Chapter 16
Uncertainty

■ Chapter Outline

■ Teaching Tips

Chapter 16 begins with a review of some basic statistics. If the class you are teaching has a statistics prerequisite, you may be able to either skip this review all together or give it a brief treatment to refresh the students' memories. The von Nuemann–Morgenstern utility material can be included as part of utility analysis (Chapter 4) if you will not have time to cover this chapter separately.

When describing attitudes toward risk, beware of the classroom demonstration. I once watched a professor come pretty close to losing $10 on a coin flip bet in an effort to demonstrate risk neutrality. When circumstances are artificial, such as classroom experiments, or there is value to posturing by participants, risk-preferring behavior is much more likely than in a true empirical test. (However, I also know two professors who were able to "win" a lot of hours of community service from students in a demonstration that showed quite clearly that the students were overconfident in their betting and did not fully understand the odds of winning casino-style games.)

One interesting application of this material is the complication caused by scope economies in regulated industries. In regulated industries, one way to determine a fair rate of return is to compare the rates of return to nonregulated industries in a similar risk class. Under such a system, a regulated telecommunications provider's allowable earnings would end up being determined in part by the level of risk that regulators believe they are exposed to. Although the portfolio effect of multiple output production can be used to create a reduction in risk by evening out revenue flows, there is also a risk-concentrating effect of using the same capital to produce multiple outputs. For example, in 1988, in Hinsdale, Illinois, a fire at a telecommunications switch caused the complete loss of service to all outputs provided by that switch. WATS lines, 800 service, residential, and business calling were all interrupted. While the switch is in operation, the telephone company receives the benefits of reduced production cost (scope economies), as well as a reduction in revenue fluctuations through diversification. However, producing all of these outputs using the same switch concentrates risk. If one type of risk is accounted for, but the other is not, the firm may be misclassified.

You can also discuss risk in the context of commodities that your students purchase. For those who travel to Jamaica or other island spring break destinations, ask how much they would be willing to pay for trip insurance. Then turn the tables and ask under what circumstances an insurer would be willing to provide such insurance and at what price relative to trip cost.

The final section on behavioral economics is filled with opportunities for classroom applications and examples because students are aware of so many real-life applications relating to seemingly inconsistent or irrational behavior. This might also be an opportunity for a macroeconomics tie-in to risk-taking by banks and mortgage lenders during the recent financial crisis.

■ Additional Applications

How Farmers Reduce Risk[1]

Farmers face both financial and production (low yield per acre) risks. Financial risks include changes in output price, physical factor input cost, and labor cost. Production risks are due to weather such as droughts, and freezes, pests, diseases, and floods. A survey of California farmers showed that they take direct actions, diversify, buy insurance, and hedge to reduce risks.

Direct approaches to reducing yield variability include installing wind machines, helicopters, and other equipment to protect crops during sudden frosts, and installing irrigation systems to protect against droughts. One-fifth of California farmers gain some risk protection from using government programs that stabilize prices (see Chapter 9). A few (1.2 percent) sign labor contracts to reduce wage fluctuations.

Nearly a quarter, 23.4 percent, of California farmers forward contract. A forward contract is signed before the growing season and usually specifies a price (or range of possible prices) to be paid upon delivery. If the contract guarantees the farmer a price of $5 per bushel, the gains or losses of higher or lower prices are borne by the buyer. Any farmer can forward contract if some other party is willing to absorb the risk.

Only 6.2 percent of farmers hedge to reduce risks. Many farmers fail to hedge because no futures or options market exists for their crops, they do not understand hedging, or they do not trust these markets. These farmers favor forward contracts because they can lock in prices for longer periods of time.

Crop insurance, which protects the farmer against unexpected drops in yield, is used by 24.4 percent of California farmers. Crop insurance is only available for some crops. Because federal crop insurance programs were designed primarily for farmers in the Midwest, they are not always attractive to California farmers. A farmer can collect only if the loss exceeds a certain percentage of the average yield. Because California farmers face less yield variability and are less likely to collect on the insurance, the federal rates are often not attractive for many of them. Only 60 percent of the farmers who do buy crop insurance buy it every year. About one-third buy it only in years where they expect adverse weather conditions. If they have better information than federal insurers, this practice can lead to adverse selection.

[1]Based on Steven C. Blank and Jeffrey McDonald, "How California Agricultural Producers Manage Risk," *California Agriculture,* 49(2), March-April 1995:9–12, and Venner and Blank (1995).

California farmers' main approach to avoiding financial and yield risks is to diversify across crops or between agricultural and nonagricultural activities. Nearly half of the farmers diversify across agricultural activities. Nearly two-thirds (63 percent) of producers receive nonagricultural income, accounting for 47 percent of their total family income. Diversification helps protect against both financial and production risks.

1. How has the globalization of agricultural markets affected U.S. farmers' need for insurance?
2. Suppose a new weather forecasting method was derived that was 50 percent more accurate than the current method. How would this affect the insurance market?
3. What is the relationship between risk and length of product cycle?

