10. The effective wage rate for the firm is reduced to \$2 per unit. Re-solving with the new lower rate yields

$$K/L = 2/2$$
$$K = L$$
$$60 = 10(L)L$$
$$L^* = 2.44$$
$$K^* = 2.44$$

Although the lower rate increases employment in this industry, employment is likely to fall in the capital industry, because fewer capital goods are demanded with subsidized wage rates.

11. Using the equation for scope economies given in Section 7.5 of the chapter, scope economies exist if $SC > 0$. In this case, scope economies do exist as the following expression is greater than zero for all values of both outputs.

$$SC = [25 + q_1 + 35 + q_2 - (45 + q_1 + q_2)]/45 + q_1 + q_2$$

12. The optimal combination of is one unit of capital good to one worker hour, as the production function is a Leotiff production function.

13. With different price for capital and labor, the optimal combination is still one unit of capital good to one worker hour, as the Leotiff production function's input combination is invariant to change in input price. The production function has a constant return to scale, the difference between this production and the one in Question 12 is that the constant a has been raised to the power of c, but that does not change the its return to scale.

■ Answers to Questions and Problems in the Text

*Jim Dearden's audio presentations are available online, at www.aw-bc.com/perloff

1. Solution in text.

2. Let q equal the number of offices cleaned per hour. Then $C = 2q$ (15 minutes of labor is required per office). Variable cost is also $2q$ because there are no fixed costs. Average variable cost and marginal cost are \$2. See Figure 7.4.

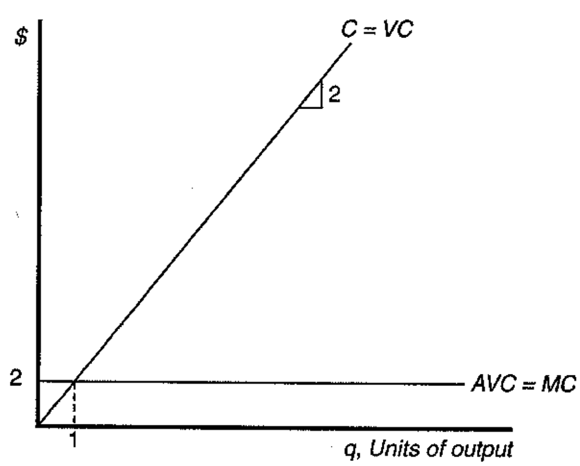

Figure 7.4

3. Solution in text.

4. Solution in text.

5. The expansion path is $B/C = 1$, or $B = C$.

6. If there are constant returns to scale, the long-run expansion path will be a straight line, indicating a constant capital–labor ratio. As the firm expands under constant returns, proportional increases in both inputs yield the same proportional increase in output. For example, the doubling of both inputs doubles output.

7. When the wage increases, the firm will use a more capital-intensive input mix. The expansion path becomes steeper. See Figure 7.5.

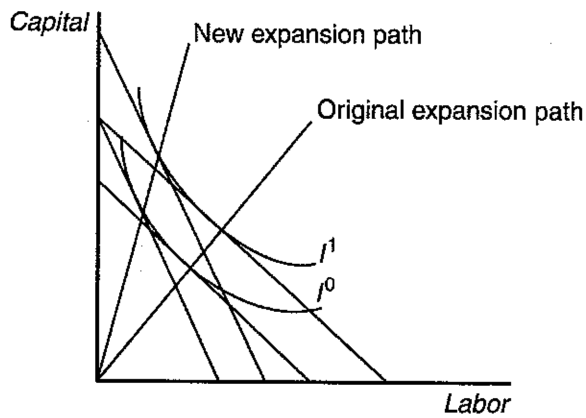

Figure 7.5

8. When the long-run curve is sloping downward, the short-run curve touches the long-run curve to the left of its minimum. When the long-run curve is upward sloping, the short-run curve touches the long-run curve to the right of its minimum. At the minimum of the long-run curve, the short-run curve touch the long-run curve at its minimum.

