

# CHAPTER 7

## The Cost of Production

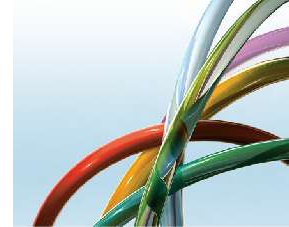


### CHAPTER OUTLINE

- 7.1 Measuring Cost: Which Costs Matter?
  - 7.2 Costs in the Short Run
  - 7.3 Costs in the Long Run
  - 7.4 Long-Run versus Short-Run Cost Curves
  - 7.5 Production with Two Outputs—Economies of Scope
  - 7.6 Dynamic Changes in Costs—The Learning Curve
  - 7.7 Estimating and Predicting Cost
- Appendix: Production and Cost Theory—A Mathematical Treatment

Prepared by:  
Fernando Quijano, Illustrator

# 7.1 Measuring Cost: Which Costs Matter?



## Economic Cost versus Accounting Cost

- **accounting cost** Actual expenses plus depreciation charges for capital equipment.
- **economic cost** Cost to a firm of utilizing economic resources in production.

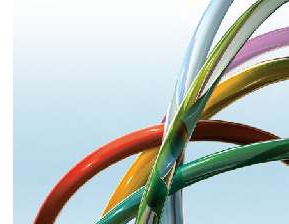
## Opportunity Cost

- **opportunity cost** Cost associated with opportunities forgone when a firm's resources are not put to their best alternative use.

The concept of opportunity cost is particularly useful in situations where alternatives that are forgone do not reflect monetary outlays.

$$\text{Economic cost} = \text{Opportunity cost}$$

## Sunk Costs



- **sunk cost** Expenditure that has been made and cannot be recovered.

Because a sunk cost cannot be recovered, it should not influence the firm's decisions.

For example, consider the purchase of specialized equipment for a plant. Suppose the equipment can be used to do only what it was originally designed for and cannot be converted for alternative use. The expenditure on this equipment is a sunk cost. *Because it has no alternative use, its opportunity cost is zero.* Thus it should not be included as part of the firm's economic costs.

A *prospective* sunk cost is an *investment*. Here the firm must decide whether that investment in specialized equipment is *economical*.

## **EXAMPLE 7.1**

### **CHOOSING THE LOCATION FOR A NEW LAW SCHOOL BUILDING**

The Northwestern University Law School has long been located in Chicago, along the shores of Lake Michigan. However, the main campus of the university is located in the suburb of Evanston. In the mid-1970s, the law school began planning the construction of a new building.

The downtown location had many prominent supporters. They argued in part that it was cost-effective to locate the new building in the city because the university already owned the land. A large parcel of land would have to be purchased in Evanston if the building were to be built there.

Does this argument make economic sense? No. It makes the common mistake of failing to appreciate opportunity cost. From an economic point of view, it is very expensive to locate downtown because the opportunity cost of the valuable lakeshore location is high: That property could have been sold for enough money to buy the Evanston land with substantial funds left over.

In the end, Northwestern decided to keep the law school in Chicago. This was a costly decision. It may have been appropriate if the Chicago location was particularly valuable to the law school, but it was inappropriate if it was made on the presumption that the downtown land had no cost.

## Fixed Costs and Variable Costs



- **total cost (TC or C)** Total economic cost of production, consisting of fixed and variable costs.
- **fixed cost (FC)** Cost that does not vary with the level of output and that can be eliminated only by shutting down.
- **variable cost (VC)** Cost that varies as output varies.

Fixed cost does not vary with the level of output—it must be paid even if there is no output. *The only way that a firm can eliminate its fixed costs is by shutting down.*

## SHUTTING DOWN



Shutting down doesn't necessarily mean going out of business.

By reducing the output of that factory to zero, the company could eliminate the costs of raw materials and much of the labor, but it would still incur the fixed costs of paying the factory's managers, security guards, and ongoing maintenance. The only way to eliminate those fixed costs would be to close the doors, turn off the electricity, and perhaps even sell off or scrap the machinery.

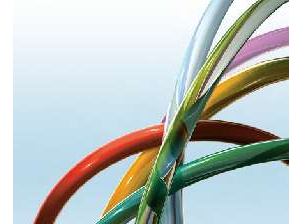
## FIXED OR VARIABLE?

How do we know which costs are fixed and which are variable?

Over a very short time horizon—say, a few months—most costs are fixed. Over such a short period, a firm is usually obligated to pay for contracted shipments of materials.

Over a very long time horizon—say, ten years—nearly all costs are variable. Workers and managers can be laid off (or employment can be reduced by attrition), and much of the machinery can be sold off or not replaced as it becomes obsolete and is scrapped.

## Fixed versus Sunk Costs



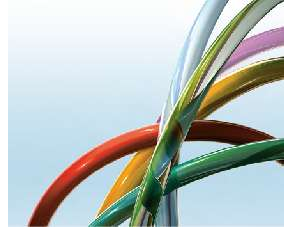
Shutting down doesn't necessarily mean going out of business. Fixed costs can be avoided if the firm shuts down a plant or goes out of business.

Sunk costs, on the other hand, are costs that have been incurred and *cannot be recovered*.

When a firm's equipment is too specialized to be of use in any other industry, most if not all of this expenditure is sunk, i.e., cannot be recovered.

Why distinguish between fixed and sunk costs? Because fixed costs affect the firm's decisions looking forward, whereas sunk costs do not. Fixed costs that are high relative to revenue and cannot be reduced might lead a firm to shut down—eliminating those fixed costs and earning zero profit might be better than incurring ongoing losses. Incurring a high sunk cost might later turn out to be a bad decision (for example, the unsuccessful development of a new product), but the expenditure is gone and cannot be recovered by shutting down. Of course a *prospective* sunk cost is different and, as we mentioned earlier, would certainly affect the firm's decisions looking forward.

## AMORTIZING SUNK COSTS



- **amortization** Policy of treating a one-time expenditure as an annual cost spread out over some number of years.

Amortizing large capital expenditures and treating them as ongoing fixed costs can simplify the economic analysis of a firm's operation. As we will see, for example, treating capital expenditures this way can make it easier to understand the tradeoff that a firm faces in its use of labor versus capital.

For simplicity, we will usually treat sunk costs in this way as we examine the firm's production decisions. When distinguishing sunk from fixed costs does become essential to the economic analysis, we will let you know.



## **EXAMPLE 7.2      SUNK, FIXED, AND VARIABLE COSTS: COMPUTERS, SOFTWARE, AND PIZZAS**

It is important to understand the characteristics of production costs and to be able to identify which costs are fixed, which are variable, and which are sunk.

Good examples include the personal computer industry (where most costs are variable), the computer software industry (where most costs are sunk), and the pizzeria business (where most costs are fixed).

Because computers are very similar, competition is intense, and profitability depends on the ability to keep costs down. Most important are the cost of components and labor.

A software firm will spend a large amount of money to develop a new application. The company can recoup its investment by selling as many copies of the program as possible.

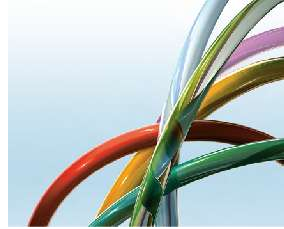
For the pizzeria, sunk costs are fairly low because equipment can be resold if the pizzeria goes out of business. Variable costs are low—mainly the ingredients for pizza and perhaps wages for a workers to produce and deliver pizzas.

# Marginal and Average Cost

TABLE 7.1 A FIRM’S COSTS

RATE OF OUTPUT (UNITS PER YEAR)	FIXED COST (DOLLARS PER YEAR)	VARIABLE COST (DOLLARS PER YEAR)	TOTAL COST (DOLLARS PER YEAR)	MARGINAL COST (DOLLARS PER UNIT)	AVERAGE FIXED COST (DOLLARS PER UNIT)	AVERAGE VARIABLE COST (DOLLARS PER UNIT)	AVERAGE TOTAL COST (DOLLARS PER UNIT)
	(FC) (1)	(VC) (2)	(TC) (3)	(MC) (4)	(AFC) (5)	(AVC) (6)	(ATC) (7)
0	50	0	50	—	—	—	—
1	50	50	100	50	50	50	100
2	50	78	128	28	25	39	64
3	50	98	148	20	16.7	32.7	49.3
4	50	112	162	14	12.5	28	40.5
5	50	130	180	18	10	26	36
6	50	150	200	20	8.3	25	33.3
7	50	175	225	25	7.1	25	32.1
8	50	204	254	29	6.3	25.5	31.8
9	50	242	292	38	5.6	26.9	32.4
10	50	300	350	58	5	30	35
11	50	385	435	85	4.5	35	39.5

## MARGINAL COST (MC)



- **marginal cost (MC)** Increase in cost resulting from the production of one extra unit of output.

Because fixed cost does not change as the firm's level of output changes, marginal cost is equal to the increase in variable cost or the increase in total cost that results from an extra unit of output. We can therefore write marginal cost as

$$MC = \Delta VC / \Delta q = \Delta TC / \Delta q$$

## AVERAGE TOTAL COST (ATC)

- **average total cost (ATC)** Firm's total cost divided by its level of output.
- **average fixed cost (AFC)** Fixed cost divided by the level of output.
- **average variable cost (AVC)** Variable cost divided by the level of output.

## 7.2 Costs in the Short Run



### The Determinants of Short-Run Cost

The change in variable cost is the per-unit cost of the extra labor  $w$  times the amount of extra labor needed to produce the extra output  $\Delta L$ . Because  $\Delta VC = w\Delta L$ , it follows that

$$MC = \Delta VC / \Delta q = w\Delta L / \Delta q$$

The extra labor needed to obtain an extra unit of output is  $\Delta L / \Delta q = 1/MP_L$ . As a result,

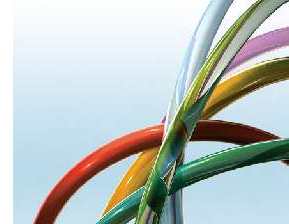
$$MC = w / MP_L \quad (7.1)$$

### DIMINISHING MARGINAL RETURNS AND MARGINAL COST

Diminishing marginal returns means that the marginal product of labor declines as the quantity of labor employed increases.

As a result, when there are diminishing marginal returns, marginal cost will increase as output increases.