■ Discussion Questions

1. How can you determine how risk averse someone is?
2. Do you know someone who is strictly risk preferring? What evidence do you have?
3. I'm convinced that one of my colleagues would try to drive home blindfolded if offered a large enough bet. Why would he do that? What must be his attitude toward risk?
4. Should governments prohibit gambling? Should governments run legalized gambling lotteries?
5. Winnings from gambling (whether legal or not) are subject to U.S. income tax. You may, however, deduct your losses up to the extent of your winnings. How do these tax laws affect an individual's willingness to gamble or the size of the gamble?
6. Are both the standard deviation and beta measures of risk?
7. Some states forbid insurance rate discrimination on the basis of gender. Without such laws, insurance companies would charge young male drivers higher rates because they are more likely to be involved in accidents. Discuss the efficiency and equity implications of such antidiscrimination laws.
8. Why would a company offer a "money-back guarantee if not completely satisfied" if the firm knows that it is impossible to satisfy all consumers completely?

■ Additional Questions and Problems

1. Suppose you are selling kites on the boardwalk at the New Jersey shore as your summer job. Every fourth person that passes you selects a kite at random and buys it. One-half of your kites sell for $8, and the other half sell for $4. If 100 people per hour pass by, what is your expected revenue for six hours work?
2. In Problem 1, of the people who bought kites, what are the variance and standard deviation of revenue per person?
3. Many states run lotteries of various kinds. One of the most popular is a daily number game where an individual buys a $1 ticket with a number of their choosing between 000 and 999. The payoff is typically $500. Would a strictly risk-neutral person buy such a ticket? What payoff would ensure that at least some people who are risk averse would buy a ticket?

4. Draw a utility function such that the person is risk preferring with respect to a small gamble, but risk averse with respect to a large gamble.

5. What would be the price of fair insurance for a $20,000 motor home for one year, assuming that during that year there is a 0.02 percent chance that it will be destroyed in an accident, leaving a $3,000 salvage value and no chance of any partial loss? Assume that the owner keeps the salvage value.

6. Suppose it costs a corporation an additional $500 in transactions costs to have two executives fly separately rather than together. If they were both killed, they would lose $5 million in profits. Given that a probability of any single flight crashing of 0.000000432, should a risk-neutral firm separate the executives?

7. Why do most investment counselors typically recommend that their clients alter their portfolio of stocks and bonds as they age?

8. Would you want to purchase auto insurance from a company that claims to turn down no one and charges all drivers the same price?

9. Why don't employers spend whatever is required to eliminate all risks from jobs (such as construction work)? Would employees want the employer to do so?

10. Twenty years ago, almost no one wore helmets while skiing. Now many skiers wear them. What could have caused such a change?

11. Using information from the utility function in the figure (in the Application: Gambling) on page 574 of the text, what would be Sylvia's preference between the following two games: (i) receiving W_3 with certainty; (ii) receiving W_1 or W_5 with equal probabilities?

12. In Figure 16.4 of the text, what is the optimal choice of the risk-averse owner if the probability of high demand and low demand is 90 percent and 10 percent, respectively?

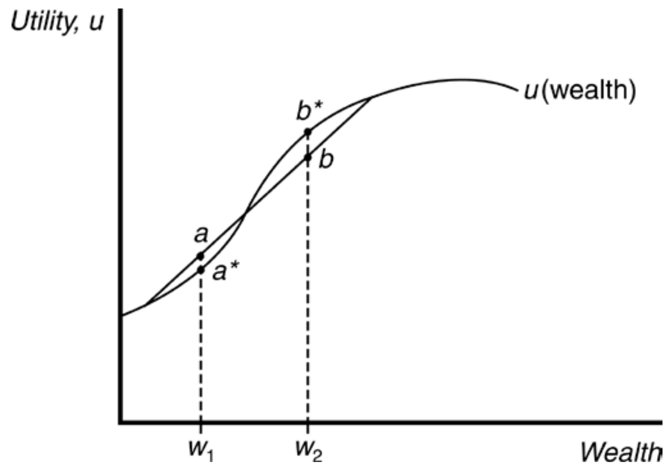
■ Answers to Additional Questions and Problems

1. Expected revenue would be the product of expected sales (1,003.25) and expected price (0.53 $4 + 0.53 $8), or $150.

2. The variance is $0.5(8 – 6)^2 + 0.5(4 – 6)^2 = 4$. The standard deviation is $2.

3. A person who is strictly risk neutral would not buy a ticket because the expected value of the ticket is only 50 cents (0.0013500). The payoff would have to be at least $1,001 to ensure that some risk-averse individuals would play. This, however, would result in losses for the lottery since, on average, they would pay out more than they took in.

4. Such a utility function would be opposite of the Friedman-Savage Utility function. The expected utility of a small gamble (*a*) is greater than if the outcome were certain (a^*), while the expected utility of a large gamble (*b*) is less than if the outcome were certain (b^*).



5. The expected value of the motor home is $19,660 = (0.02 × $3,000 + 0.98 × $20,000). Thus fair insurance for the motor home would cost $340 = (0.02 × $17,000), where $17,000 is the net loss from an accident.

