given the information, an optimal strategy for the rulers may be: (i) spend little resource in catapult research and development; (ii) buy latest technology from other countries; (iii) publicly announce the deployment of catapult. Suppose catapult research and development required a substantial fixed cost, then this strategy is particularly suitable for small countries. Since new technology was not well protected, public announcement of research and development would encourage free riding of other countries. On the other hand, credible announcement of catapult deployment would help to discourage possible attacks.

10. Audio-PowerPoint answer by James Dearden is also available (14A Thugs).
    a. The safe owner (*S*) moves first and decides whether to "open" the safe or "don't open" the safe. Then the thug decides to "kill" or "don't kill."

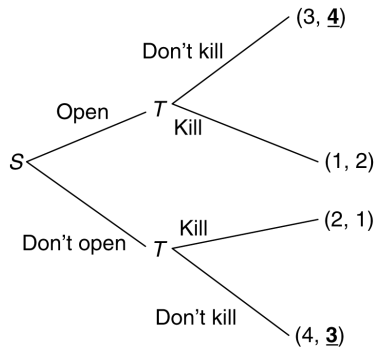


Figure 14.2

    b. The subgame perfect Nash equilibrium is "don't open" for the safe owner and "don't kill" for the thug irrespective of whether the owner opens the safe or not. So the safer owner will not open the safe and the thug will not kill.
    c. The thug's threat is not credible. Therefore the safe owner should not believe it.
    d. He will not open the safe.

11. Audio-PowerPoint answer by James Dearden is also available (14B Film Release).
    a. The pure strategy Nash equilibrium in this game is for both Warner Bros. and the T-3 producer to release their movies on July 4. Given that the T-3 producer releases its movie on July 4, the best choice for Warner Bros. is to release its movie on July 4. On the other hand, given that Warner Bros. releases its movie on July 4, the best choice for the T-3 producer is to release its movie on July 4.
    b. The release on July 18 by the T-3 producer and on July 4 by Warner Bros. maximizes joint profit. Note that $90 + 30 = 120$ is the greatest sum of profits.
    c. The maximum Warner Bros. is willing to pay is the difference between its profit when both movies are released on July 4th, and its payoff when it has bought the release of T-3 (released on July 18) and Matrix (released on July 4), i.e., $90 - 50 = 40$. The profit the T-3 producer earns if it does not sell its right of release is 50. Therefore the minimum price the T-3 producer accepts for the sale of its right of release is 50. Since the maximum that Warner Bros. is willing to pay (40) is less than the minimum that the T-3 producer is willing to accept (50), there is no mutually beneficial price at which trade can take place.
    d. Warner Bros. will release its movie on July 4th and T-3 on July 18.

12. The advertising might be strategic to increase Microsoft's own market share by attracting customers from competitors. On the other hand, it does not necessarily reflect its "fear" of competitors. The advertising might target new customers. Overall, the optimal advertising is determined jointly by the marginal cost and marginal benefit.

13. Figure 14.3(a) offers an extensive form representation of the sequential game when Mimi moves first. Using backward induction we conclude that Jeff will not choose actions that are double-crossed because alternatives offer Jeff better payoffs (4 vs. 2 and 1 vs. 0). Given Jeff's preferred actions, Mimi receives the same payoff of -1 regardless of what she does. Since supporting Jeff will not improve Mimi's payoff, she may choose not to support him.

If Jeff moves first, the game looks as in Figure 14.3(b). When Mimi makes a decision, she chooses actions that give her the highest payoffs; we crossed actions that Mimi will not choose. Then Jeff realizes that if he looks for a job, his payoff will be 2, and if he does not, his payoff will be 0. Maximizing his payoff, Jeff chooses to look for a job. This means that the subgame perfect Nash equilibrium is the strategy look-support with the outcome of (4, 2).