# The Shapes of the Cost Curves



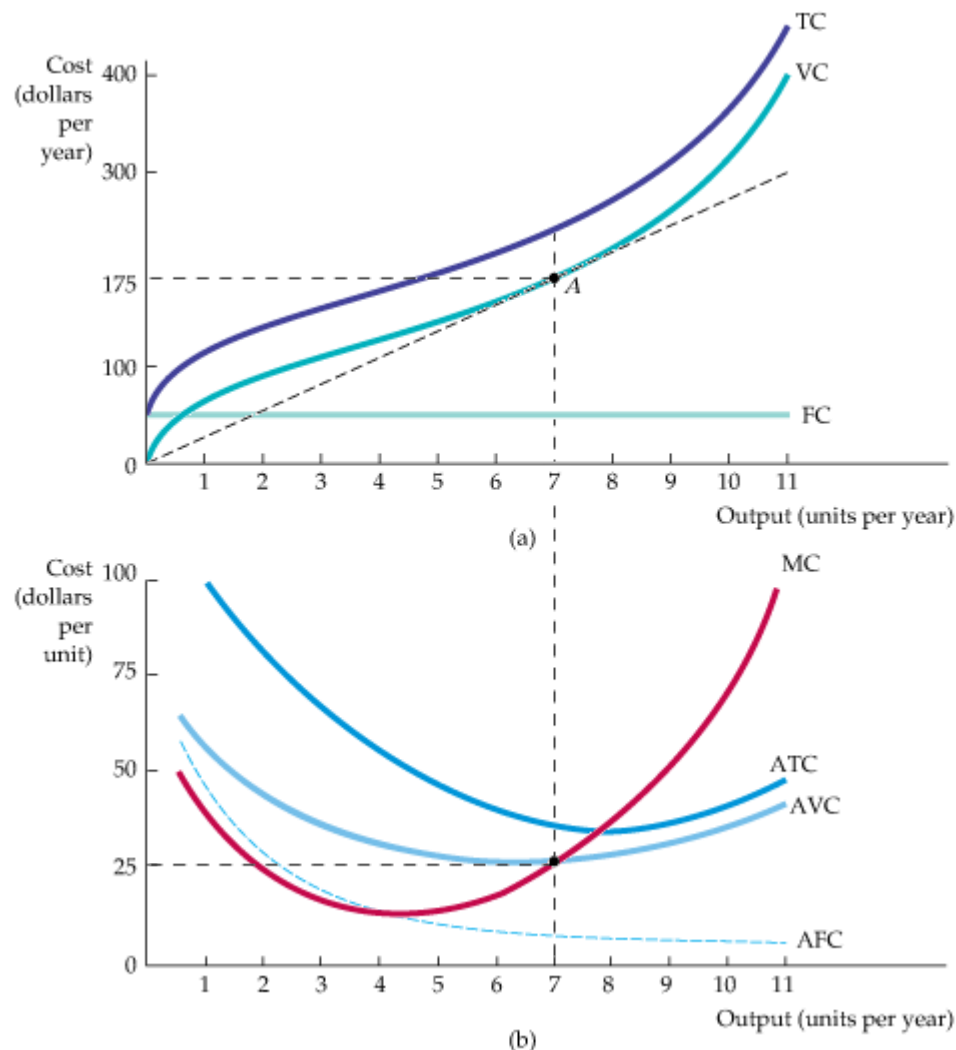
## FIGURE 7.1

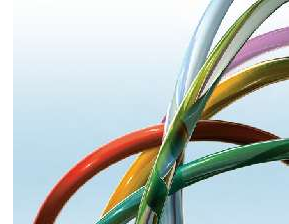
### COST CURVES FOR A FIRM

In **(a)** total cost TC is the vertical sum of fixed cost FC and variable cost VC.

In **(b)** average total cost ATC is the sum of average variable cost AVC and average fixed cost AFC.

Marginal cost MC crosses the average variable cost and average total cost curves at their minimum points.





## THE AVERAGE-MARGINAL RELATIONSHIP

Marginal and average costs are another example of the average-marginal relationship described in Chapter 6 (with respect to marginal and average product).

Because average total cost is the sum of average variable cost and average fixed cost and the AFC curve declines everywhere, the vertical distance between the ATC and AVC curves decreases as output increases.

## TOTAL COST AS A FLOW

Total cost is a flow: the firm produces a certain number of units *per year*. Thus its total cost is a flow—for example, some number of dollars per year. For simplicity, we will often drop the time reference, and refer to total cost in dollars and output in units.

Knowledge of short-run costs is particularly important for firms that operate in an environment in which demand conditions fluctuate considerably. If the firm is currently producing at a level of output at which marginal cost is sharply increasing, and if demand may increase in the future, management might want to expand production capacity to avoid higher costs.

## EXAMPLE 7.3 THE SHORT-RUN COST OF ALUMINUM SMELTING

The production of aluminum begins with the mining of bauxite. The process used to separate the oxygen atoms from aluminum oxide molecules, called smelting, is the most costly step in producing aluminum. The expenditure on a smelting plant, although substantial, is a sunk cost and can be ignored. Fixed costs are relatively small and can also be ignored.

**TABLE 7.2 PRODUCTION COSTS FOR ALUMINUM SMELTING (\$/TON)  
(BASED ON AN OUTPUT OF 600 TONS/DAY)**

PER-TON COSTS THAT ARE CONSTANT FOR ALL OUTPUT LEVELS	OUTPUT $\leq$ 600 TONS/DAY	OUTPUT $\geq$ 600 TONS/DAY
Electricity	\$316	\$316
Alumina	369	369
Other raw materials	125	125
Plant power and fuel	10	10
Subtotal	\$820	\$820
<b>PER-TON COSTS THAT INCREASE WHEN OUTPUT EXCEEDS 600 TONS/DAY</b>		
Labor	\$150	\$225
Maintenance	120	180
Freight	50	75
Subtotal	\$320	\$480
<b>Total per-ton production costs</b>	<b>\$1140</b>	<b>\$1300</b>

## EXAMPLE 7.3 THE SHORT-RUN COST OF ALUMINUM SMELTING

For an output  $q$  up to 600 tons per day, total variable cost is  $\$1140q$ , so marginal cost and average variable cost are constant at  $\$1140$  per ton. If we increase production beyond 600 tons per day by means of a third shift, the marginal cost of labor, maintenance, and freight increases from  $\$320$  per ton to  $\$480$  per ton, which causes marginal cost as a whole to increase from  $\$1140$  per ton to  $\$1300$  per ton. What happens to average variable cost when output is greater than 600 tons per day? When  $q > 600$ , total variable cost is given by:

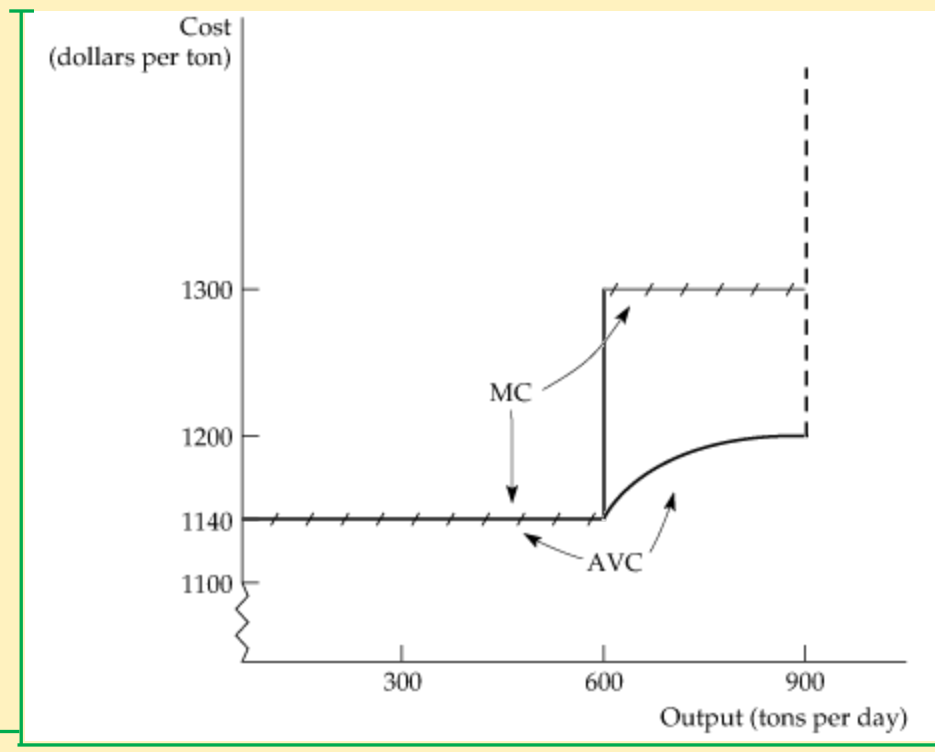
$$TVC = (1140)(600) + 1300(q - 600) = 1300q - 96,000$$

Therefore average variable cost is  $AVC = 1300 - \frac{96,000}{q}$

**FIGURE 7.2**

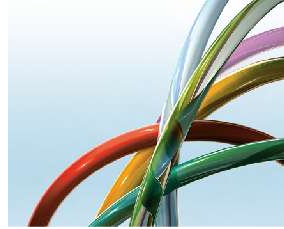
### THE SHORT-RUN VARIABLE COSTS OF ALUMINUM SMELTING

The short-run average variable cost of smelting is constant for output levels using up to two labor shifts. When a third shift is added, marginal cost and average variable cost increase until maximum capacity is reached.





## 7.3 Cost in the Long Run



### The User Cost of Capital

- **user cost of capital** Annual cost of owning and using a capital asset, equal to economic depreciation plus forgone interest.

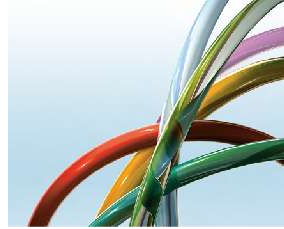
The user cost of capital is given by the *sum of the economic depreciation and the interest (i.e., the financial return) that could have been earned had the money been invested elsewhere*. Formally,

$$\text{User Cost of Capital} = \text{Economic Depreciation} + (\text{InterestRate})(\text{Value of Capital})$$

We can also express the user cost of capital as a *rate* per dollar of capital:

$$r = \text{Depreciation rate} + \text{Interest rate}$$

# The Cost-Minimizing Input Choice



We now turn to a fundamental problem that all firms face: *how to select inputs to produce a given output at minimum cost.*

For simplicity, we will work with two variable inputs: labor (measured in hours of work per year) and capital (measured in hours of use of machinery per year).

## THE PRICE OF CAPITAL

The price of capital is its user cost, given by  $r = \text{Depreciation rate} + \text{Interest rate}$ .

## THE RENTAL RATE OF CAPITAL

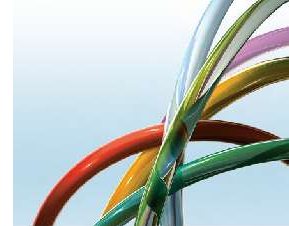
- **rental rate** Cost per year of renting one unit of capital.

If the capital market is competitive, *the rental rate should be equal to the user cost,  $r$ .* Why? Firms that own capital expect to earn a competitive return when they rent it. *This competitive return is the user cost of capital.*

*Capital that is purchased can be treated as though it were rented at a rental rate equal to the user cost of capital.*

## The Isocost Line

- **isocost line** Graph showing all possible combinations of labor and capital that can be purchased for a given total cost.



To see what an isocost line looks like, recall that the total cost  $C$  of producing any particular output is given by the sum of the firm's labor cost  $wL$  and its capital cost  $rK$ :

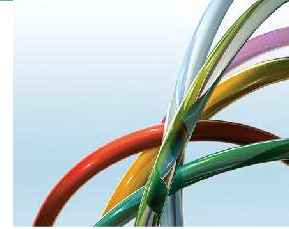
$$C = wL + rK \quad (7.2)$$

If we rewrite the total cost equation as an equation for a straight line, we get

$$K = C/r - (w/r)L$$

It follows that the isocost line has a slope of  $\Delta K/\Delta L = -(w/r)$ , which is the ratio of the wage rate to the rental cost of capital.