6. They should fly together, as the expected loss in profits due to a possible crash is only about $2.

7. As individuals age, their ability to make up losses due to large swings in financial markets is reduced, making them more risk averse. Typically, fund managers recommend higher risk (variance), higher expected return investments for long-term investors, as market fluctuations will be averaged out over time. For investors near retirement, lower return, lower variance investments are preferred.

8. An adverse selection problem occurs if the insurance company does not turn away bad drivers. Because prices are set by the insurance company such that the expected payout is less than the sum of all premiums, good drivers end up subsidizing bad drivers if premiums are uniform. As a good driver, you would be better off insuring with a firm that turns away drivers with a history of previous accidents.

9. Employers do not remove all risk from work environments for two reasons. First, it may not be possible to eliminate all risk, no matter what the expenditure. Second, it may be cheaper to pay employees to accept the risk in the form of a differential rather than abate it. Employees have different levels of risk aversion. Those employees who are highly risk averse will prefer lower wage, safe jobs. Less risk-averse workers with a high marginal utility of income will prefer to accept some risk in exchange for higher pay.

10. There are several possible reasons. The first is that skiing might be more dangerous than it was 20 years ago (an increase in the probability of injury). Second, the risk may be the same, but skiers may be more aware of the risks (better information). Finally, skiers may be more risk averse than they were 20 years ago and so are less willing to risk an injury.

11. Sylvia will be indifferent between the two games, as the expected value of the bet is equal to the payoff with certainty.

12. The expected payoff with 90 percent and 10 percent probability is $40 \times 90\% + 0 \times 10\% = 36 > 35$. Hence the owner will choose to invest in this business.

■ Answers to Exercises in the Text

1.1 A probability is a number between 0 and 1 that indicates the likelihood that a particular outcome will occur. If 5 houses will burn and 7 houses will be damaged by high winds, then 12 houses will burn or be damaged by high wind. The probability a house catches fire or is damaged by high winds is 12/1000 or 1.2 percent.

1.2 Assuming that the painting is not insured against fire, its expected value is

$$\$550 = (0.2 \times \$1{,}000) + (0.1 \times \$0) + (0.7 \times \$500).$$

1.3 The expected value is

EV = (0.25)(400) + (0.75)(200) = 100 + 150 = $250.

The variance is

Variance = 0.25 $(400 - 250)^2$ + 0.75 $(200 - 250)^2$ = 0.25(22,500) + 0.75(2,500) = 5,625 + 1,875 = 7,500.

1.4 The expected value is the value of each possible outcome times the probability of that outcome:

$$EV = \Pr(\text{no piers})(4000) + \Pr(\text{piers})(-1000)$$
$$EV = 0.75(4000) + 0.25(-1000)$$
$$EV = \$2{,}750.$$

If the study indicates piers are required, then *EZ* will not accept the contract and will incur no losses:

$$EV = \Pr(\text{no piers})(4000) + \Pr(\text{piers})(0)$$
$$EV = 0.75(4000) + 0.25(0)$$
$$EV = \$3{,}000.$$

The most that *EZ* will pay for the study is the difference in the expected value of the contract with the study and the expected value of the contract without the study:

$$\$3{,}000 - \$2{,}750 = \$250.$$

1.5 For individuals, gambling at a casino and buying a stock might not be too different. Both serve to redistribute wealth within a society. For the society as whole, stock market places an important role in the financial market, which influences the general economy directly. On the other hand, gambling is generally considered an undesirable good, or a “bad,” as it does not contribute to the productivity of the society.

1.6 The expected punishment for violating traffic laws is θV, where θ is the probability of being caught and fined and V is the fine. If people care only about the expected punishment (that is, there’s no additional psychological pain from the experience), increasing the expected punishment by increasing θ or V works equally well in discouraging bad behavior. The government prefers to increase the fine, V, which is costless, rather than to raise θ, which is costly due to the extra police, district attorneys, and courts required.

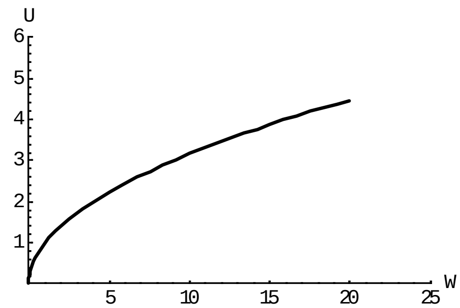
1.7 The probability of being caught must be at least 0.625, because $800 × 0.625 = $500.

2.1 A fair bet is one whose expected value is zero. The expected value (EV) is the value of each possible outcome times the probability of that outcome:

EV = Pr(one or two dots)*(3) – Pr(not one or two dots)*(2)
EV = (0.333)*(3) – Pr(0.666)*(2)
EV = –$0.33.

So, the bet is not fair.