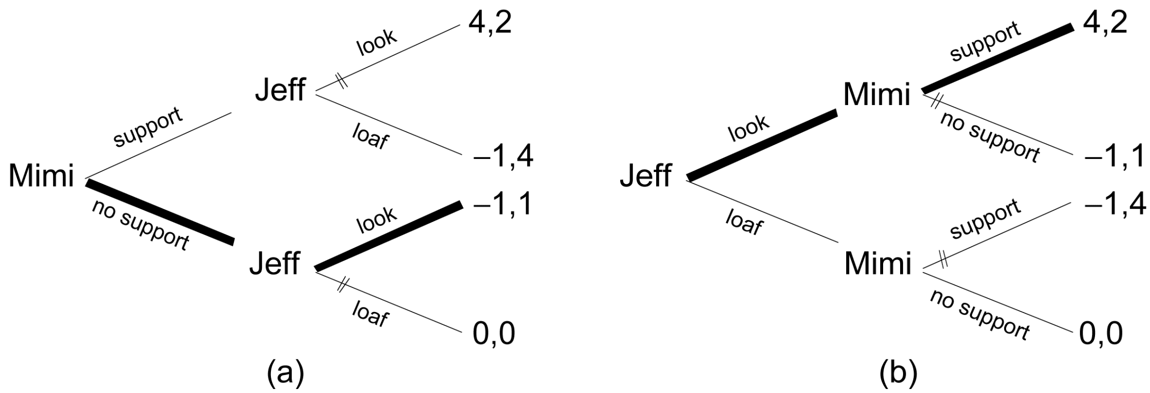


Figure 14.3

14. Suppose the payoff is 1 if you win, 0 if you tie, and −1 if you lose. The payoff matrix will be the following:

		Rock	Sotheby's Paper	Scissors
Christie's	Rock	0, 0	−1, 1	1, −1
	Paper	1, −1	0, 0	−1, 1
	Scissors	−1, 1	1, −1	0, 0

Assuming the 11-year-old girls provide the correct insight and the rival would be very likely to take their advice (of selecting scissors), then a pure strategy of rock should be recommended (assuming the rival didn't know that we knew their consultation with the girls).

15. It is not clear what the answer to the second part of this question is because we don't know the payoffs.

16. The payoff matrix is:

		B: Swerve	B: Don't Swerve
A	Swerve	1, 1	0, 2
	Don't swerve	2, 0	−10, −10

The two Nash equilibria are the cells with outcomes (2, 0) and (0, 2).

17. A Nash equilibrium is a set of strategies such that, holding the strategies of all other firms constant, no firm can obtain a higher profit by choosing a different strategy. If driver 1 swerves and driver 2 does not swerve, then neither driver can increase his payoff. For example, if driver 1 instead does not swerve, then his payoff decreases from 0 to −2. If driver 2 instead swerves, then his payoff decreases from 2 to 1. Therefore, neither driver has an incentive to change his behavior. The game is symmetric, so if it is a Nash equilibrium for driver 1 to swerve and driver 2 to not swerve, then it is also a Nash equilibrium for driver 1 to not swerve and driver 2 to swerve.

18. See Figure 14.4. Taliban decides if kidnapping an Italian journalist will pay off. If Taliban kidnaps the journalist, Italy should pay because if it pays, Italy gets the journalist back (payoff of 1), but if Italy does not pay, the journalist is not going to be freed (payoff of −1 for Italy). If Italy pays, Taliban's payoff is 5 (5 Taliban prisoners are released) and if Italy does not pay, the payoff for Taliban is −5 (5 Taliban prisoners are not released). If Taliban does not kidnap the journalist, the payoff to Italy is always 0, whether or not Italy chooses to release Taliban prisoners; hence Italy is not motivated to pay. For Taliban, the payoffs are 5 if Italy pays (i.e., releases Taliban prisoners) and −5 if Italy does not pay (i.e., does not release prisoners). Backward induction suggests that the subgame perfect Nash equilibrium is the strategy kidnap-pay with outcome (5, 1). This is exactly what happened. The other governments were upset because the situation demonstrated to Taliban that a government may act as a rational agent. Hence Taliban may decide that it is beneficial to "play" this "game" in the future again.