# Choosing Inputs



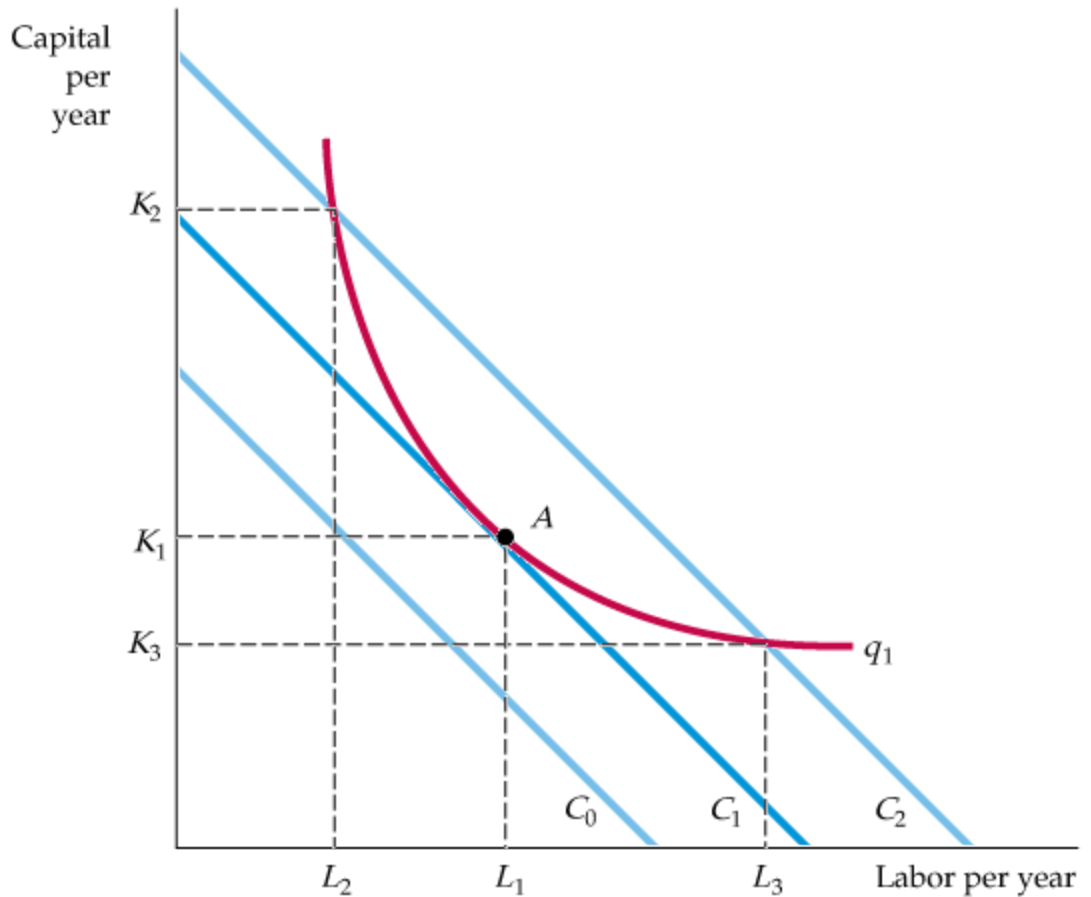
**FIGURE 7.3**

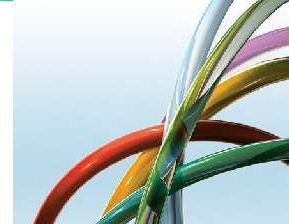
## PRODUCING A GIVEN OUTPUT AT MINIMUM COST

Isocost curves describe the combination of inputs to production that cost the same amount to the firm.

Isocost curve  $C_1$  is tangent to isoquant  $q_1$  at  $A$  and shows that output  $q_1$  can be produced at minimum cost with labor input  $L_1$  and capital input  $K_1$ .

Other input combinations— $L_2, K_2$  and  $L_3, K_3$ —yield the same output but at higher cost.





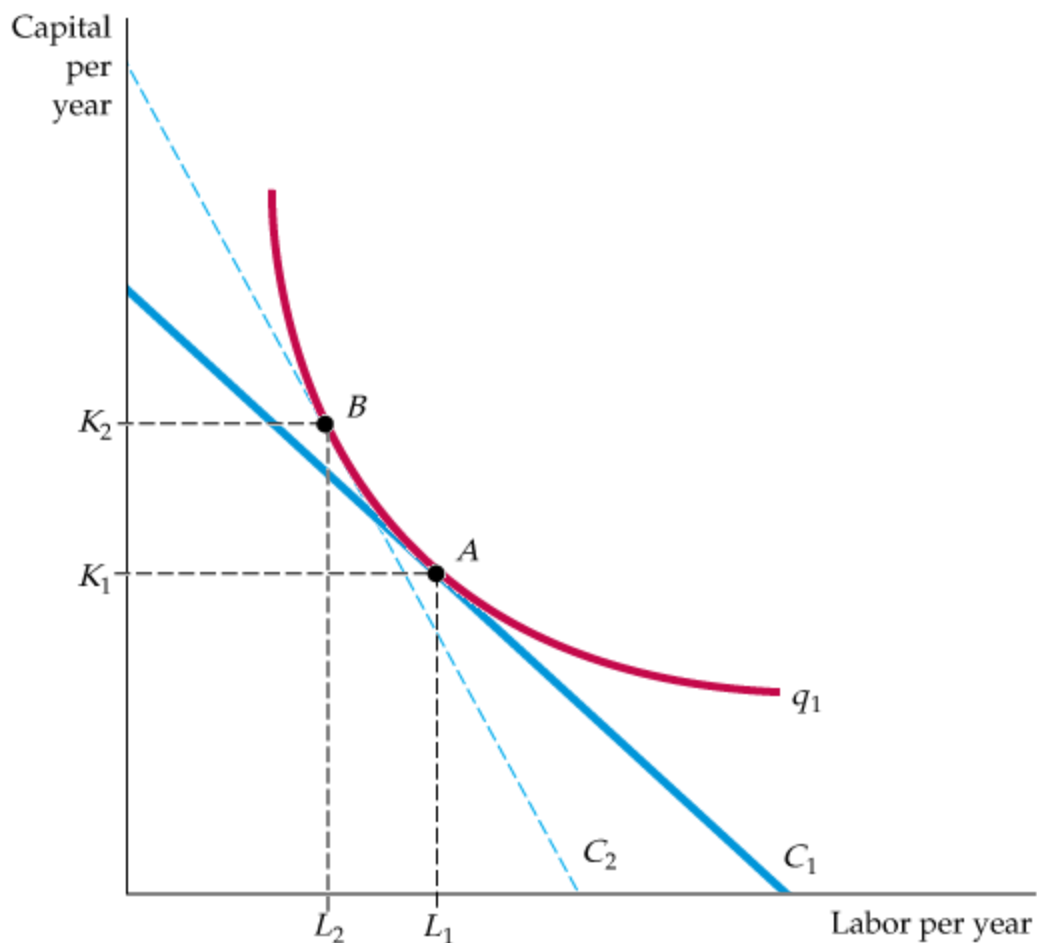
## FIGURE 7.4

### INPUT SUBSTITUTION WHEN AN INPUT PRICE CHANGES

Facing an isocost curve  $C_1$ , the firm produces output  $q_1$  at point  $A$  using  $L_1$  units of labor and  $K_1$  units of capital.

When the price of labor increases, the isocost curves become steeper.

Output  $q_1$  is now produced at point  $B$  on isocost curve  $C_2$  by using  $L_2$  units of labor and  $K_2$  units of capital.





Recall that in our analysis of production technology, we showed that the marginal rate of technical substitution of labor for capital (MRTS) is the negative of the slope of the isoquant and is equal to the ratio of the marginal products of labor and capital:

$$\text{MRTS} = -\Delta K / \Delta L = \text{MP}_L / \text{MP}_K \quad (7.3)$$

It follows that when a firm minimizes the cost of producing a particular output, the following condition holds:

$$\text{MP}_L / \text{MP}_K = w / r$$

We can rewrite this condition slightly as follows:

$$\text{MP}_L / w = \text{MP}_K / r \quad (7.4)$$

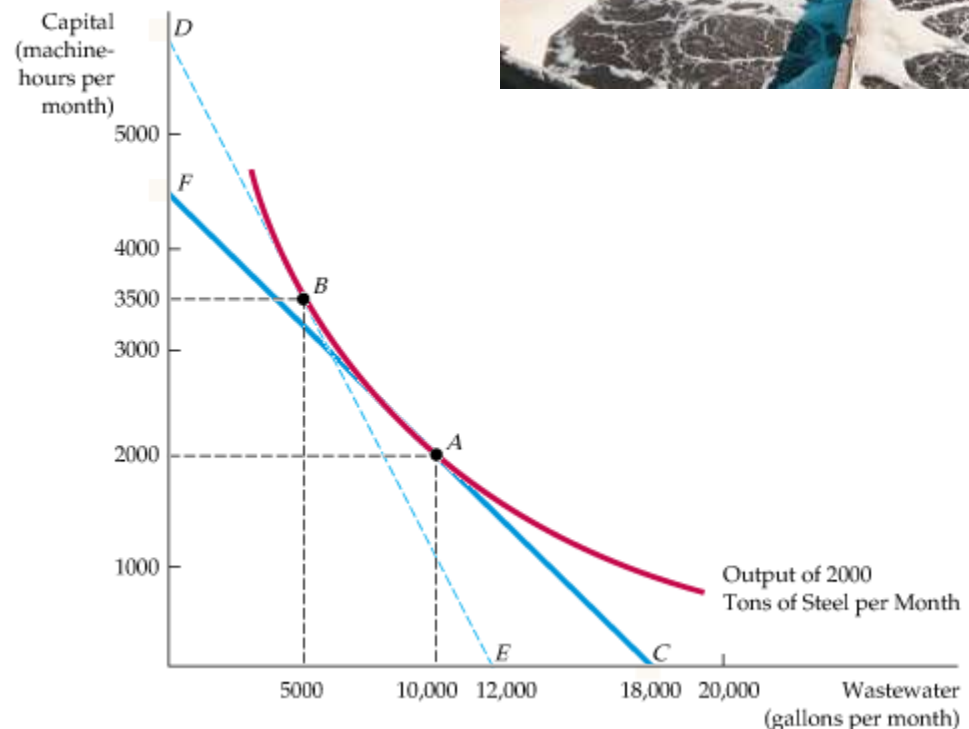
## EXAMPLE 7.4 THE EFFECT OF EFFLUENT FEES ON INPUT CHOICES

An effluent fee is a per-unit fee that the steel firm must pay for the effluent that goes into the river.

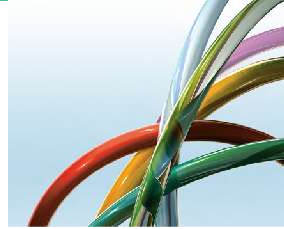
**FIGURE 7.5**  
**THE COST-MINIMIZING RESPONSE TO AN EFFLUENT FEE**

When the firm is not charged for dumping its wastewater in a river, it chooses to produce a given output using 10,000 gallons of wastewater and 2000 machine-hours of capital at *A*.

However, an effluent fee raises the cost of wastewater, shifts the isocost curve from *FC* to *DE*, and causes the firm to produce at *B*—a process that results in much less effluent.