2.2 The figure plots $U(W) = \sqrt{W}$. Since it is a concave function, Jen is a risk-averse individual.



2.3 The expected value is the value of each possible outcome times the probability of that outcome. For Stock A, this is

EV = (50%)($100) + (50%)($200)
EV = $150.

For Stock B, this is

EV = (50%)($50) + (50%)($250)
EV = $150.

The variance is the probability-weighted average of the squared mean differences in the outcomes values. For Stock A, this is

$$\text{Variance} = (50\%)(100 - 150)^2 + (50\%)(200 - 150)^2$$
$$\text{Variance} = 2{,}500.$$

For Stock B, this is

$$\text{Variance} = (50\%)(50 - 150)^2 + (50\%)(250 - 150)^2$$
$$\text{Variance} = 10{,}000.$$

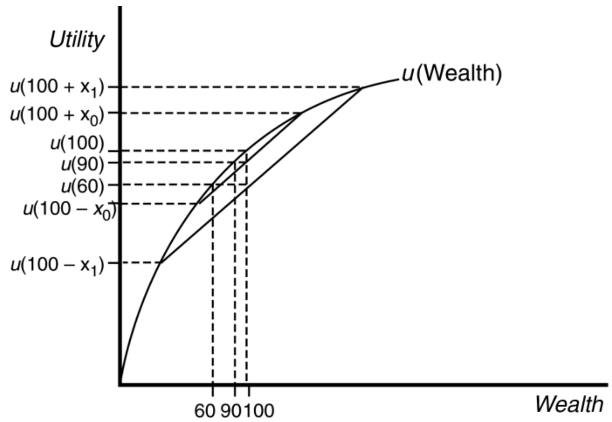
Utility is a set of numerical values that reflect the relative ranking of various bundles of goods. For Stock A, this is

$$U =(50\%)(100)^{0.5} + (50\%)(200)^{0.5}$$
$$U = 12.071$$

For Stock B, this is

$$U = (50\%)(50)^{0.5} + (50\%)(250)^{0.5}$$
$$U = 11.441.$$

2.4 When x increases from x_0 to x_1, the chord showing expected utility shifts downward. Initially the risk premium is \$10. After the increase in x, the risk premium increases to \$40.



2.5 Irma's expected utility of 133 at point f (where her expected wealth is \$64) is the same as her utility from a certain wealth of Y.

2.6 $U(100) = Ln(100) = 4.6$. Note that the expected value of the gamble is $0.5 \times 120 + 0.5 \times 80 = 100$. Therefore, the expected value of the gamble is equal to 100 and thus the gamble is fair. We know that the utility of the gamble is the expected utility of the gamble and it is

$$0.5U(120) + 0.5U(80) = 0.5 \times Ln(120) + 0.5 \times Ln(80)$$
$$= 4.5845.$$

We know

$$U(97.98) = 4.5845.$$

Therefore, the risk premium is

$$100 - 97.98 = 2.02.$$

2.7 Hugo's expected wealth is $EW = \left(\frac{2}{3} \times 144\right) + \left(\frac{1}{3} \times 225\right)$
$$= 96 + 75 = 171.$$

His expected utility is

$$\begin{aligned} EU &= \left[\tfrac{2}{3} \times U(144)\right] + \left[\tfrac{1}{3} \times U(225)\right] \\ &= \left[\tfrac{2}{3} \times \sqrt{144}\right] + \left[\tfrac{1}{3} \times \sqrt{225}\right] \\ &= \left[\tfrac{2}{3} \times 12\right] + \left[\tfrac{1}{3} \times 15\right] = 13. \end{aligned}$$

He would pay up to an amount P to avoid bearing the risk, where $U(EW - P)$ equals his expected utility from the risky stock, EU. That is, $U(EW - P) = U(171 - P) = \sqrt{171 - P} = 13 = EU$. Squaring both sides, we find that that $171 - P = 169$, or $P = 2$. That is, Hugo would accept an offer for his stock today of \$169 (or more), which reflects a risk premium of \$2.

2.8 Yes, Mary is risk averse because she has a declining marginal utility of wealth ($MU_W = 1/3\ W^{-2/3}$).

The risk premium is the amount that a risk-averse person would pay to avoid taking a risk. The expected value (*EV*) of the gamble is

$$EV = 0.50\ (29{,}791) + 0.50\ (24{,}389) = \$27{,}091.$$

Mary's expected utility (*EU*) from the gamble is

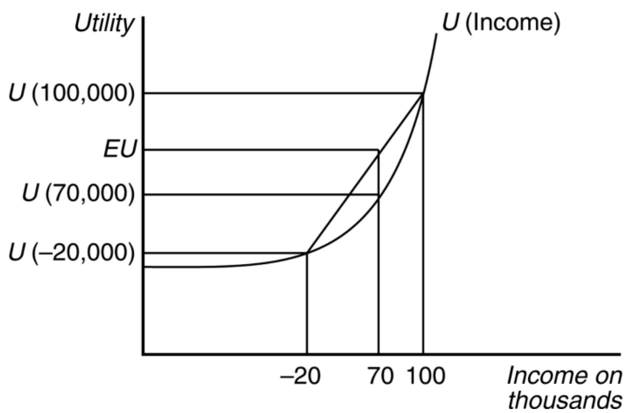
$$EU = 0.50\ (29{,}791^{1/3}) + 0.50\ (24{,}389^{1/3}) = 30.$$

2.9 No. Risk-neutral individuals are indifferent between certainty and a fair bet. Thus they may or may not purchase fair insurance, but would not be willing to purchase unfair insurance. Because the risk premium for a risk-neutral person is zero, he or she is unwilling to pay anything to avoid taking a risk.

