9. When significant scale economies are present, large firms will have an advantage over small firms because increases in inputs yield more than proportional increases in output. For example, in the automobile industry, the minimum efficient scale of production is approximately 300,000 units per plant. Thus a small manufacturer producing just a few thousand cars would use more resources per car than a larger firm.

10. This production function exhibits increasing returns to scale. There are two ways to see this. First, we can simply note that the sum of the exponents exceeds one, indicating increasing returns. Second, we can plug in a sample value and check the outcome. When $M = K = L = 4$, $Q = 8$. If we double all inputs, such that $M = K = L = 8$, $Q = 22.63$, which is more than double the original output level.

11. When capital and labor are fixed in the short run, production is fixed as well. There would be no isoquants indicating a tradeoff, similar to fixed proportion production, except that the levels and the proportions would be set. Professional sports teams fall into this category, in that agreements between the players' associations and team owners stipulate roster sizes. Thus the teams cannot reduce or increase the size of the roster over the course of the season (though it is possible to change who the players are).

12. The expansion path of this production function is the 45 degree line since the two inputs will always equal.

13. The students get tired at the end of a long lecture and have problem concentrating in class, indicating a decreasing return to scale (length of the lecture). By giving a short break, professors break the long lecture into two shorter ones, and help mitigate the degree of decreasing return to scale.

Answers to Questions and Problems in the Text

*Jim Dearden's audio presentations are available online, at www.aw-bc.com/perloff

1. Solution in text.

2. See Figure 6.3. After the sixth unit, marginal product of labor falls to zero. Total product remains at 6 units, and average product of labor falls after the sixth unit.

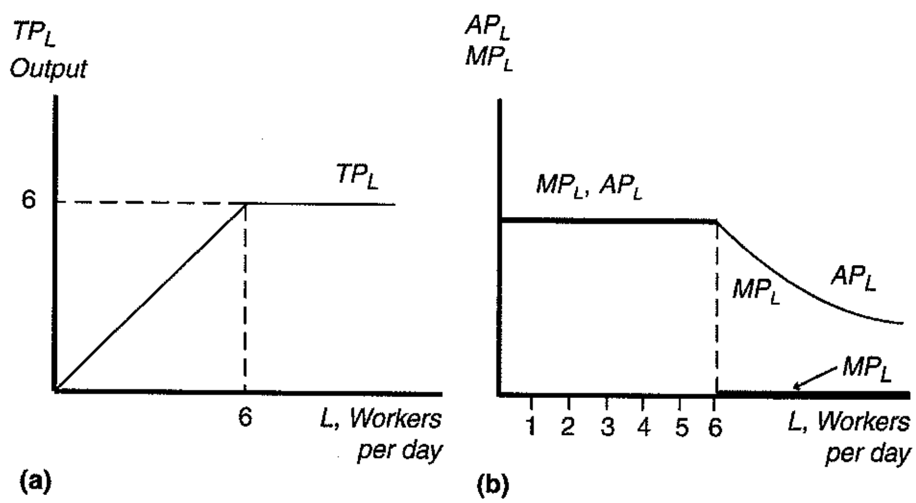

Figure 6.3

3. An indifference curve shows all combinations of goods that result in the same level of utility; an isoquant shows all the combinations of inputs that result in a given level of output.

4. If an isoquant were thick, it would imply that the addition of both capital and labor from a point on the inside edge of an isoquant to the outer edge of the same isoquant would not increase output.

5. a. See Figure 6.4a.
   b. See Figure 6.4b and 6.4c.