## Cost Minimization with Varying Output Levels



- **expansion path** Curve passing through points of tangency between a firm's isocost lines and its isoquants.

The firm can hire labor  $L$  at  $w = \$10/\text{hour}$  and rent a unit of capital  $K$  for  $r = \$20/\text{hour}$ . Given these input costs, we have drawn three of the firm's isocost lines. Each isocost line is given by the following equation:

$$C = (\$10/\text{hour})(L) + (\$20/\text{hour})(K)$$

The expansion path is a straight line with a slope equal to

$$K/L = (50 - 25)/(100 - 50) = \frac{1}{2}$$

## The Expansion Path and Long-Run Costs

To move from the expansion path to the cost curve, we follow three steps:

1. Choose an output level represented by an isoquant. Then find the point of tangency of that isoquant with an isocost line.
2. From the chosen isocost line, determine the minimum cost of producing the output level that has been selected.
3. Graph the output-cost combination.

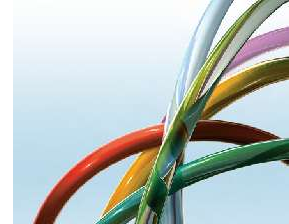
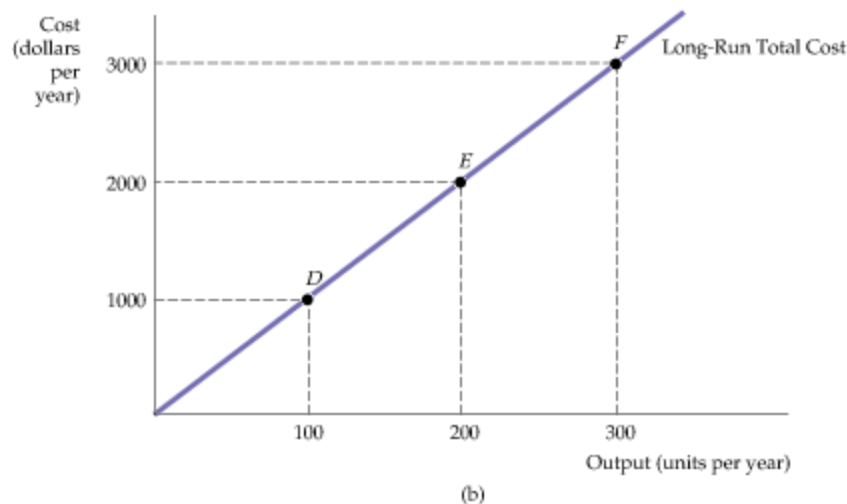
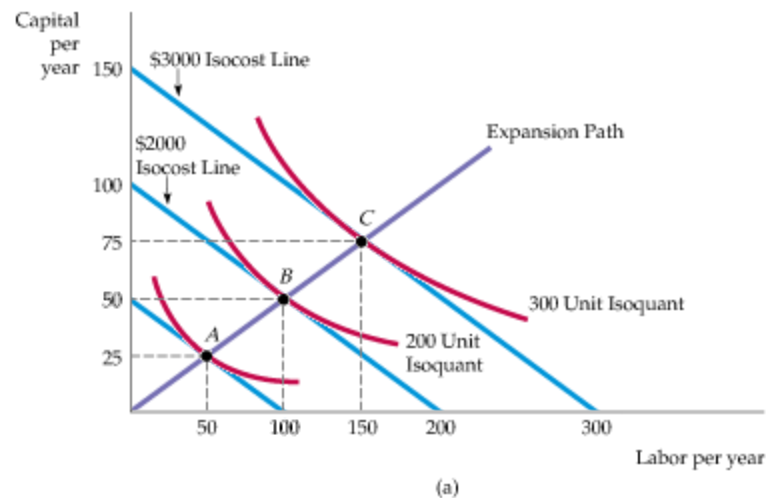


## FIGURE 7.6

### A FIRM'S EXPANSION PATH AND LONG-RUN TOTAL COST CURVE

In **(a)**, the expansion path (from the origin through points *A*, *B*, and *C*) illustrates the lowest-cost combinations of labor and capital that can be used to produce each level of output in the long run—i.e., when both inputs to production can be varied.

In **(b)**, the corresponding long-run total cost curve (from the origin through points *D*, *E*, and *F*) measures the least cost of producing each level of output.



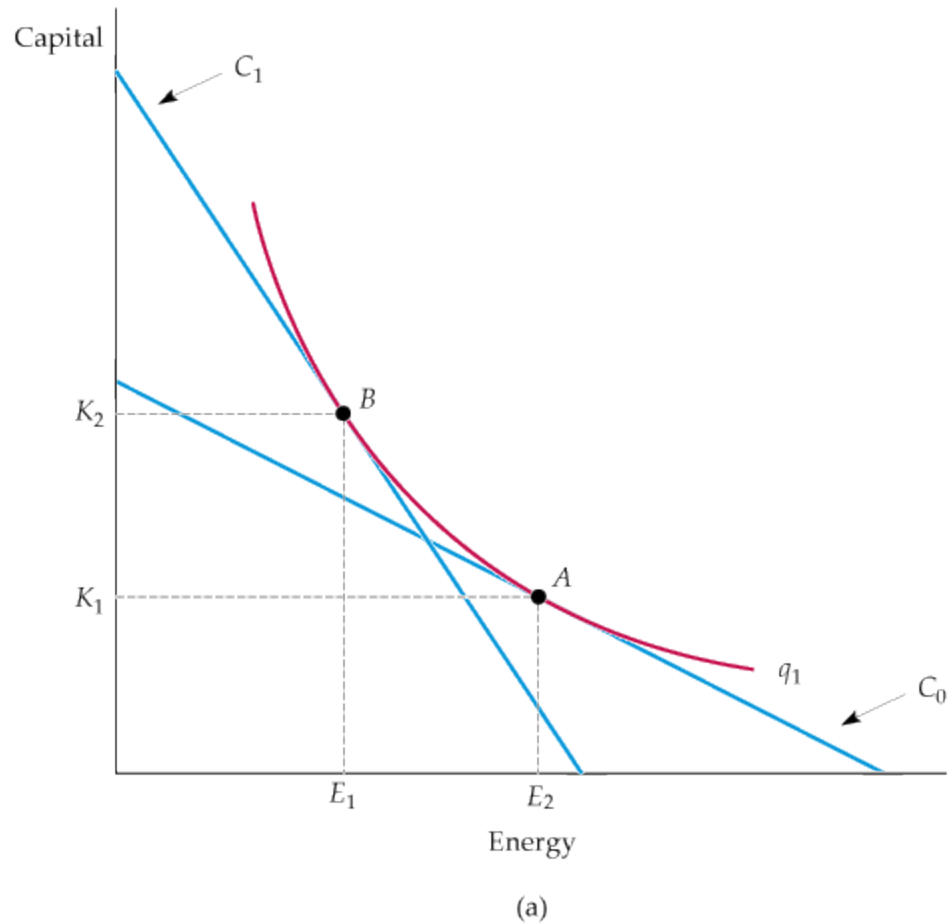
## EXAMPLE 7.5 REDUCING THE USE OF ENERGY

**FIGURE 7.7a**

### ENERGY EFFICIENCY THROUGH CAPITAL SUBSTITUTION FOR LABOR

Greater energy efficiency can be achieved if capital is substituted for energy.

This is shown as a movement along isoquant  $q_1$  from point  $A$  to point  $B$ , with capital increasing from  $K_1$  to  $K_2$  and energy decreasing from  $E_2$  to  $E_1$  in response to a shift in the isocost curve from  $C_0$  to  $C_1$ .

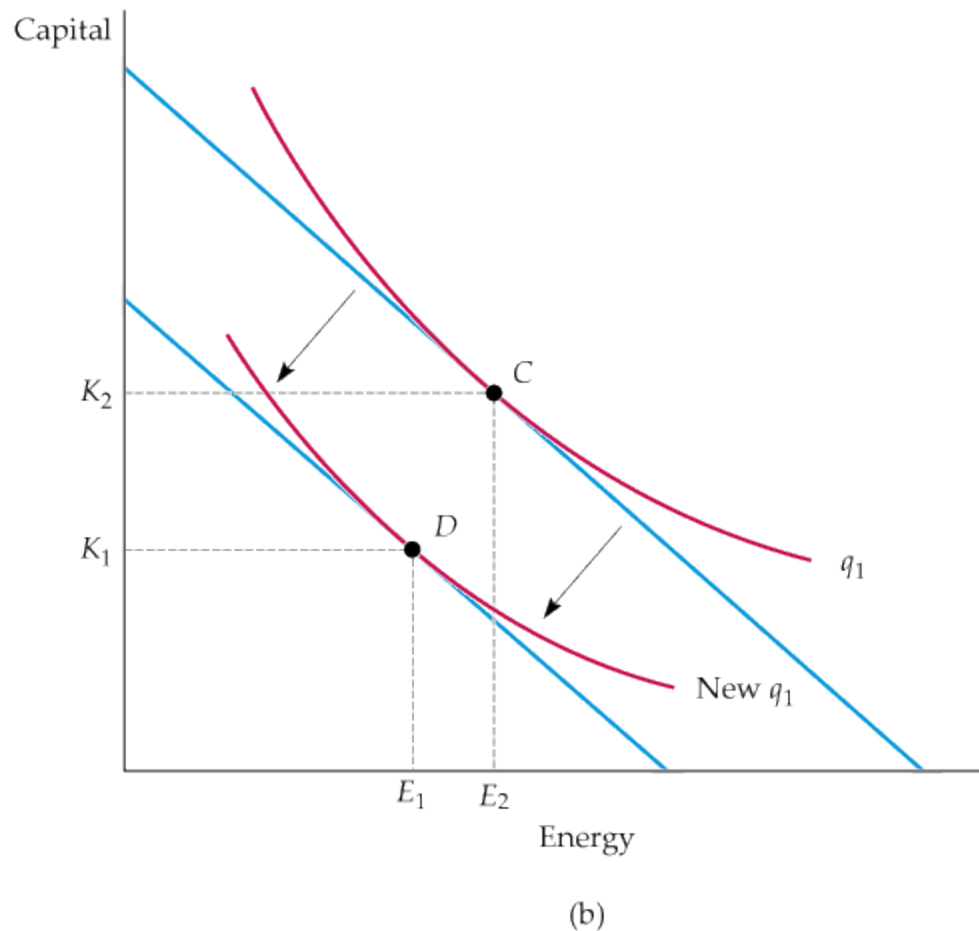


## EXAMPLE 7.5 REDUCING THE USE OF ENERGY

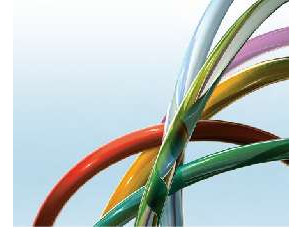
**FIGURE 7.7b**

### ENERGY EFFICIENCY THROUGH TECHNOLOGICAL CHANGE

Technological change implies that the same output can be produced with smaller amounts of inputs. Here the isoquant labeled  $q_1$  shows combinations of energy and capital that will yield output  $q_1$ ; the tangency with the isocost line at point  $C$  occurs with energy and capital combinations  $E_2$  and  $K_2$ . Because of technological change the isoquant shifts inward, so the same output  $q_1$  can now be produced with less energy and capital, in this case at point  $D$ , with energy and capital combination  $E_1$  and  $K_1$ .