2.10 a. $EV = 0.6(100{,}000) + 0.4(–20{,}000) = \$52{,}000$.
Var = $[0.6(\$48{,}000^2) + 0.4(–72{,}000^2)] = \$3456$ million.

b. Yes, she would accept the offer because the expected value of the harvest ($52,000) is below that of the sure thing offer of $70,000.

c. One reason is that Ethan may not understand the laws of probability and is unable to calculate the expected value. He may also not know the probabilities involved. If he believes that the probability of good weather is greater than it actually is, he may believe that he is making a good investment even if he is risk averse. Finally, he may be risk preferring. If his expected utility from the harvest is greater than the utility of $70,000 with certainty, he will make the purchase. See figure below.



2.11 Job 1's expected value is 30, its variance is 100, and its standard deviation is 10. Job 2's expected value is 36, its variance is 189, and its standard deviation is 13.74. Job 3's expected value is 30, its variance is 0, and its standard deviation is 0. We cannot conclude anything about the job Joanna will select if she's risk averse because Job 2 has the highest expected value but also the highest variance. If she's risk-neutral, then she'll select the job with the highest expected value, which is Job 2.

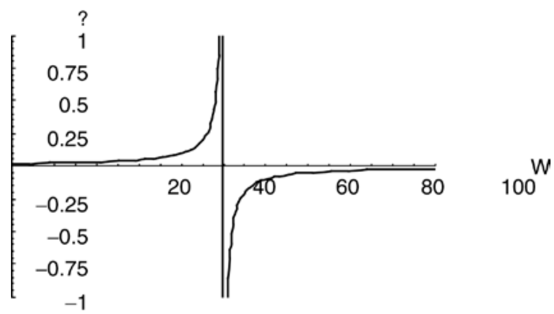
2.12 Since

$$\frac{dU}{dW} = 100 - 2W$$
$$\frac{d^2U}{dW^2} = -2,$$

the Arrow-Pratt measure is

$$\rho(W) = -\frac{\frac{d^2U}{dW^2}}{\frac{dU}{dW}} = \frac{2}{100-2W}.$$

The measure is shown graphically below. As it is apparent from the graph, the measure is positive and increasing in wealth when $W < 50$.



2.13 a. Expected utilities for street parking are:

$$\text{EU}_{\text{Carolyn}} = 0.5(80{,}000-10{,}000)^{0.4} + 0.5(80{,}000)^{0.4} = 89.08$$
$$\text{EU}_{\text{Sanjay}} = 0.5(20{,}000-10{,}000)^{0.4} + 0.5(20{,}000)^{0.4} = 46.17$$

Let p_C be the largest amount that Carolyn is willing to pay for a garage, and let p_S be the maximum amount that Sanjay is willing to pay; then:

$$(80{,}000 - p_C)^{0.4} = \text{EU}_{\text{Carolyn}} = 89.08 \quad\Rightarrow\quad p_C = 5100.11$$
$$(20{,}000 - p_S)^{0.4} = \text{EU}_{\text{Sanjay}} = 46.17 \quad\Rightarrow\quad p_S = 5515.13$$

b. $p_C = 5100.11 < 5515.13 = p_S$ because Sanjay is more risk averse than Carolyn.

The Arrow-Pratt measure of risk aversion at any wealth level W is:

$$r(W) = -\frac{U''(W)}{U'(W)} = \frac{0.24W^{-1.6}}{0.4W^{-0.6}} = \frac{0.6}{W},$$

which is a decreasing function of the wealth level W.

Thus Sanjay is more risk averse than Carolyn because Carolyn is wealthier than Sanjay.

3.1 If she sends them together, the expected utility is $qU(0) + (1 – q)U(\$2000)$. If they are sent separately and losses are independent events, she might lose both, one, or neither. The expected utility is $(1 – q^2)U(\$2000) + 2q(1 – q)U(1000) + (q^2)U(\$0)$.

3.2 Fair insurance is a bet between an insurer and a policyholder in which the value of the bet to the policyholder is zero. In this example, the cost of a \$1.00 insurance payment if successfully sued is \$0.05263 if not sued (from a 5 percent probability of being successfully sued divided by a 95 percent probability of not being successfully sued). Because Helen is risk averse, she fully insures by buying enough insurance to eliminate risk altogether. With this amount of insurance, she has the same amount of wealth whether sued successfully or not. The expected value of Helen's profits (*EV*) is

$$EV = 0.95(100) + 0.05(10)$$
$$EV = \$95.5.$$

Thus, at a cost of \$0.05263 if not sued per \$1.00 insurance paid if successfully sued, Helen will purchase insurance such that she receives an insurance payment of \$85.5 if successfully sued (from an expected value of \$95.5 minus profit if successfully sued of \$10). That is, Helen will spend \$4.50 on insurance (from a price of \$0.05263 multiplied by insurance coverage of \$75.5).