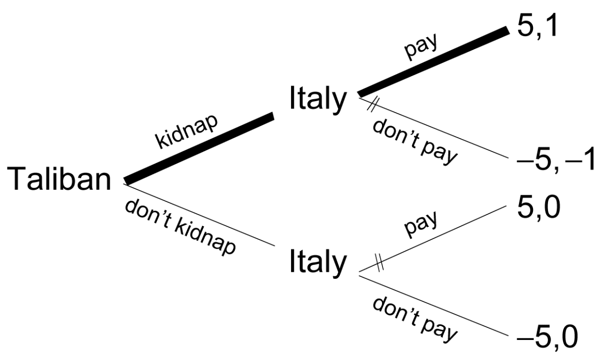


Figure 14.4

19. We start by checking for dominant strategies. (See Figure 14.5) Given the payoff matrix, Toyota always does at least as well by entering the market. If GM enters, Toyota earns 10 by entering and 0 by staying out of the market. If GM does not enter, Toyota earns 250 if it enters and 0 otherwise. Thus entering is Toyota's dominant strategy. GM does not have a dominant strategy. It wants to enter if Toyota does not enter (earning 200 rather than 0), and it wants to stay out if Toyota enters (earning 0 rather than −40). Because GM knows that Toyota will enter (entering is Toyota's dominant strategy), GM stays out of the market. Toyota's entering and GM's not entering is a Nash equilibrium. Given the other firm's strategy, neither firm wants to change its strategy. Next we examine how the subsidy affects the payoff matrix and dominant strategies. The subsidy does not affect Toyota's payoff, so Toyota still has a dominant strategy: It enters the market. With the subsidy, GM's payoff if it enters increases by 50: GM earns 10 if both enter and 250 if it enters and Toyota does not. With the subsidy, entering is a dominant strategy for GM. Thus both firms' entering is a Nash equilibrium.

20. If GM gets no subsidy but can move first, it faces a situation as described in Figure 14.5. Backward induction suggests that the Nash equilibrium is a pair of strategies "don't enter" for GM and "enter" for Toyota, with the outcome of 250 for Toyota and 0 for GM.

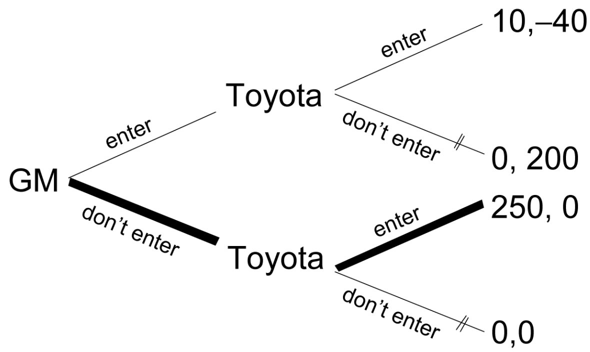


Figure 14.5

21. If firms only care about current profits, then the firms will not use any intertemporal strategies. They will solve a simple single-period prisoners' dilemma game in each period.

22. The game tree in Figure 14.6 illustrates why the incumbent may install the robotic arms to discourage entry even though its total cost rises. If the incumbent fears that a rival is poised to enter, it invests to discourage entry. The incumbent can invest in equipment that lowers its marginal cost. With the lowered marginal cost, it is credible that the incumbent will produce larger quantities of output, which discourages entry. The incumbent's monopoly (no-entry) profit drops from $900 to $500 if it makes the investment because the investment raises its total cost. If the incumbent doesn't buy the robotic arms, the rival enters because it makes $300 by entering and nothing if it stays out of the market. With entry, the incumbent's profit is $400. With the investment, the rival loses $36 if it enters, so it stays out of the market, losing nothing. Because of the investment, the incumbent earns $500. Nonetheless, earning $500 is better than earning only $400, so the incumbent invests.