# 7.4 Long-Run versus Short-Run Cost Curves



## The Inflexibility of Short-Run Production

**FIGURE 7.8**

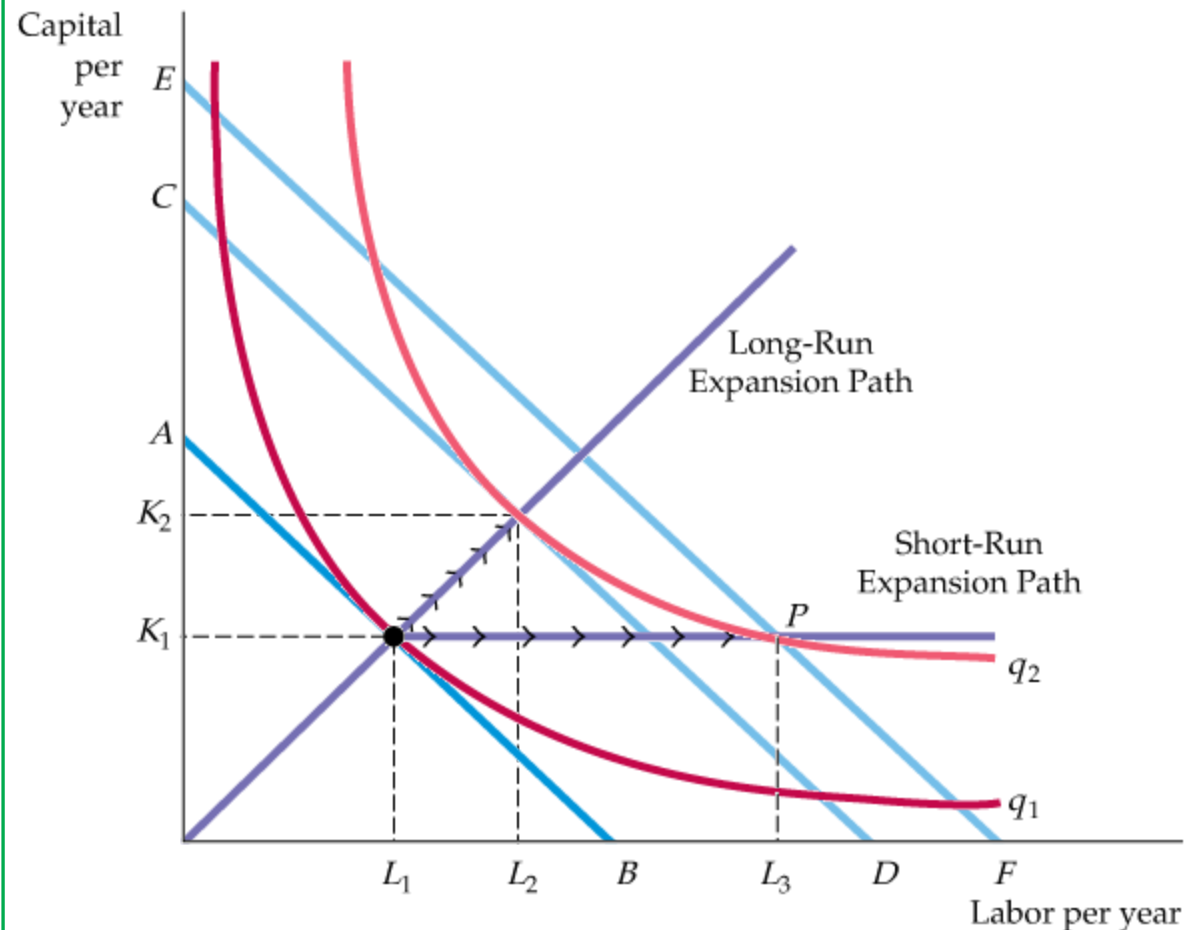
### THE INFLEXIBILITY OF SHORT-RUN PRODUCTION

When a firm operates in the short run, its cost of production may not be minimized because of inflexibility in the use of capital inputs.

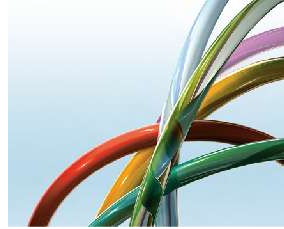
Output is initially at level  $q_1$ , (using  $L_1$ ,  $K_1$ ).

In the short run, output  $q_2$  can be produced only by increasing labor from  $L_1$  to  $L_3$  because capital is fixed at  $K_1$ .

In the long run, the same output can be produced more cheaply by increasing labor from  $L_1$  to  $L_2$  and capital from  $K_1$  to  $K_2$ .



## Long-Run Average Cost



In the long run, the ability to change the amount of capital allows the firm to reduce costs.

The most important determinant of the shape of the long-run average and marginal cost curves is the relationship between the scale of the firm's operation and the inputs that are required to minimize its costs.

- **long-run average cost curve (LAC)** Curve relating average cost of production to output when all inputs, including capital, are variable.
- **short-run average cost curve (SAC)** Curve relating average cost of production to output when level of capital is fixed.
- **long-run marginal cost curve (LMC)** Curve showing the change in long-run total cost as output is increased incrementally by 1 unit.



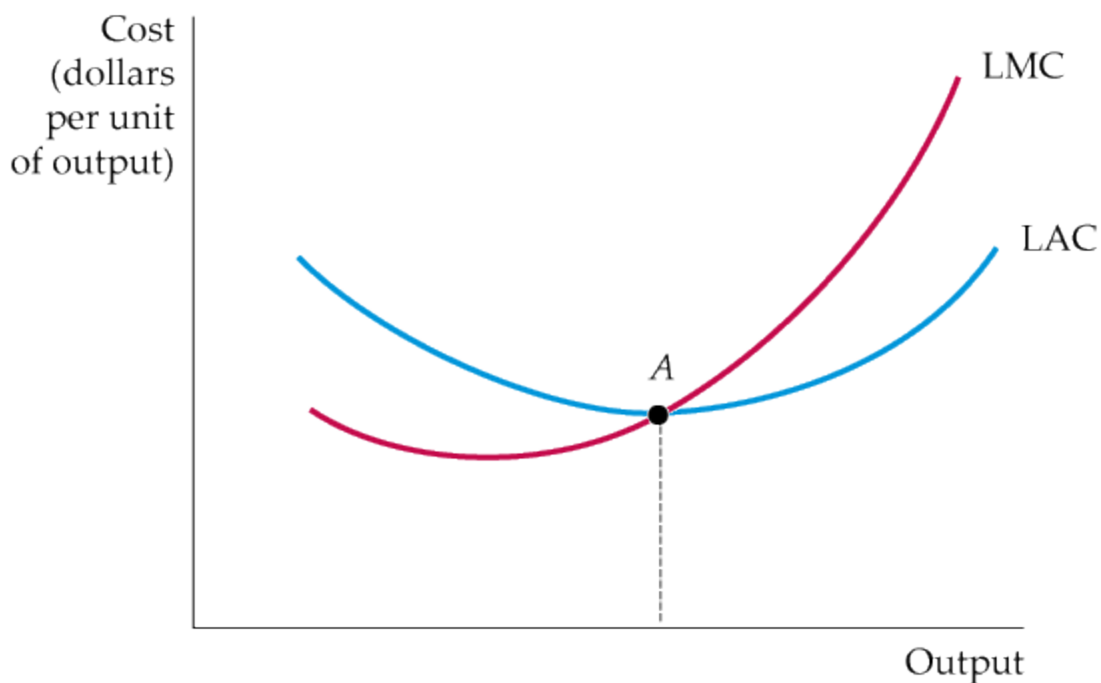
## FIGURE 7.9

### LONG-RUN AVERAGE AND MARGINAL COST

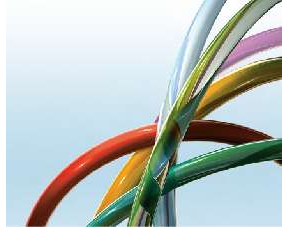
When a firm is producing at an output at which the long-run average cost LAC is falling, the long-run marginal cost LMC is less than LAC.

Conversely, when LAC is increasing, LMC is greater than LAC.

The two curves intersect at A, where the LAC curve achieves its minimum.



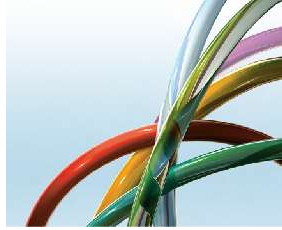
# Economies and Diseconomies of Scale



As output increases, the firm's average cost of producing that output is likely to decline, at least to a point.

This can happen for the following reasons:

1. If the firm operates on a larger scale, workers can specialize in the activities at which they are most productive.
2. Scale can provide flexibility. By varying the combination of inputs utilized to produce the firm's output, managers can organize the production process more effectively.
3. The firm may be able to acquire some production inputs at lower cost because it is buying them in large quantities and can therefore negotiate better prices. The mix of inputs might change with the scale of the firm's operation if managers take advantage of lower-cost inputs.

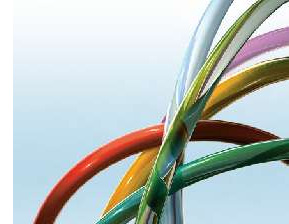


At some point, however, it is likely that the average cost of production will begin to increase with output.

There are three reasons for this shift:

1. At least in the short run, factory space and machinery may make it more difficult for workers to do their jobs effectively.
2. Managing a larger firm may become more complex and inefficient as the number of tasks increases.
3. The advantages of buying in bulk may have disappeared once certain quantities are reached. At some point, available supplies of key inputs may be limited, pushing their costs up.





- **economies of scale** Situation in which output can be doubled for less than a doubling of cost.
- **diseconomies of scale** Situation in which a doubling of output requires more than a doubling of cost.

*Increasing Returns to Scale:* Output more than doubles when the quantities of all inputs are doubled.

*Economies of Scale:* A doubling of output requires less than a doubling of cost.