3.3 a. Fair insurance is a bet between an insurer and a policyholder in which the value of the bet to the policyholder is zero. In this example, the insurance company will make a \$70,000 payment with probability 0.02, so the cost should be \$1,400.

b. Jill's expected utility (*EU*) without insurance is

$$EU = 0.98*4(160{,}000)^{0.5} + 0.02*4(70{,}000)^{0.5}$$
$$EU = 1{,}592.$$

Her utility with income of \$145,000 (from \$160,000-\$15,000) with certainty is

$$EU = 4(145{,}000)^{0.5}$$
$$EU = 1{,}523.25.$$

So Jill should not buy the insurance.

c. This level of utility (1,592) can be obtained with income of \$158,404 with certainty, from

$$1{,}592 = 4(X)^{0.5}$$
$$398 = X^{0.5}$$
$$X = \$158{,}404.$$

Thus, the most Jill would be willing to pay for the insurance policy is \$1,596, from \$160,000 minus income of \$158,404 with certainty.

3.4 a. *Risk Neutral: Y* is the wealth

$$U_{INS} = Y – 150.$$

The expected utility of no insurance = $(1/36)(Y – 400) + (35/36)Y = Y – 11.11$. A risk-neutral person doesn't buy insurance.

Risk Averse and poor: Y is the wealth and is equal to 4,000.

$$U_{INS} = (4{,}000 – 150)^{0.5} = 62.04$$

The expected utility of no insurance = $(1/36)(0)^{0.5} + (35/36)(4{,}000)^{0.5} = 61.49$. A risk-averse and poor person purchases insurance.

Risk Averse and wealthy: Y is the wealth and equal to 500,000.

$$U_{INS} = (500{,}000 - 150)^{0.5} = 707$$

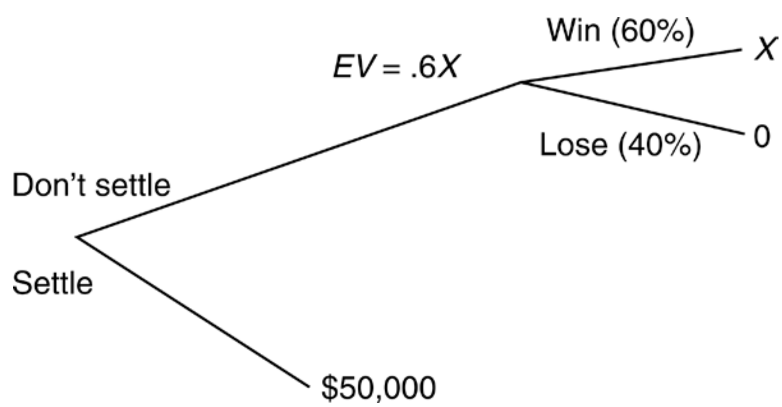
The expected utility of no insurance = $(1/36)(500{,}000 - 4000)^{0.5} + (35/36)(500{,}000)^{0.5} = 726.67$. A risk-averse and wealthy person does not purchase insurance.

b. The loss for a wealthy person is inconsequential, and he or she behaves like a risk-neutral person.

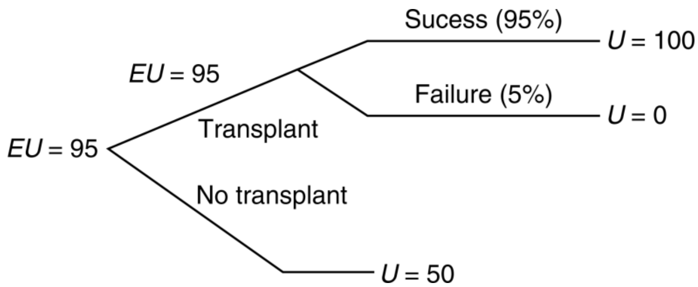
3.5 If individuals know that the government will provide subsidies to homeowners with losses, they have an incentive to purchase less insurance. In the figure, initially, the individual has an expected utility of $U(w_2')$, because he or she receives w_0 if there is a flood, and w_3 if there is no flood. This results in a risk premium of $(w_2 - w_2')$. When the government offers the subsidy, the potential loss decreases, even though the probability of a loss does not change. The expected utility chord swings upward, resulting in a higher utility level if there is a flood [$U(w_1)$]. The risk premium of the individual decreases to $w_2'' - w_2'$, decreasing the person's willingness to pay for insurance.