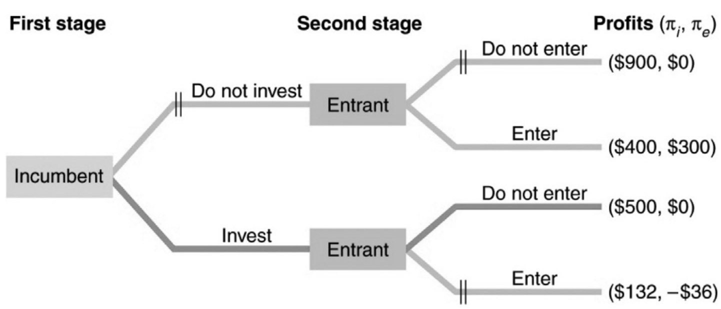


Figure 14.6

23. The incumbent firm has a *first-mover advantage,* as the game tree on the facing page illustrates. Moving first allows the incumbent or leader firm to *commit* to producing a relatively large quantity. If the incumbent does not make a commitment before its rival enters, entry occurs and the incumbent earns a relatively low profit. By committing to produce such a large output level that the potential entrant decides not to enter because it cannot make a positive profit, the incumbent's commitment discourages entry. Moving backward in time (moving to the left in the diagram), we examine the incumbent's choice. If the incumbent commits to the small quantity, its rival enters and the incumbent earns $450. If the incumbent commits to the larger quantity, its rival does not enter and the incumbent earns $800. Clearly, the incumbent should commit to the larger quantity because it earns a larger profit and the potential entrant chooses to stay out of the market. Their chosen paths are identified by the darker blue in the figure.

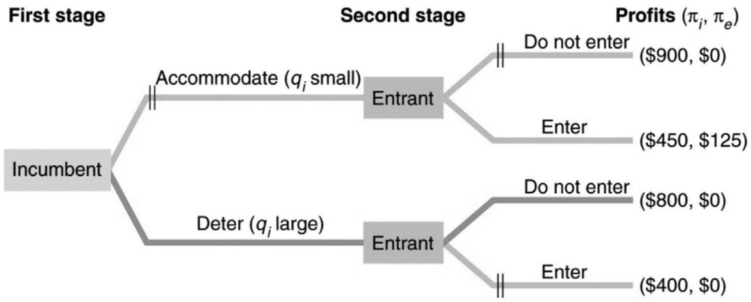


Figure 14.7

24. It is worth more to the monopoly to keep the potential entrant out than it is worth to the potential entrant to enter, as the figure shows. Before the pollution-control device requirement, the entrant would pay up to \$3 to enter, whereas the incumbent would pay up to $\pi_i - \pi_d = \$7$ to exclude the potential entrant. The incumbent's profit is \$6 if entry does not occur, and it loses \$1 if entry occurs. Because the new firm would lose \$1 if it enters, it does not enter. Thus the incumbent has an incentive to raise costs by \$4 to both firms. The incumbent's profit is \$6 if it raises costs rather than \$3 if it does not.

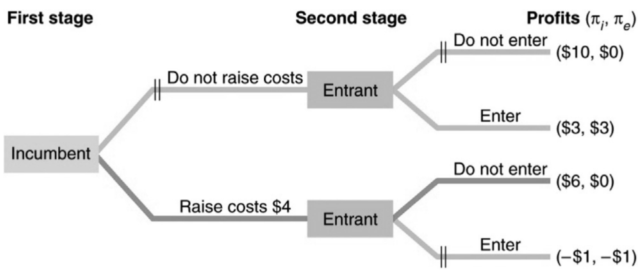
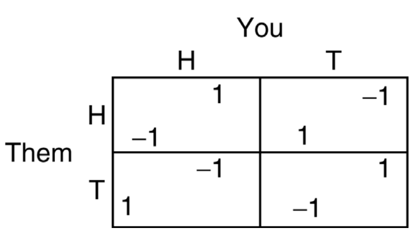


Figure 14.8

25. The payoff matrix is shown below. Given these payoffs, mixed strategies are required. There is no Nash equilibrium because in any given cell the outcome (winner) changes if one player changes strategies. Thus in any given cell the loser would always prefer to switch given the opponent's strategy.