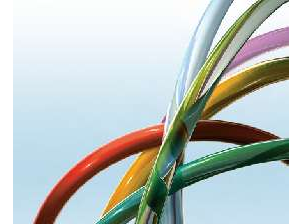
Economies of scale are often measured in terms of a cost-output elasticity,  $E_C$ .  $E_C$  is the percentage change in the cost of production resulting from a 1-percent increase in output:

$$E_C = (\Delta C/C)/(\Delta q/q) \quad (7.5)$$

To see how  $E_C$  relates to our traditional measures of cost, rewrite equation as follows:

$$E_C = (\Delta C/\Delta q)/(C/q) = MC/AC \quad (7.6)$$

# The Relationship between Short-Run and Long-Run Cost

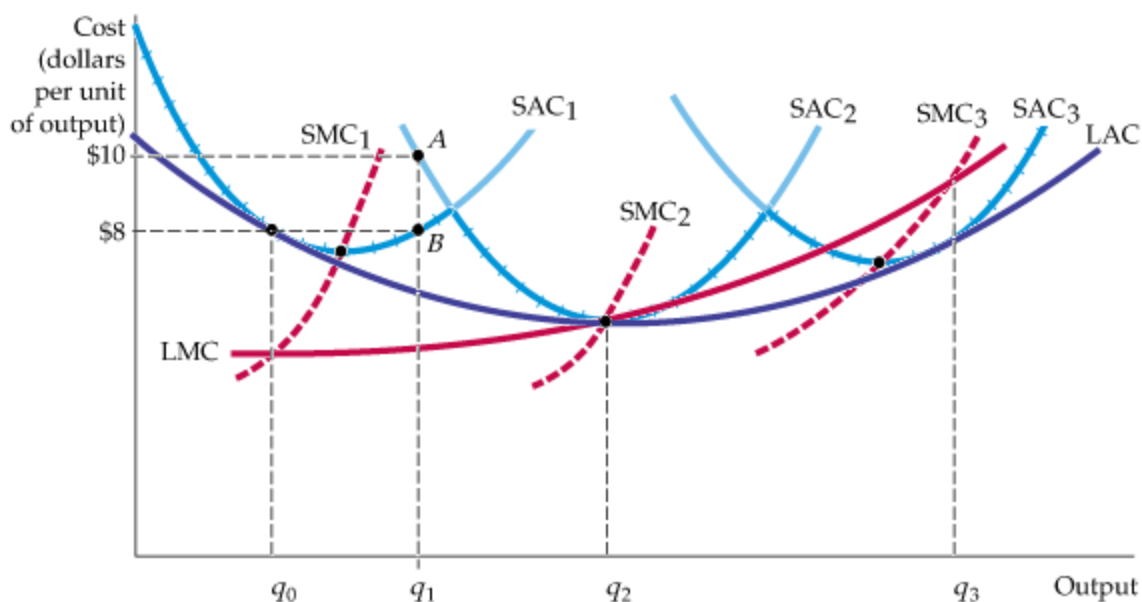


**FIGURE 7.10**

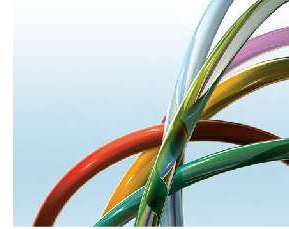
## LONG-RUN COST WITH ECONOMIES AND DISECONOMIES OF SCALE

The long-run average cost curve LAC is the envelope of the short-run average cost curves  $SAC_1$ ,  $SAC_2$ , and  $SAC_3$ .

With economies and diseconomies of scale, the minimum points of the short-run average cost curves do not lie on the long-run average cost curve.



# 7.5 Production with Two Outputs—Economies of Scope



## Product Transformation Curves

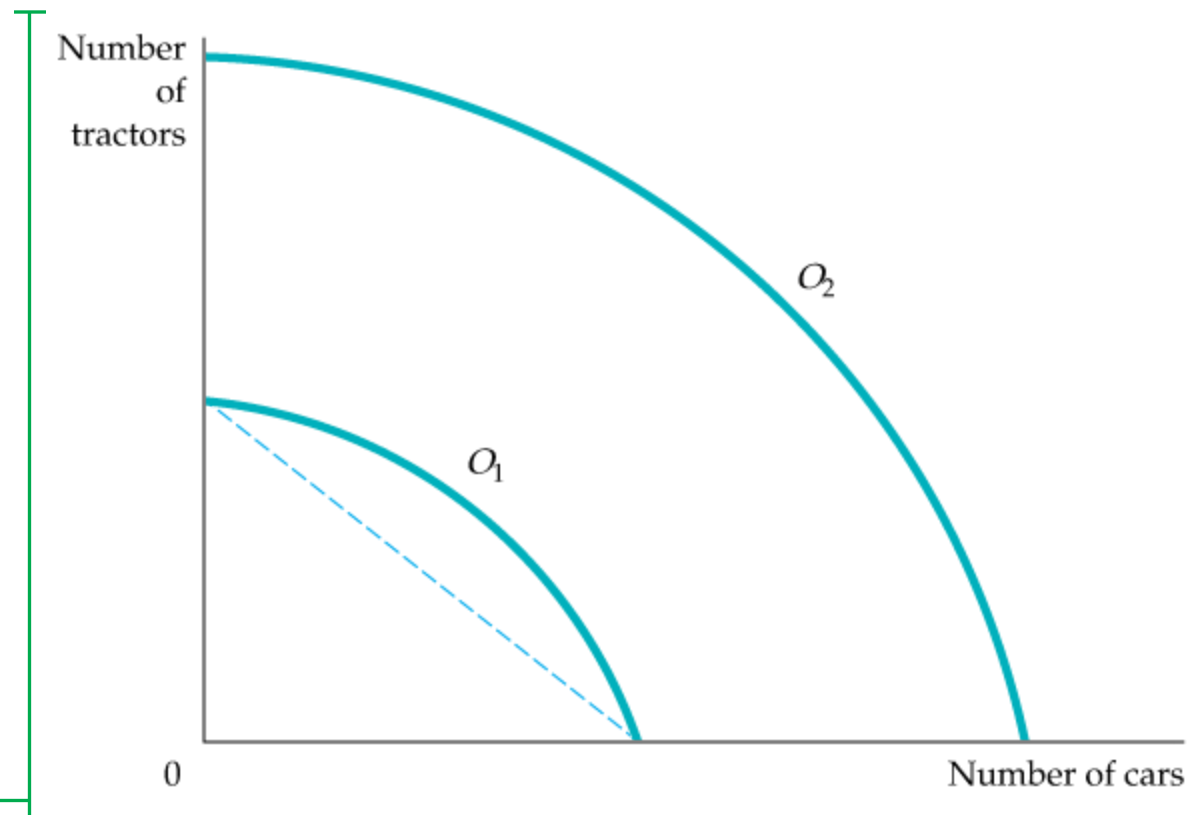
- **product transformation curve** Curve showing the various combinations of two different outputs (products) that can be produced with a given set of inputs.

**FIGURE 7.11**

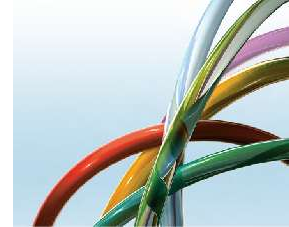
### PRODUCT TRANSFORMATION CURVE

The product transformation curve describes the different combinations of two outputs that can be produced with a fixed amount of production inputs.

The product transformation curves  $O_1$  and  $O_2$  are bowed out (or concave) because there are economies of scope in production.



## Economies and Diseconomies of Scope



- **economies of scope** Situation in which joint output of a single firm is greater than output that could be achieved by two different firms when each produces a single product.
- **diseconomies of scope** Situation in which joint output of a single firm is less than could be achieved by separate firms when each produces a single product.

## The Degree of Economies of Scope

To measure the *degree* to which there are economies of scope, we should ask what percentage of the cost of production is saved when two (or more) products are produced jointly rather than individually.

$$SC = \frac{C(q_1) + C(q_2) - C(q_1, q_2)}{C(q_1, q_2)} \quad (7.7)$$

- **degree of economies of scope (SC)** Percentage of cost savings resulting when two or more products are produced jointly rather than Individually.

## **EXAMPLE 7.6     ECONOMIES OF SCOPE IN THE TRUCKING INDUSTRY**

In the trucking business, several related products can be offered, depending on the size of the load and the length of the haul. This range of possibilities raises questions about both economies of scale and economies of scope.

The scale question asks whether large-scale, direct hauls are more profitable than individual hauls by small truckers. The scope question asks whether a large trucking firm enjoys cost advantages in operating direct quick hauls and indirect, slower hauls.

Because large firms carry sufficiently large truckloads, there is usually no advantage to stopping at an intermediate terminal to fill a partial load.

Because other disadvantages are associated with the management of very large firms, the economies of scope get smaller as the firm gets bigger.

The study suggests, therefore, that to compete in the trucking industry, a firm must be large enough to be able to combine loads at intermediate stopping points.



## 7.6 Dynamic Changes in Costs— The Learning Curve

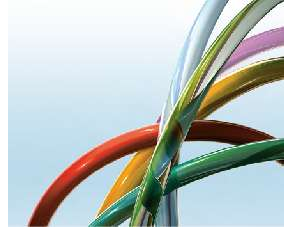


As management and labor gain experience with production, the firm's marginal and average costs of producing a given level of output fall for four reasons:

1. Workers often take longer to accomplish a given task the first few times they do it. As they become more adept, their speed increases.
2. Managers learn to schedule the production process more effectively, from the flow of materials to the organization of the manufacturing itself.
3. Engineers who are initially cautious in their product designs may gain enough experience to be able to allow for tolerances in design that save costs without increasing defects. Better and more specialized tools and plant organization may also lower cost.
4. Suppliers may learn how to process required materials more effectively and pass on some of this advantage in the form of lower costs.

● **learning curve** Graph relating amount of inputs needed by a firm to produce each unit of output to its cumulative output.

# Graphing the Learning Curve

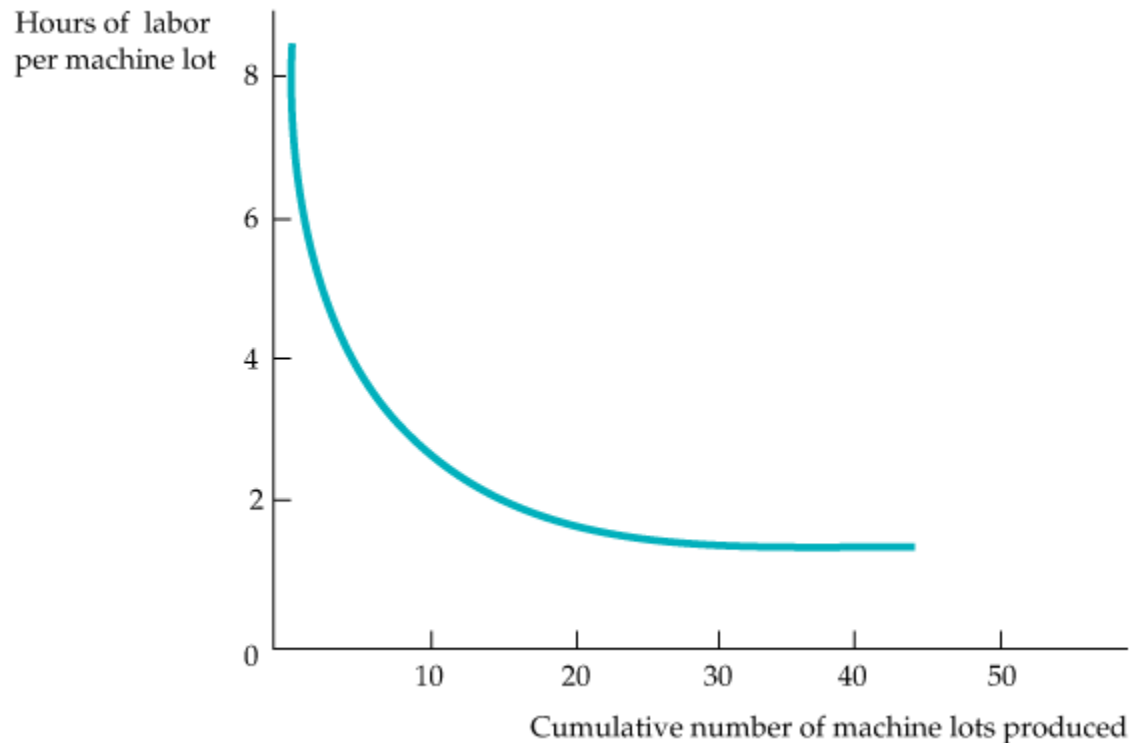


**FIGURE 7.12**

## THE LEARNING CURVE

A firm's production cost may fall over time as managers and workers become more experienced and more effective at using the available plant and equipment.