4.1 If they were married, Andy would receive half the potential earnings whether they stayed married or not. As a result, Andy will receive \$12,000 in present-value terms from Kim's additional earnings. Because the returns to the investment exceed the cost, Andy will make this investment (unless a better investment is available). However, if they stay unmarried and split, Andy's expected return on the investment is the probability of staying together, $\frac{1}{2}$, times Kim's half of the returns if they stay together, \$12,000. Thus, Andy's expected return on the investment, \$6,000, is less than the cost of the education, so Andy is unwilling to make that investment (regardless of other investment opportunities).

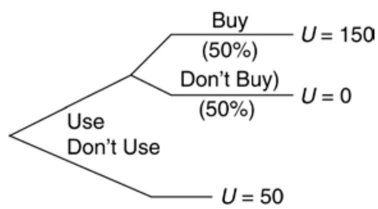
4.2 The plaintiff must believe that *X* is at least \$83,333 (\$50,000/0.6) in order for the expected value of not settling to exceed the certain settlement. If the plaintiff is risk averse, he would accept a smaller offer of settlement to avoid taking the risk.



4.3 In the decision tree below, the individual decides to have the transplant because the expected utility from the transplant is greater than remaining on dialysis.



4.4 In the decision tree below, the individual decides to use the roundup-resistant beets because the expected utility from the roundup-resistant beet is greater than not using it.



4.5 Guatam's expected profits (π) without advertising if he makes the investment is

$$\pi = 0.4(100) - 0.6(100)$$
$$\pi = -\$20.$$

Thus, he will not invest and have profit of \$0. The minimum probability of high demand for which Guatam will advertise and invest is p such that

$$p(100 - 50) - (1 - p)(100 + 50) \geq 0$$
$$50p - 150 + 150p \geq 0$$
$$200p \geq 150$$

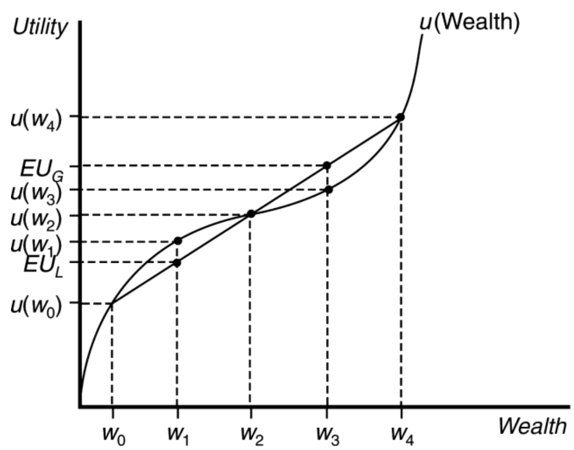
$$p \geq 0.75$$

Thus, advertising will have to raise the probability of high demand to at least 75 percent for Guatam to be willing to invest.

5.1 It is not consistent because the two experiments have identical payoffs. The second choice probably is more popular in Scenario *B* for most individuals because they are starting from a higher level. They reason that only one flip of a coin is required from them to be able to keep all of their winnings, and if they lose, they still have \$10,000. In the first scenario, beginning from a lower level, the \$2,500 sure thing is too tempting for many to resist.

5.2 According to prospect theory, people are concerned about gains and losses—the changes in wealth—rather than the level of wealth, as in expected utility theory. People start with a reference point and consider lower outcomes as losses and higher ones as gains, using their initial endowment as a reference point. To determine the value of a gamble, individuals use decision weights, where the weight function assigns different weights than the original probabilities of the gamble. For example, people might assign disproportionately high weights to rare events. The value function in prospect theory passes through the reference point because gains and losses are determined relative to the initial situation. The value curve is concave to the horizontal, outcome axis. Finally, the value function's curve is asymmetric with respect to gains and losses, with people treating gains and losses differently, in contrast to the predictions of expected utility theory. For example, the value function's curve might show a bigger impact to a loss than to a comparably sized gain.

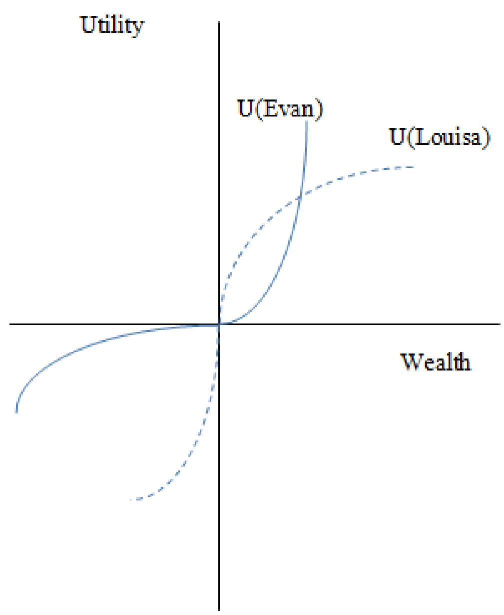
5.3 If the individual has wealth w_2 and faces the possibility of a loss to w_0, he or she is risk averse, as the expected utility EU_L is less than the utility of w_1 with certainty. The same individual is risk preferring with respect to a possible gain to w_4, as the expected utility EU_G is greater than the utility of w_3 with certainty.