26. Let the probability that a firm sets a low price be θ_1 for Firm 1 and θ_2 for Firm 2. If the firms choose their prices independently, then $\theta_1\theta_2$ is the probability that both set a low price, $(1 - \theta_1)(1 - \theta_2)$ is the probability that both set a high price, $\theta_1(1 - \theta_2)$ is the probability that Firm 1 prices low and Firm 2 prices high, and $(1 - \theta_1)\theta_2$ is the probability that Firm 1 prices high and Firm 2 prices low. Firm 2's expected payoff is $E(\pi_2) = 2\theta_1\theta_2 + (0)\theta_1(1 - \theta_2) + (1 - \theta_1)\theta_2 + 6(1 - \theta_1)(1 - \theta_2) = (6 - 6\theta_1) - (5 - 7\theta_1)\theta_2$. Similarly, Firm 1's expected payoff is $E(\pi_1) = (0)\theta_1\theta_2 + 7\theta_1(1 - \theta_2) + 2(1 - \theta_1)\theta_2 + 6(1 - \theta_1)(1 - \theta_2) = (6 - 4\theta_2) - (1 - 3\theta_2)\theta_1$. Each firm forms a belief about its rival's behavior. For example, suppose that Firm 1 believes that Firm 2 will choose a low price with a probability $\hat{\theta}_2$. If $\hat{\theta}_2$ is less than $\frac{1}{3}$ (Firm 2 is relatively unlikely to choose a low price), it pays for Firm 1 to choose the low price because the second term in $E(\pi_1)$, $(1-\hat{\theta}_2)\theta_1$, is positive, so as θ_1 increases, $E(\pi_1)$ increases. Because the highest possible θ_1 is 1, Firm 1 chooses the low price with certainty. Similarly, if Firm 1 believes $\hat{\theta}_2$ is greater than $\frac{1}{3}$, it sets a high price with certainty ($\theta_1 = 0$).

If Firm 2 believes that Firm 1 thinks $\hat{\theta}_2$ is slightly below $\frac{1}{3}$, Firm 2 believes that Firm 1 will choose a low price with certainty, and hence Firm 2 will also choose a low price. That outcome, $\theta_2 = 1$, however, is not consistent with Firm 1's expectation that $\hat{\theta}_2$ is a fraction. Indeed, it is only rational for Firm 2 to believe that Firm 1 believes Firm 2 will use a mixed strategy if Firm 1's belief about Firm 2 makes Firm 1 unpredictable. That is, Firm 1 uses a mixed strategy only if it is *indifferent* between setting a high or a low price. It is indifferent only if it believes $\hat{\theta}_2$ is exactly $\frac{1}{3}$. By similar reasoning, Firm 2 will use a mixed strategy only if its belief is that Firm 1 chooses a low price with probability $\hat{\theta}_1 = \frac{5}{7}$. Thus the only possible Nash equilibrium is $\theta_1 = \frac{5}{7}$ and $\theta_2 = \frac{1}{3}$.

27. In the battle of sexes game the payoff matrix is:

		Wife	
		Mountain	Ocean
Husband	Mountain	2, 1	0, 0
	Ocean	0, 0	1, 2

This game has two Nash equilibria (mountain, mountain) and (ocean, ocean), which means that the game theory cannot offer a unique solution and cannot predict what the couple will end up doing. If, however, this is a repeated game (the couple makes the same decision before every vacation), the husband and wife may agree that they alternate vacation spots. Then once the initial vacation is chosen, all the following vacation locations can be predicted.

28. Audio-PowerPoint answer by James Dearden is also available (13B Chicken Pie).

a.