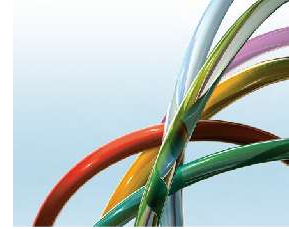
The learning curve shows the extent to which hours of labor needed per unit of output fall as the cumulative output increases.



The learning curve is based on the relationship

$$L = A + BN^{-\beta} \quad (7.8)$$

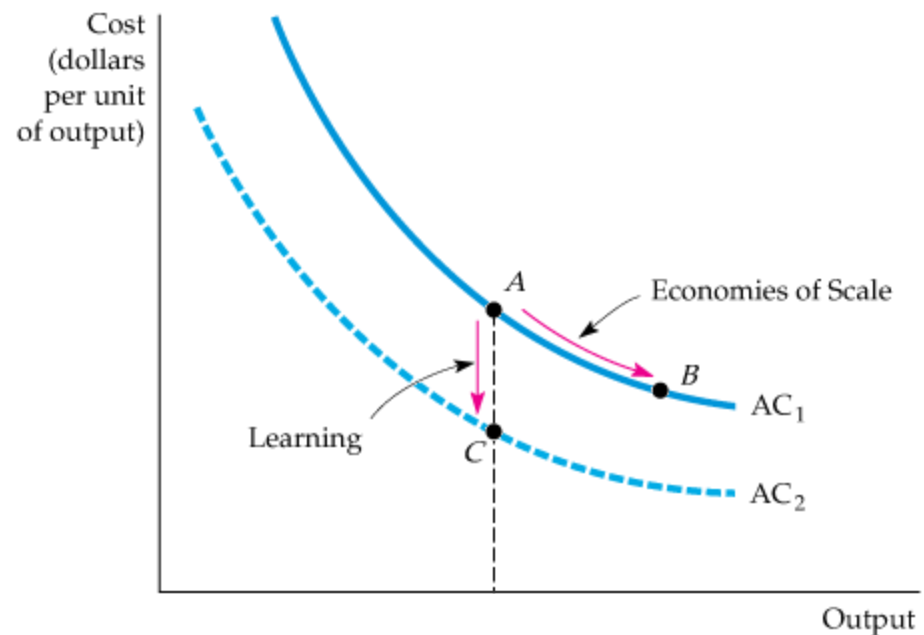
# Learning versus Economies of Scale



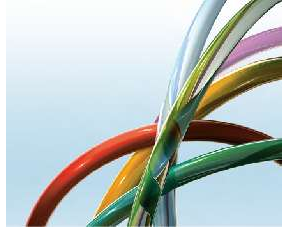
**FIGURE 7.13**

## ECONOMIES OF SCALE VERSUS LEARNING

A firm's average cost of production can decline over time because of growth of sales when increasing returns are present (a move from  $A$  to  $B$  on curve  $AC_1$ ), or it can decline because there is a learning curve (a move from  $A$  on curve  $AC_1$  to  $C$  on curve  $AC_2$ ).







A firm producing machine tools knows that its labor requirement per machine for the first 10 machines is 1.0, the minimum labor requirement  $A$  is equal to zero, and  $\beta$  is approximately equal to 0.32. Table 7.3 calculates the total labor requirement for producing 80 machines.

**TABLE 7.3** PREDICTING THE LABOR REQUIREMENTS OF PRODUCING A GIVEN OUTPUT

CUMULATIVE OUTPUT ( $N$ )	PER-UNIT LABOR REQUIREMENT FOR EACH 10 UNITS OF OUTPUT ( $L$ )*	TOTAL LABOR REQUIREMENT
10	1.00	10.0
20	.80	18.0 = (10.0 + 8.0)
30	.70	25.0 = (18.0 + 7.0)
40	.64	31.4 = (25.0 + 6.4)
50	.60	37.4 = (31.4 + 6.0)
60	.56	43.0 = (37.4 + 5.6)
70	.53	48.3 = (43.0 + 5.3)
80	.51	53.4 = (48.3 + 5.1)

\*The numbers in this column were calculated from the equation  $\log(L) = -0.322 \log(N/10)$ , where  $L$  is the unit labor input and  $N$  is cumulative output.

## EXAMPLE 7.7 THE LEARNING CURVE IN PRACTICE

Learning-curve effects can be important in determining the shape of long-run cost curves and can thus help guide management decisions. Managers can use learning-curve information to decide whether a production operation is profitable and, if so, how to plan how large the plant operation and the volume of cumulative output need be to generate a positive cash flow.

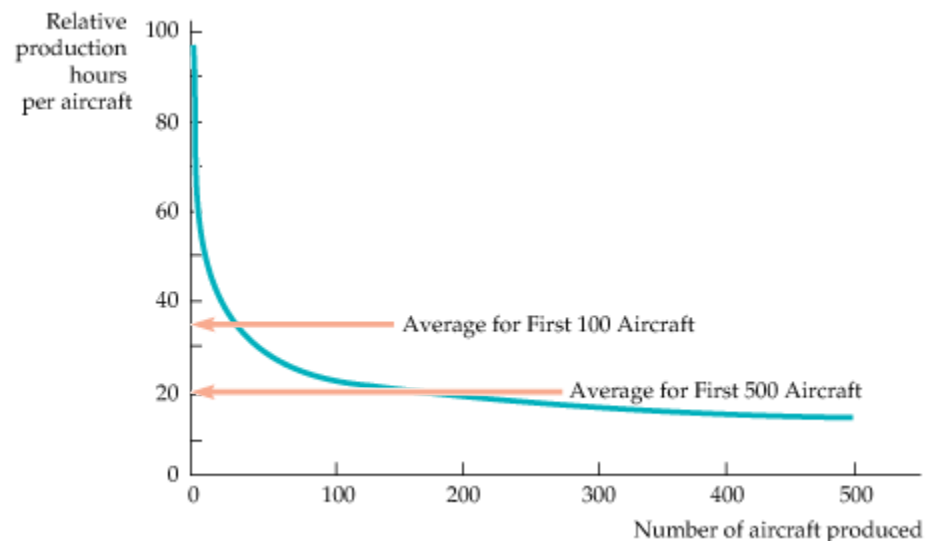


**FIGURE 7.14**

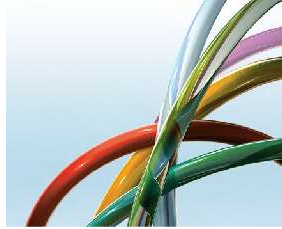
### LEARNING CURVE FOR AIRBUS INDUSTRIE

The learning curve relates the labor requirement per aircraft to the cumulative number of aircraft produced.

As the production process becomes better organized and workers gain familiarity with their jobs, labor requirements fall dramatically.



## 7.7 Estimating and Predicting Cost



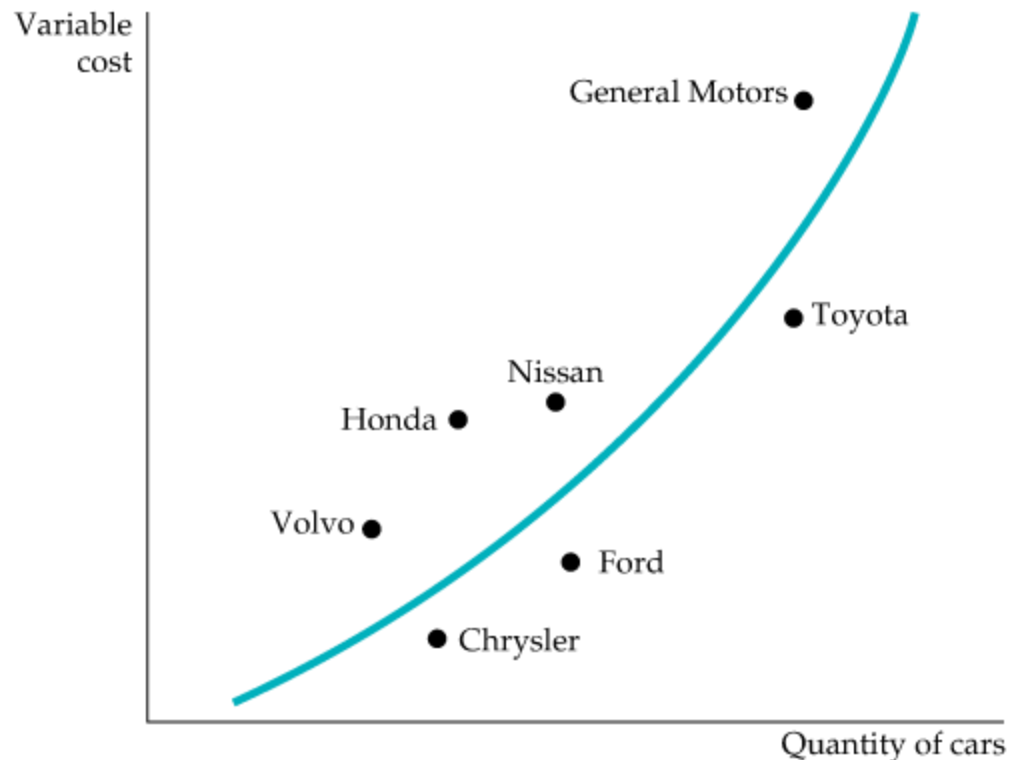
- **cost function** Function relating cost of production to level of output and other variables that the firm can control.

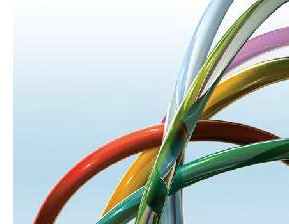
**FIGURE 7.15**

### VARIABLE COST CURVE FOR THE AUTOMOBILE INDUSTRY

An empirical estimate of the variable cost curve can be obtained by using data for individual firms in an industry.

The variable cost curve for automobile production is obtained by determining statistically the curve that best fits the points that relate the output of each firm to the firm's variable cost of production.





To predict cost accurately, we must determine the underlying relationship between variable cost and output. The curve provides a reasonably close fit to the cost data.

But what shape is the most appropriate, and how do we represent that shape algebraically?