5.4 According to prospect theory, people are concerned about gains and losses rather than the level of wealth. People start with a reference point and consider lower outcomes as losses and higher ones as gains. Evan's and Louisa's utility functions are consistent with prospect theory because they treat gains and losses differently, in contrast to predictions of expected utility theory.

Evan is risk-seeking with respect to gains and risk averse with respect to losses, so his utility function is the solid line.

Louisa is risk-seeking with respect to losses and risk averse with respect to gains, so her utility function is the dashed line.



5.5 Kahneman and Tversky's (1979) prospect theory is an alternative theory of decision-making under uncertainty that can explain some of the choices people make that are inconsistent with expected utility theory.

In expected utility theory, if an individual does not take a gamble, then his utility is $U(W)$, where W is initial wealth. Expected utility (EU) with the gamble is

$$EU = \theta U(W - A) + (1 - \theta)\, U\,(W + B).$$

The individual will take the gamble if

$$EU > U(W).$$

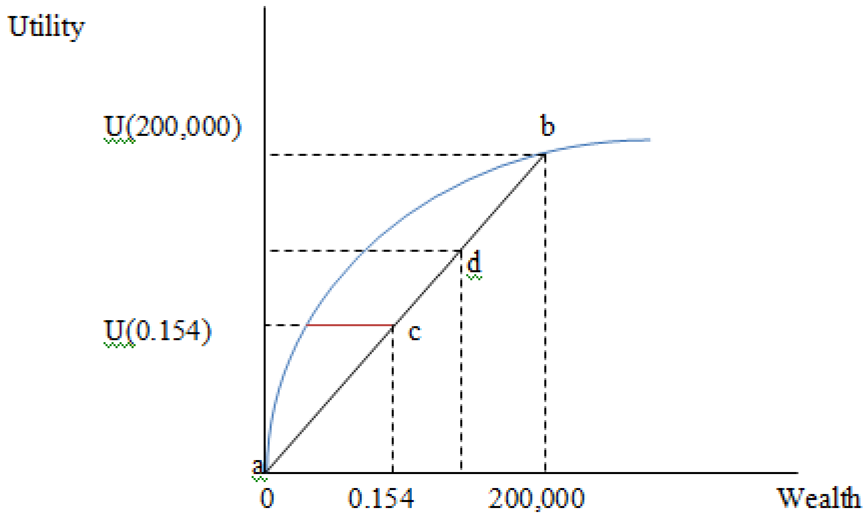
In prospect utility theory, people are concerned about gains and losses—changes in wealth—rather than the level of wealth, as in expected utility theory. People start with a reference point and consider lower outcomes as losses and higher ones as gains. To determine the value from taking a gamble, individuals do not calculate the expectation using the probabilities θ and $(1 - \theta)$, as they would with expected utility theory. Rather, an individual would use decision weights $w(\theta)$ and $w(1 - \theta)$, where the w function assigns different weights than the original probabilities. Furthermore, in prospect theory, the value function's curve is asymmetric, potentially with a bigger impact to a loss than to a comparably-sized gain. An individual gambles if the value from not gambling, $V(0)$, is less than her evaluation of the gamble, which is the weighted average of her values in the two outcomes:

$$V(0) < [w(\theta)V(-A) + (1 - \theta)\, V(B)].$$

Because prospect theory differs from expected utility theory in both the valuation of outcomes and how they are weighted, it is not possible to state conditions for which someone who acts as described in prospect theory is always more or less likely to take a gamble than someone who acts as described in expected utility theory. For example, even if $w(\theta)$ were less than θ, it may be the case that someone who acts as described in prospect theory is less likely to take a gamble than someone who acts as described in expected utility theory if the value function reflects loss aversion, where people hate making losses more than they like making gains.

6.1 Fair insurance is a bet between an insurer and a policyholder in which the value of the bet to the policyholder is zero. For insurance to be fair, its expected value must be zero. The expected value (EV) of the insurance is the probability of dying in a plane crash (θ) multiplied by the insurance payment in the event of a crash ($200,000) plus the probability of not dying in a plane crash $(1 - \theta)$ multiplied by no insurance payoff is

$$EV = [\theta(200{,}000)] + [(1 - \theta)(0)]$$
$$EV = [(0.00000077)(200{,}000)] + [(1 - 0.99999923)(0)]$$
$$EV = \$0.154.$$



Risk premium is the amount that a risk-averse person would pay to avoid taking a risk. The actual risk premium can be represented with a horizontal line showing the difference between the level of wealth with certainty that generates the same level of utility as the insurance.

Suppose the figure above shows a passenger's utility from income of $0 (point a), utility from income of $200,000 (point b), and expected utility from a 0.00000077 probability of receiving $200,000 and a 0.99999923 probability of receiving $0.00 (point c).

A risk-averse person might by unfair flight insurance because she incorrectly perceives the probability of dying in a plane crash to be much higher than is actually the case. Thus, she believes the expected value and the expected utility from the insurance are much higher, represented by a point on the cord between points *a* and *b* that is above and to the right of point *c*, such as point *d*.