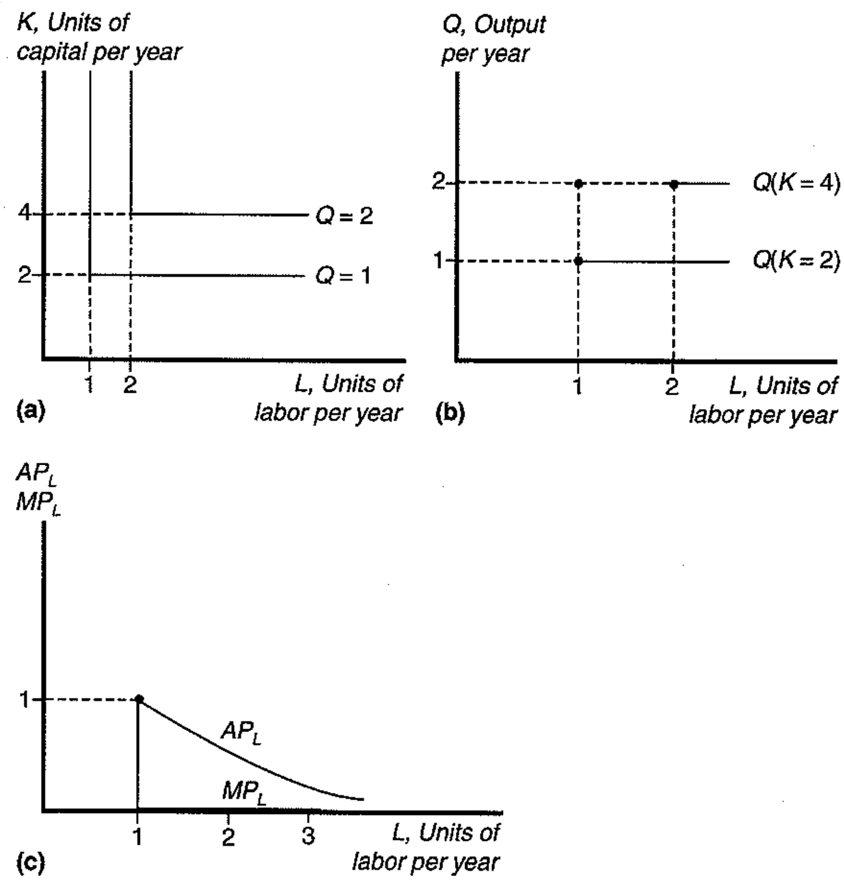

Figure 6.4

6. Solution in text.

7. Michelle's production process illustrates diminishing marginal returns to labor. This diminishing return to extra labor may be due to too many workers sharing too few machines or to crowding.

8. a. See Figure 6.5a

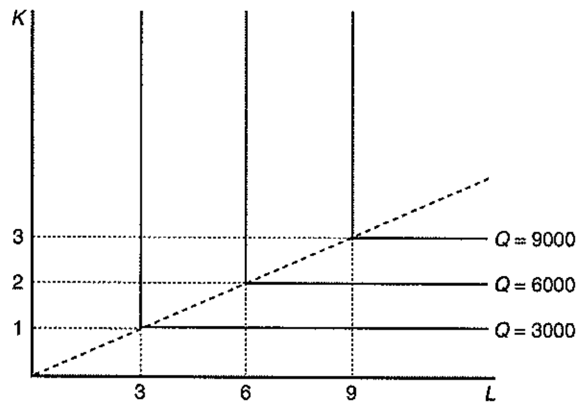

Figure 6.5a

b. See Figure 6.5b. Assume the number of copy machines K is fixed at 1. Then production function is $Q = 1000 * \min(L, 3)$. For $L \leq= 3$, $Q = 1000L$; for $L > 3$, $Q = 3000$.