9. The subsidy effectively reduces the wage rate, flattening the isocost line. With the new, lower effective wage, firms will use more labor and less capital to produce any given level of output.

10. Because the tax is unrelated to how much output is produced, marginal cost and average variable cost are unaffected. The annual fee has the effect of increasing fixed costs, which shifts the long- and short-run total cost curves upward.

11. $C = VC + F$; $AC = AVC + AF = VC/q + F/q$. By going to supermarket, consumer can lower the fixed cost F, thus also lower AC.

12. Solution in text.

13. See Figure 7.6. Paddles and nets are perfect complements that the company always sells one net with two paddles. The expansion path is a straight line with slope 2. The change in relative prices of paddles and nets does not change the expansion path.

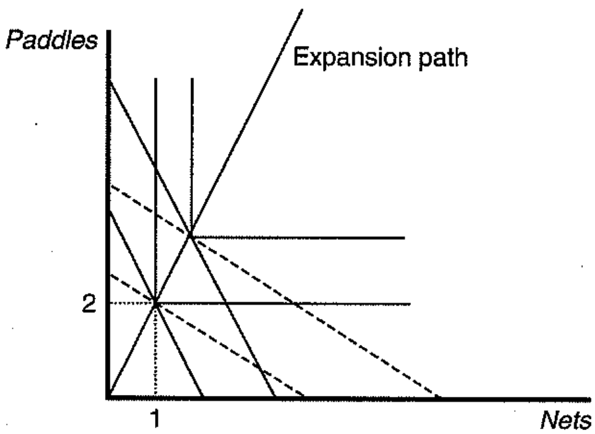

Figure 7.6

14. Solution in text.

15. a. See Figure 7.7a.
    b. See Figure 7.7a. The firm chooses labor-machine technology.
    c. See Figure 7.7b.

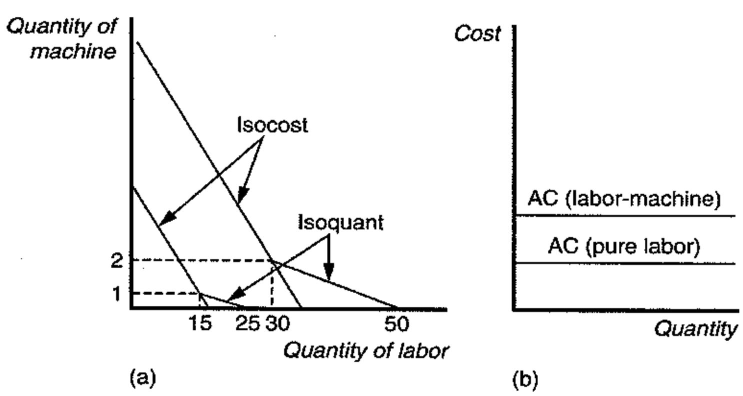

Figure 7.7

16. Solution provided in Jim Dearden's audio presentation.

17. Solution provided in Jim Dearden's audio presentation.

18. Solution provided in Jim Dearden's audio presentation.

19. Solution in text.

20. If her production possibilities frontier is *PPF*1, she can pick 6 pints of mushrooms and 4 pints of strawberries in one day. If there were no scope economies, she could only pick 2.67 pints of mushrooms per day if she picks 4 pints of strawberries. Assuming she works 8-hour days, scope economies produce the equivalent of an additional 3.33 hours of work, valued at $16.65.

21. a. $AFC = 10/q$. $MC = 10$. $AVC = 10$. $AC = 10/q + 10$. See Figure 7.8.

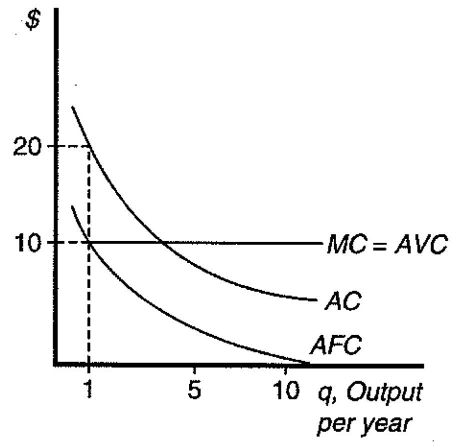

Figure 7.8

b. $AFC = 10/q$. $MC = 2q$. $AVC = q$. $AC = 10/q + q$. See Figure 7.9.