		Emil's Diner						
		0	1	2	3	4	5	6
Bobby's Diner	**0**	(–120, –120)	(–240, 0)	(–240, 0)	(–240, 0)	(–240, 0)	(–240, 0)	(–240, 0)
	1	(0, –240)	(–50, –50)	(–100, 0)	(–100, 0)	(–100, 0)	(–100, 0)	(–100, 0)
	2	(0, –240)	(0, –100)	(0, 0)	(0, 0)	(0, 0)	(0, 0)	(0, 0)
	3	(0, –240)	(0, –100)	(0, 0)	(30, 30)	(60, 0)	(60, 0)	(60, 0)
	4	(0, –240)	(0, –100)	(0, 0)	(0, 60)	(40, 40)	(80, 0)	(80, 0)
	5	(0, –240)	(0, –100)	(0, 0)	(0, 60)	(0, 80)	(30, 30)	(60, 0)
	6	(0, –240)	(0, –100)	(0, 0)	(0, 60)	(0, 80)	(0, 60)	(0, 0)

b. The Nash equilibriua are as follows:

Bobby	**Emil**
2	2
3	3

c. The inverse demand curve is

$$P = 6 - 0.05Q,$$

thus

$$MR = 6 - 0.1Q.$$

The profit is maximized where

$$MR = MC \rightarrow 6 - 0.1Q = 2 \rightarrow Q = 40, \quad P = 4, \quad \text{profit} = \pi = 160 - 80 = 80.$$

d. At the (3, 3) Nash equilibrium, each diner's profit is 30. However, if they collude, they can set the price at 4, the monopoly price, get the monopoly profit and then divide it between themselves and in this way increase their profit to 40 for each.

29. Audio-PowerPoint answer by James Dearden is also available (13C Warranties).

a. $\pi_i = R_i - C_i = 27000w_i/(w_A + w_B) - 2000w_i$.

b. (numbers in $ thousands).

		Acura				
		1	**2**	**3**	**4**	**5**
	1	(11.5, 11.5)	(7, 14)	(4.75, 14.25)	(3.4, 13.6)	(2.5, 12.2)
	2	(14, 7)	(9.5, 9.5)	(6.8, 10.2)	(5, 10)	(3.7, 9.3)
Volvo	**3**	(14.25, 4.75)	(10.2, 6.8)	(7.5, 7.5)	(5.6, 7.4)	(4.2, 6.9)
	4	(13.6, 3.4)	(10, 5)	(7.4, 5.6)	(5.5, 5.5)	(4, 5)
	5	(12.5, 2.5)	(9.3, 3.7)	(6.9, 4.2)	(5, 4)	(3.5, 3.5)

c. The Nash equilibrium is for both to offer three years warranty. Offering three years warranty is the dominant strategy for each company.

d. They both offer the same warranty, because this is best response to each other's strategy.

e. If they collude, they will still provide three-year warranties. Following other strategies makes one better off and the other worse off.

f.

		Acura				
		1	**2**	**3**	**4**	**5**
	1	(12.5, 11.5)	(8, 14)	(5.75, 14.25)	(4.4, 13.6)	(3.5, 12.5)
	2	(16, 7)	(11.5, 9.5)	(8.8, 10.2)	(7, 10)	(5.7, 9.3)
Volvo	**3**	(17.25, 4.75)	(13.2, 6.8)	(10.5, 7.5)	(8.6, 7.4)	(7.2, 6.9)
	4	(17.6, 3.4)	(14, 5)	(11.4, 5.6)	(9.5, 5.5)	(8, 5)
	5	(17.5, 2.5)	(14.3, 3.7)	(11.9, 4.2)	(10, 4)	(8.5, 3.5)

The Nash equilibrium is now for Volvo to offer five years warranty and for Acura to offer three years. Now the Volvo profit is larger compared with when its costs were higher.