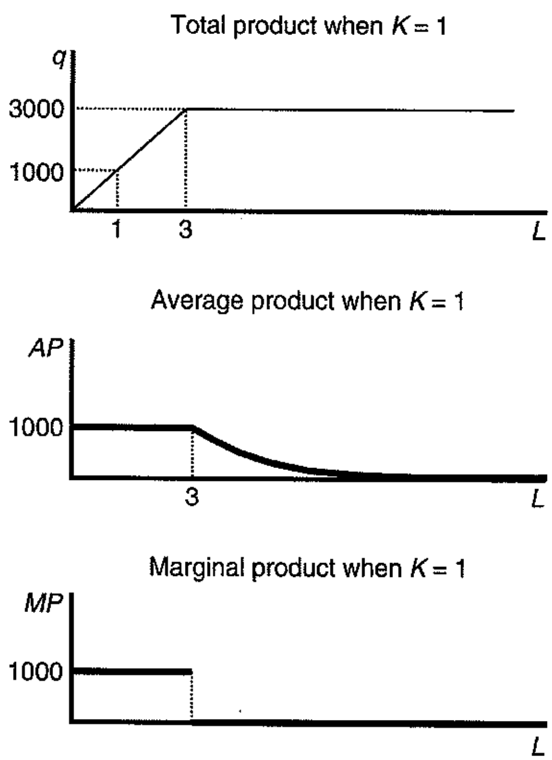

Figure 6.5b

9. The law of diminishing marginal products indicates that, if a firm keeps increasing one input while holding all other inputs and technology constant, the corresponding increases in output will become smaller eventually. For example, when the number of machines is fixed, with too many workers sharing too few machines, each extra worker produces less extra output.

10. Solution in text.

11. The input are helicopters and pilots, the output is delivery of relief. Since the output will not double while one input is doubled, the production process exhibits diminishing marginal product of capital. Since helicopters do not fly themselves, the admiral was likely suggesting that there were enough pilots available to fly them and in that case, the production process had nearly constant returns to scale.

12. *This question will be confusing if students assume all the printers are identical.* Each printer is embodied with a different number of units of capital. For example an all-in-one inkjet copier is very different from a high-speed laser copier with collating and stapling features. If each technology uses one printer and one worker, the production functions will be of the form $Q = \min(L, K)$ and the isoquants will all have the typical L-shape. The corners will be along different rays out of the origin as illustrated in Figure 6.6.

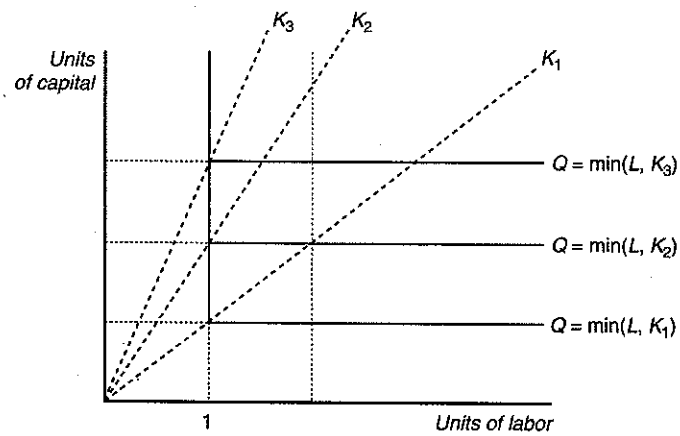

Figure 6.6

13. The amount of capital needed is proportional to the output. If the amount of labor to operate the catapult did not vary substantially with the size of the projectile, the marginal productivity of capital and scale economies will decrease.

14. See Figure 6.7. The technological progress is not neutral. Because of the technology progress, less labor was required to spin the same amount of cotton in order to produce the same amount of product.