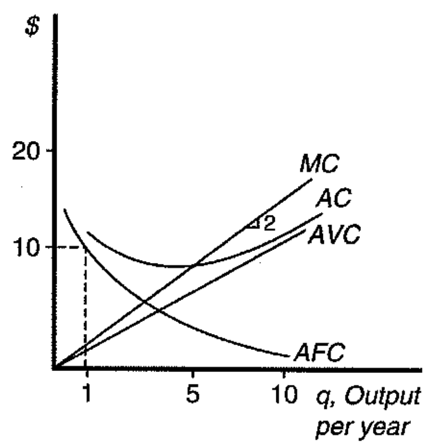

Figure 7.9

c. $AFC = 10/q$. $MC = 10 - 8q + 3q^2$. $AVC = 10 - 4q + q^2$. $AC = 10/q + 10 - 4q + q^2$. See Figure 7.10.

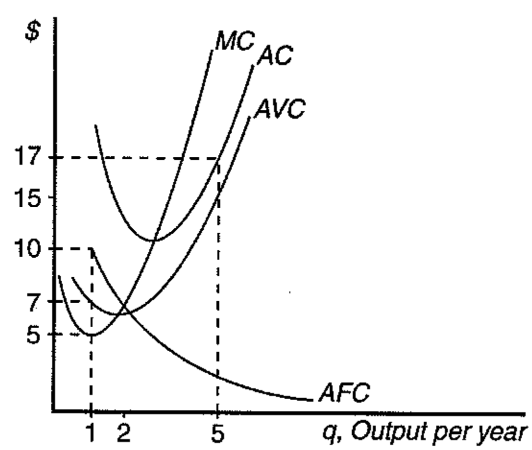

Figure 7.10

22. $AC(q) = 0.55q^{0.67} + 800q^{-2}$

$$\frac{\partial AC(q)}{\partial q} = 0.3685q^{-0.33} - 1600q^{-3} = 0$$
$$q^{2.67} = 4341.93$$

therefore

$$q \approx 23$$

with the tax:

$$AC(q) = 0.55q^{0.67} + 800q^{-2} + 400q^{-1}$$

and

$$\frac{\partial AC(q)}{\partial q} = 0.3685q^{-0.33} - 1600q^{-3} - 400q^{-2} = 0 \quad q \approx 68.$$

23. Solution in text.

24. $C = q$, $MC = AVC = AC = 1$ for q less than or equal to 80 per day. $C = 80 + 1.5\,(q - 80) = 1.5q - 40$, $MC = 1.5$, $AVC = AC = 1.5 - 40/q$ for all q greater than 80 per day. See Figure 7.11a and 7.11b.