30. Solution is also provided in Jim Dearden's audio presentation.

a. The expected profit function for university i is:

$$\pi_i = v_i \frac{m_i}{m_i + m_j} - m_i \qquad (i = A, j = B, \text{ or } i = B, j = A).$$

The F.O.C. for university A is:

$$\frac{\partial \pi_A}{\partial m_A} = \frac{v_A m_A}{(m_A + m_B)^2} - 1 = 0. \tag{1}$$

The F.O.C. for university B is:

$$\frac{\partial \pi_B}{\partial m_B} = \frac{v_B m_B}{(m_A + m_B)^2} - 1 = 0. \tag{2}$$

The solution to equations (1) and (2) is:

$$\begin{cases} m_A^* = \dfrac{v_A^2 v_B}{(v_A + v_B)^2} \\ m_B^* = \dfrac{v_B^2 v_A}{(v_A + v_B)^2}, \end{cases}$$

i.e., in the Nash equilibrium, university A wins $m_A^* = \frac{v_A^2 v_B}{(v_A + v_B)^2}$, and university B wins

$$m_B^* = \frac{v_B^2 v_A}{(v_A + v_B)^2}.$$

If $v_A = v_B = v$ in the Nash equilibrium, each school wins $m_A^* = m_B^* = \frac{v}{4}$. Since $\dfrac{\frac{v}{4}}{\frac{v}{4} + \frac{v}{4}} = \dfrac{1}{2}$, so each school wins one-half of its game.

b. Since $m_A^* = m_B^* = \dfrac{v}{4}$, then $\dfrac{\partial m_A^*}{\partial v} = \dfrac{\partial m_B^*}{\partial v} = \dfrac{1}{4} > 0$, therefore if $v_A = v_B$ increases, each school spends more on its teams. Moreover, since $\dfrac{\frac{v}{4}}{\frac{v}{4} + \frac{v}{4}} = \dfrac{1}{2}$ still holds after the increase, each school continues to win one-half of its games.

c. This is not a zero-sum game.

31. a. It seems in this model the demand does not depend on the price that companies charge but on how much they spend on advertisement.

The market share of Firm A is

$$q_A/(q_A + q_B) = [a + b(A_A + A_B)^{0.5}]/\{2[a + b(A_A + A_B)^{0.5}]\} = 0.5.$$

Therefore, irrespective of how much each firm spends on advertisement, each firm's market share will not change.

b. Each firm maximizes its profit, given the other firm's level of expenditures on advertisement and the price it charges:

$$d\pi_i/dA_i = 0.5bp_i(A_i + A_j)^{-0.5} - 1 = 0,$$
$$i = A, \quad B.$$

Solving the two equations we get that

$$P_A = P_B = P$$

and

$$A_A = A_B = 0.125b^2P^2.$$

As b increases, the expenditure on advertisement will also increase.

32. The table below illustrates this game. "*S*" denotes taking steroids and "*NS*" denotes not taking steroids. If they both take steroids they tie and the payoff for each is 10 − 6 = 4. If one takes steroids and one does not, the person who has taken steroids wins and gets 20 − 6 = 14, and the other person loses and gets 0. If they both don't take steroids, they tie again and each gets 10.

		200-Meter Star	
		S	***NS***
100-Meter Star	***S***	(4, 4)	(14, 0)
	NS	(0, 14)	(10, 10)

a. The Nash equilibrium is for both to take steroids, even though they will both be better off if they don't take steroids. This is a prisoner's dilemma.
b. Suppose the utility of the 100-meter star receives from taking the steroids is −12, while it is −6 for the 200 meter star. The payoff table for this game is illustrated below.

		200-Meter Star	
		S	***NS***
100-Meter Star	***S***	(−2, 4)	(8, 0)
	NS	(0, 14)	(10, 10)

Now the Nash equilibrium is "don't take steroids" for the 100-meter star and "take steroids" for the 200-meter star. Both players have dominant strategies, but cooperation by not taking steroids does not improve both. This is not a prisoner's dilemma game.

33. This game has two possible outcomes, one in which Westley drinks from the poisoned glass and Vizzini drinks from the glass that is not poisoned, and in the other outcome the opposite happens. Each of the outcomes can happen with the probability of 50%.