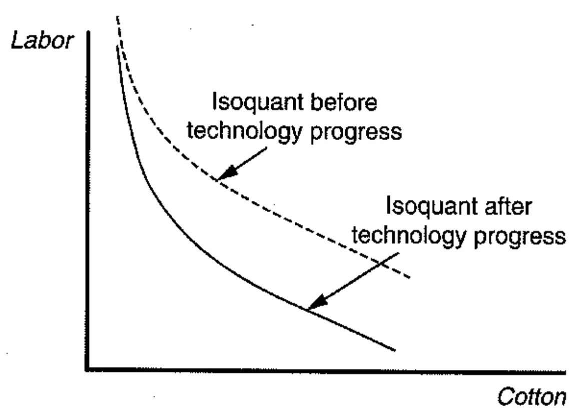

Figure 6.7

15. See Figure 6.8.

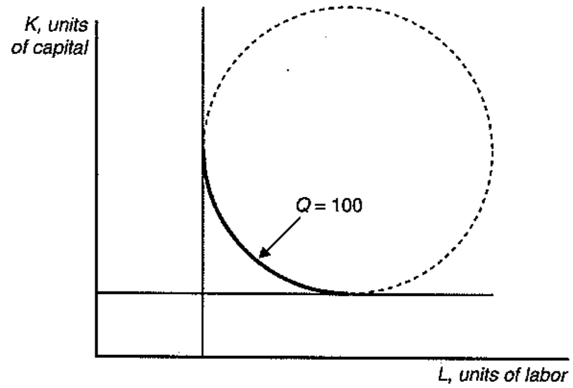

Figure 6.8

16. In the short run, the marginal product of the first few workers will be greater, as they are now able to produce more than they could with the old machine on a per-person basis. However, given that the number of workers needed to correctly operate the machine has been reduced, marginal labor productivity will fall sooner than with the old machine. In order to determine the precise changes in the shapes of the curves, more information is needed about how much labor is saved, and how much more each worker can produce. This is partly dependent on the technology (i.e., how the workers interact with the machine). Returns to scale will also depend on the technology.

17. See Figure 6.7 in the text. Diminishing marginal returns is a short-run phenomenon, caused by the invariability of the fixed input, and they occur regardless of returns to scale. Returns to scale is a long-run phenomenon, which must be evaluated with all inputs variable. In Figure 6.5a, returns to scale are constant, yet the isoquants are convex, due to the diminishing marginal returns of the inputs.

18. The marginal product of labor will increase if the firm experiences falling output and reduces its work force. If the work force remained constant and less output were produced, marginal product would fall.

19. Solution in text.

20. While this would be true for the productivity of labor at the amount of output observed, it may not be true for all output levels. Firm 1 may be capital intensive giving it a higher average product of labor while Firm 2 is labor intensive giving it a higher average product of capital. If different production technologies are used, at another output level, Firm 2 may be more productive.

21. Solution provided in Jim Dearden's audio presentation.

22. Solution in text.

23. $Q = L + K$.

24. Solution in text.

25. a. No diminishing marginal returns to labor. With K fixed at any level, marginal product of labor is constant at 10.

$$\frac{\partial^2 Q}{\partial L^2} = 0$$

b. Diminishing marginal returns to labor. With capital fixed at 2 units, the production function becomes $Q = 1.414L^{1/2}$. Marginal product of labor, calculated using the derivative formula, is $MP_L = 0.707L^{-1/2}$, which decreases as L is increased.

$$\frac{\partial^2 Q}{\partial L^2} = -0.3535L^{-\frac{3}{2}} < 0$$

26. a. This production always displays constant return to scale.
b. The Cobb-Douglas production function has decreasing, constant, or increasing returns to scale as $\alpha + \beta$ is less than, equal to, or greater than 1.
c. This production function has decreasing, constant, or increasing returns to scale as $\alpha + \beta$ is less than, equal to, or greater than 1.
d. The CES production function has decreasing, constant, or increasing returns to scale as d is less than, equal to, or greater than 1.

27. Solution in text.

28. Solution provided in Jim Dearden's audio presentation.

29. Solution in text.

30. If $f(xL, xK) = x^{\gamma} f(L, K)$ then differentiating with respect to L yields $xf_1'(xL, xK) = x^{\gamma} f_1'(L, K)$ and $f_1'(xL, xK) = x^{\gamma-1} f_1'(L, K)$ and differentiating with respect to K yields $f_2'(xL, xK) = x^{\gamma-1} f_2'(L, K)$.

31. From 5.30 above with $\gamma = 1$, $f_1'(xL, xK) = f_1'(L, K)$ and $f_2'(xL, xK) = f_2'(L, K)$ which implies

$$\frac{f_1'(xL, xK)}{f_2'(xL, xK)} = \frac{f_1'(L, K)}{f_2'(L, K)}$$

and the *MRTS* is independent of x.

32. If $f(xL, xK) = x^{\gamma} f(L, K)$ then differentiating with respect to x yields $Lf_1'(xL, xK) + Kf_2'(xL, xK) = \gamma x^{\gamma-1} f(L, K)$ Set $x = 1$ and $Lf_1'(L, K) + Kf_2'(L, K) = \gamma f(L, K)$.

33. If one divides the production function by $(a+b)^{1/\rho}$ and also multiplies by this term, one derives the second expression, where c is the "share" (lies between 0 and 1).

34. Solution provided in Jim Dearden's audio presentation.

35. Solution provided in Jim Dearden's audio presentation.