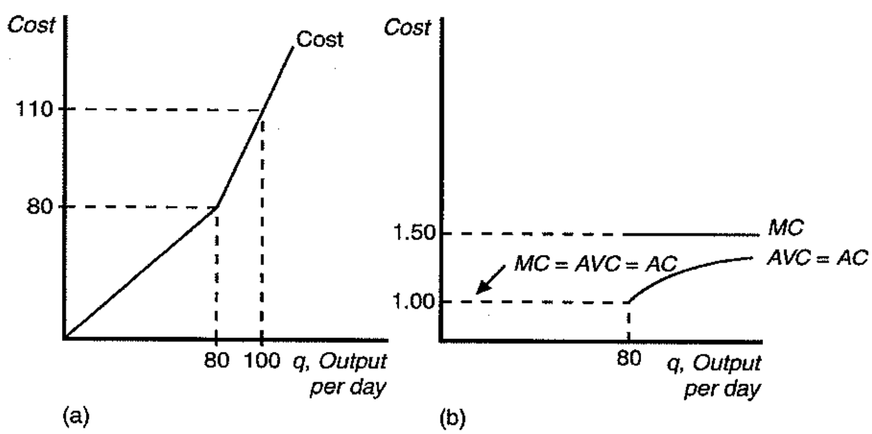

Figure 7.11

25. a. When $b < F/q^2 + 10/q + q$, all cost values are positive.
    b. The average cost curve is U-shaped. AC is minimized at $dAC/dq = -Fq^{-2} - b + 2q = 0$.
    c. MC crosses AC when the functions are equal. $MC = AC$ where $10 - 2bq + 3q^2 = F/q + 10 - bq + q^2$. $MC = AVC$ where $10 - 2bq + 3q^2 = 10 - bq + q^2$.
    d. AVC is minimized where $dAVC/dq = 0$.

$$dAVC/dq = -b + 2q = 0$$
$$\text{or } b = 2q$$
$$MC = AVC$$

where

$$10 - 2bq + 3q^2 = 10 - bq + q^2.$$

Substituting $2q$ for b on both sides yields

$$10 - q^2 = 10 - q^2$$

26. a. You must set $dAC/dq = 0$ for each firm. The minimum point of AC_1 is at $q = 2$. At plant 2, the minimum point is at $q = 1$.
    b. The firm should produce 3 units in plant 1, and 1 unit in plant 2.

27. If the wage increases, the firm should use more capital and less labor at all output levels; thus, the expansion path increases in slope.

28. In this case, the production exhibits decreasing returns to scale, resulting in an increasing cost, or upward-sloping long-run cost function.

29. Solution in text.

30. In the short run, suppose capital is fixed at $\bar{K}$. $F = r\bar{K} = 20\bar{K}$; $VC = wL = 10L$.
    Total cost $C = F + VC = 20\bar{K} + 10L$.
    $q = 10L^{0.32}\ \bar{K}.^{0.56} => L = (0.1q\bar{K}^{-0.56})^{1/0.32} = (0.1q\bar{K}^{-0.56})^{3.125}$
    $AVC = VC/q = 10L/q$
    Suppose $\bar{K} = 1$. $L = (0.1q)^{3.125}$; $VC = 10L = 0.0075\ q^{3.125}$.
    $AVC = VC/q = 0.0075q^{2.125}$.
    $MC = dVC/dq = d(0.0075\ q^{3.125})/dq = 0.023q^{2.125}$.

31. a. See Figure 7.12.
    b. $MP_L/MP_K = (1/2q/L)/(1/2q/K) = K/L = w/r = 1/4 => L = 4K$;
    c. $c = wL + rK = L + 4K = 8K$;
    $q = 10(LK)^{1/2} = 20K => K = q/20$; $c = 8K = 2q/5$

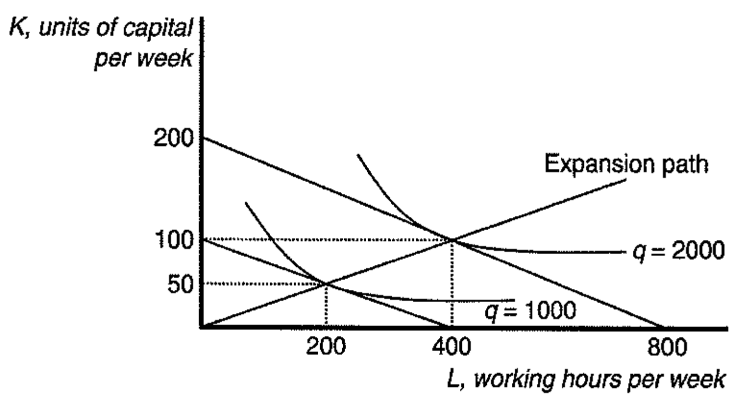

Figure 7.12

32. Using the same technology, the *MRTS* will be the same at home and abroad. Since the wage—rental ratio is the same, the capital labor ratio will also be the same at home and abroad. Cost at home will be twice as high as costs abroad.