34. Xavier makes the first move and Ying, observing Xavier's decision, makes his choice. The setup is like the Stackelberg model of noncooperative behavior. Xavier knows that Ying maximizes his utility and considers his optimal choice when making his own decision. Ying chooses how much he needs to work to maximize his utility:

$$dU_Y/dh_Y = 0 \rightarrow 9(h_X + h_Y)^{0.5} - 1 = 0$$
$$\rightarrow h_X + h_Y = 81$$
$$\rightarrow h_Y = 81 - h_X.$$

Knowing Ying's choice, Xavier maximizes his utility:

$$U_X = 18(h_X + h_Y)^{0.5} - h_X \rightarrow U_X = 18(h_X + 81 - h_X)^{0.5} - h_X$$
$$\rightarrow U_X = 162 - h_X.$$

The optimum for Xavier is to not work at all; i.e., $h_X^* = 0$, and thus the optimum for Ying will be to work $h_Y^* = 81$. Therefore Ying's threat that he will not work is not credible.

35. In the auction, Anna's, Bill's, and Cameron's optimal bids will be \$20,000, \$18,500, and \$16,800, respectively. Anna will win the auction and the price she pays will be \$18,500, and her surplus will be \$20,000 − \$18,500 = \$1500.

36. a.

		Colts	
		R	***P***
Patriots	***DR***	(+1, −1)	(−10, +10)
	DP	(−6, +6)	(+2, −2)

Where "*R*" denotes run, "*DR*" denotes defense against run, "*P*" denotes pass, and "*DP*" denotes defense against pass. There is no pure Nash equilibrium. We can find the mixed-strategy Nash equilibrium. Suppose Colts follow "*R*" by probability α and "*P*" by probability $1 - \alpha$. Then the expected payoff of following "*DR*" and "*DP*" by Patriots is:

$$E(DR) = \alpha - 10(1 - \alpha) = 11\alpha - 10$$

and

$$E(DP) = -6\alpha + 2(1 - \alpha) = -8\alpha + 2.$$

At the equilibrium we have:

$$E(DR) = E(DP) \rightarrow 11\alpha - 10 = -8\alpha + 2$$
$$\rightarrow \alpha = 12/19.$$

Suppose Patriots follow "*DR*" by probability β and "*DP*" by probability $1 - \beta$, then the expected pay off of following "*R*" and "*P*" by Colts is:

$$E(R) = -\beta + 6\,(1 - \beta)$$
$$= -7\beta + 6$$

and

$$E(P) = 10\beta - 2(1 - \beta)$$
$$= 12\beta - 2.$$

At the equilibrium we have:

$$E(R) = E(P) \rightarrow -7\beta + 6 = 12\beta - 2$$
$$\rightarrow \beta = 8/19.$$

Therefore the mixed strategy Nash equilibrium is {[12/19, 7/19], [8/19, 11/19]}.

b. The payoff matrix is now:

		Colts	
		R	***P***
Patriots	***DR***	(+1, –1)	(–10, +10)
	DP	(–6, +6)	(+2, –2)

There are no pure Nash equilibria. The maixed-strategy Nash equilibrium is as follows:

Suppose Colts follow "*R*" by probability α and "*P*" by probability $1 - \alpha$. Then the expected payoff of following "*DR*" and "*DP*" by Patriots is:

$$E(DR) = \alpha - 10(1 - \alpha) = 11\alpha - 10$$

and

$$E(DP) = -6\alpha + 2(1 - \alpha) = -8\alpha + 2.$$

At the equilibrium we have:

$$E(DR) = E(DP) \rightarrow 11\alpha - 10 = -8\alpha + 2$$
$$\rightarrow \alpha = 12/19.$$

Suppose Patriots follow "*DR*" by probability β and "*DP*" by probability $1 - \beta$, then the expected pay off of following "*R*" and "*P*" by Colts is:

$$E(R) = -\beta + 6\,(1 - \beta)$$
$$= -7\beta + 6$$

and

$$E(P) = 10\beta - 2(1 - \beta)$$
$$= 12\beta - 2.$$

At the equilibrium we have:

$$E(R) = E(P) \rightarrow -7\beta + 6 = 12\beta - 2$$
$$\rightarrow \beta = 8/19.$$

Therefore the mixed strategy Nash equilibrium is {[12/19, 7/17], [8/19, 11/19]}.