33. $MP_L/MP_K = (1/2K^{1/2}/L^{1/2})/(1/2L^{1/2}/K^{1/2}) = K/L = w/r$
    In U.S., $w = r = 10 => L = K$;
    In Mexico, $w^* = 5$; $r^* = 10 => L = 2K$
    $C_{us} = wL + rK = 20K$; $C_{mexico} = 20K$
    $q_{us} = (LK)^{1/2} = K$; $q_{mexico} = 2^{1/2}K$
    $C_{us} = 20q_{us}$; $C_{mexico} = 14.1q_{mexico}$
    When $q = 100$, for U.S., $C = 2000$, $L = K = 100$;
    for Mexico, $C = 1410$, $K = 70.7$, $L = 141$.

34. Solution in text.

35. $MP_L/MP_K = (0.7q/L)/\ (0.3q/K) = 7K/3L = w/r$
    In US, $w/r = 7/3 => L = K$; $q = L^{0.7}K^{0.3} = K$. when $q = 100$, $K = L = 100$, $c = 1000$.
    In Asia, $w/r = 7/9 => L = 3K$; $q = L^{0.7}K^{0.3} = 2.16K$. When $q = 100$, $K = 46.3$, $L = 138.9$, $c = 694.5$.
    If it had to use the same factor quantities as in the United States ($K = L = 100$), $c = 350 + 450 = 800$.

36. $C = C(w,r,e,q) = wL(w,r,e,q) + rK(w,r,e,q) + eM(w,r,e,q)$ where $L(.)$, $K(.)$ and $M(.)$ are the factor demands. From the first derivatives of the Lagrangian:

$$w = \lambda\frac{\partial f}{\partial L},\quad r = \lambda\frac{\partial f}{\partial K},\ \text{and}\ \ e = \lambda\frac{\partial f}{\partial M}$$

So

$$C(w,r,e,q) = \lambda\frac{\partial f}{\partial L}L(w,r,e,q) + \lambda\frac{\partial f}{\partial K}K(w,r,e,q) + \lambda\frac{\partial f}{\partial M}M(w,r,e,q)$$

Rearranging:

$$C(w,r,e,q) = \lambda\left[\frac{\partial f}{\partial L}L(w,r,e,q) + \frac{\partial f}{\partial K}K(w,r,e,q) + \frac{\partial f}{\partial M}M(w,r,e,q)\right]$$

From Euler's Theorem where $f(L, K, M)$ is homogeneous of degree γ:

$C(w,r,e,q) = \lambda\gamma q$ where γ is the marginal cost of production.

37. From 7.22 $\dfrac{\partial C}{\partial w} = \bar{q}\dfrac{aw^{a-1}r^{1-a}}{Aa^a(1-a)^{1-a}} > 0$ for all $a > 0$.

38. *L*: $wL + rK + \lambda\left[\bar{q}^{\rho} - L^{\rho} - K^{\rho}\right]$

FOCs

$$w - \lambda\rho L^{\rho-1} = 0$$
$$r - \lambda\rho K^{\rho-1} = 0$$
$$\bar{q}^{\rho} - L^{\rho} - K^{\rho} = 0$$

From the FOCs

$$L = w^{\frac{1}{\rho-1}}\left[w^{\frac{1}{\rho-1}} + r^{\frac{1}{\rho-1}}\right]^{-\frac{1}{\rho}} q \text{ and } K = r^{\frac{1}{\rho-1}}\left[w^{\frac{1}{\rho-1}} + r^{\frac{1}{\rho-1}}\right]^{-\frac{1}{\rho}} q$$

and

$$C(w,r,q) = q\left[w^{\frac{\rho}{\rho-1}} + r^{\frac{\rho}{\rho-1}}\right]^{\frac{1}{\rho}}$$

39. Solution provided in Jim Dearden's audio presentation.

40. Solution provided in Jim Dearden's audio presentation.