Here is one cost function that we might choose:

$$VC = \beta q \quad (7.9)$$

If we wish to allow for a U-shaped average cost curve and a marginal cost that is not constant, we must use a more complex cost function. One possibility is the *quadratic* cost function, which relates variable cost to output and output squared:

$$VC = \beta q + \gamma q^2 \quad (7.10)$$

If the marginal cost curve is not linear, we might use a cubic cost function:

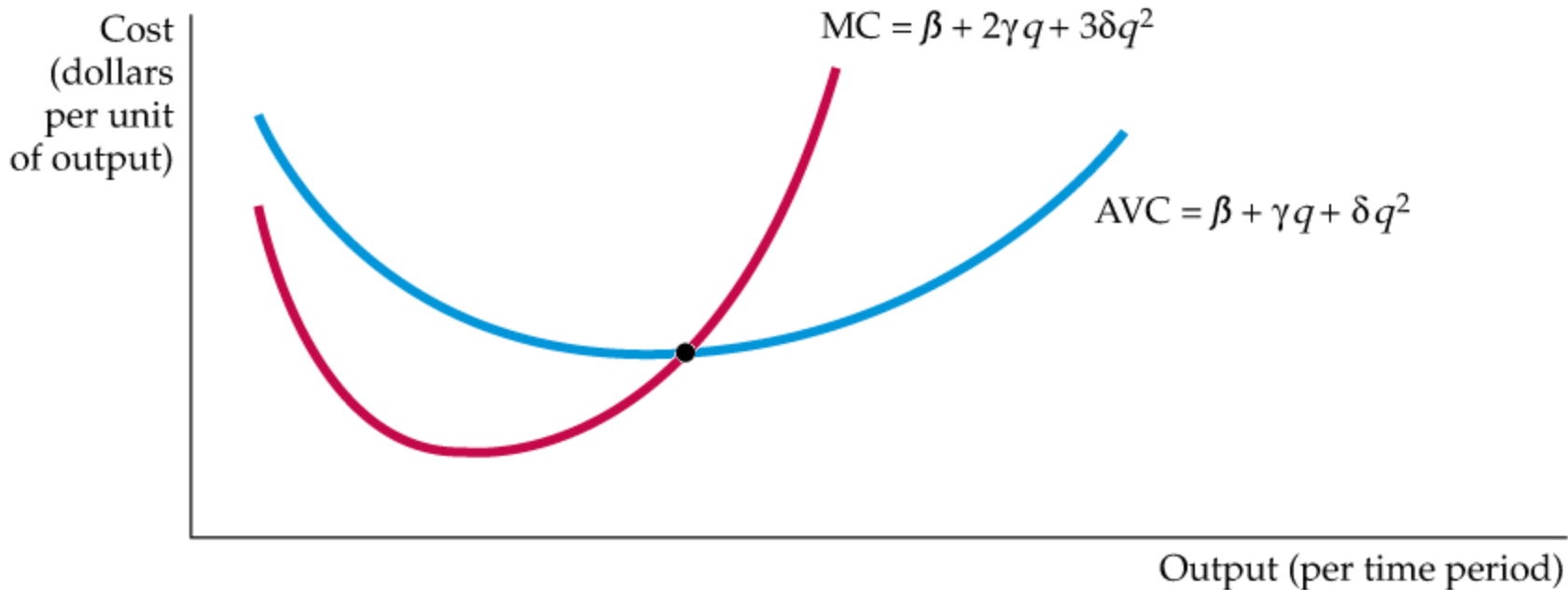
$$VC = \beta q + \gamma q^2 + \delta q^3 \quad (7.11)$$

## Cost Functions and the Measurement of Scale Economies

The *scale economies index* (SCI) provides an index of whether or not there are scale economies.

SCI is defined as follows:

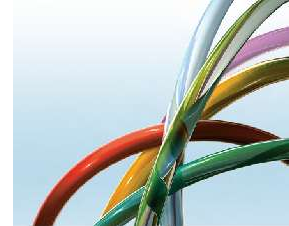
$$SCI = 1 - E_C \quad (7.12)$$



**FIGURE 7.16**

### CUBIC COST FUNCTION

A cubic cost function implies that the average and the marginal cost curves are U-shaped.



## EXAMPLE 7.8 COST FUNCTIONS FOR ELECTRIC POWER

In 1955, consumers bought 369 billion kilowatt-hours (kwh) of electricity; in 1970 they bought 1083 billion.

Was this increase due to economies of scale or to other factors?

If it was the result of economies of scale, it would be economically inefficient for regulators to “break up” electric utility monopolies.

The cost of electric power was estimated by using a cost function that is somewhat more sophisticated than the quadratic and cubic functions discussed earlier.

Table 7.4 shows the resulting estimates of the scale economies index. The results are based on a classification of all utilities into five size categories, with the median output (measured in kilowatt-hours) in each category listed.



**TABLE 7.4 SCALE ECONOMIES IN THE ELECTRIC POWER INDUSTRY**

Output (million kwh)	43	338	1109	2226	5819
Value of SCI, 1955	.41	.26	.16	.10	.04

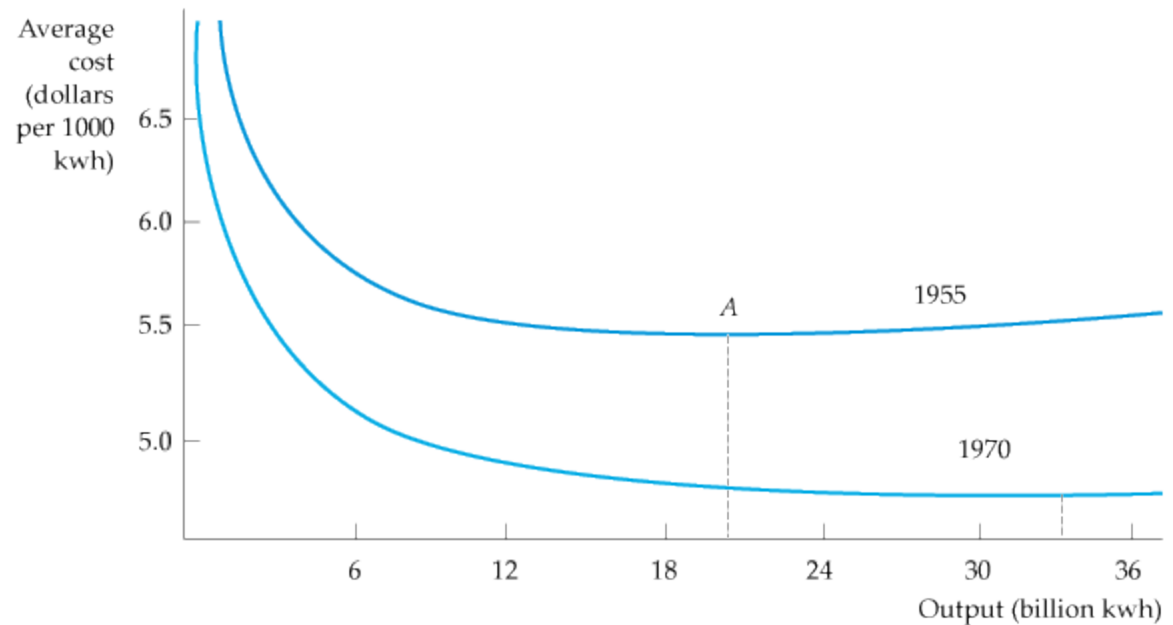
## EXAMPLE 7.8 COST FUNCTIONS FOR ELECTRIC POWER

**FIGURE 7.17**

### AVERAGE COST OF PRODUCTION IN THE ELECTRIC POWER INDUSTRY

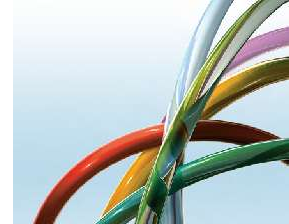
The average cost of electric power in 1955 achieved a minimum at approximately 20 billion kilowatt-hours.

By 1970 the average cost of production had fallen sharply and achieved a minimum at an output of more than 33 billion kilowatt-hours.



# Appendix to Chapter 7

## Production and Cost Theory—A Mathematical Treatment



### Cost Minimization

If there are two inputs, capital  $K$  and labor  $L$ , the production function  $F(K, L)$  describes the maximum output that can be produced for every possible combination of inputs. Writing the marginal product of capital and labor as  $MP_K(K, L)$  and  $MP_L(K, L)$ , respectively, it follows that

$$MP_K(K, L) = \frac{\partial F(K, L)}{\partial K} > 0, \quad \frac{\partial^2 F(K, L)}{\partial K^2} < 0$$

$$MP_L(K, L) = \frac{\partial F(K, L)}{\partial L} > 0, \quad \frac{\partial^2 F(K, L)}{\partial L^2} < 0$$

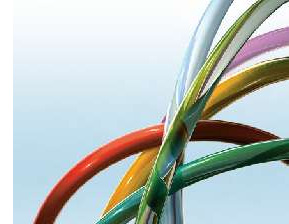
The cost-minimization problem can be written as

$$\text{Minimize } C = wL + rK \tag{A7.1}$$

subject to the constraint that a fixed output  $q_0$  be produced:

$$F(K, L) = q_0 \tag{A7.2}$$





- **Step 1:** Set up the Lagrangian.

$$\Phi = wL + rK - \lambda[F(K, L) - q_0] \quad (\text{A7.3})$$

- **Step 2:** Differentiate the Lagrangian with respect to K, L, and  $\lambda$  and set equal to zero.

$$\begin{aligned} \partial\Phi/\partial K &= r - \lambda MP_K(K, L) = 0 \\ \partial\Phi/\partial L &= w - \lambda MP_L(K, L) = 0 \\ \partial\Phi/\partial\lambda &= q_0 - F(K, L) = 0 \end{aligned} \quad (\text{A7.4})$$

- **Step 3:** Combine the first two conditions in (A7.4) to obtain

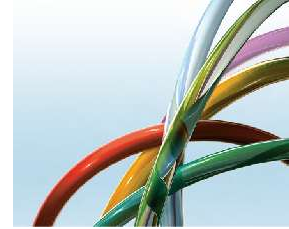
$$MP_K(K, L) / r = MP_L(K, L) / w \quad (\text{A7.5})$$

Rewrite the first two conditions in (A7.4) to evaluate the Lagrange multiplier:

$$\begin{aligned} r - \lambda MP_K(K, L) &= 0 \Rightarrow \lambda = \frac{r}{MP_K(K, L)} \\ w - \lambda MP_L(K, L) &= 0 \Rightarrow \lambda = \frac{w}{MP_L(K, L)} \end{aligned} \quad (\text{A7.6})$$

$r/MP_K(K, L)$  measures the additional input cost of producing an additional unit of output by increasing capital, and  $w/MP_L(K, L)$  the additional cost of using additional labor as an input. In both cases, the Lagrange multiplier is equal to the marginal cost of production.

## Marginal Rate of Technical Substitution



Write the isoquant:  $MP_K(K, L)dK + MP_LdL = dq = 0$  (A7.7)

Rearrange terms:  $-dK/dL = MRTS_{LK} = MP_L(K, L) / MP_K(K, L)$  (A7.8)

Rewrite the condition given by (A7.5) to get

$$MP_L(K, L)/MP_K(K, L) = w/r \quad (\text{A7.9})$$

Rewrite (A7.9):  $MP_L/w = MP_K/r$  (A7.10)

## Duality in Production and Cost Theory

The dual problem asks what combination of  $K$  and  $L$  will let us produce the most output at a cost of  $C_0$ .

$$\text{Maximize } F(K, L) \text{ subject to } wL + rK = C_0 \quad (\text{A7.11})$$

- **Step 1:** Set up the Lagrangian.

$$\Phi = F(K, L) - \mu(wL + rK - C_0) \quad (\text{A7.12})$$

- **Step 2:** Differentiate the Lagrangian with respect to  $K$ ,  $L$ , and  $\mu$  and set equal to zero:

$$\partial\Phi/\partial K = MP_K(K, L) - \mu r = 0$$

$$\partial\Phi/\partial L = MP_L(K, L) - \mu w = 0$$

$$\partial\Phi/\partial\mu = wL - rK + C_0 = 0$$

(A7.13)

- **Step 3:** Combine the first two equations:

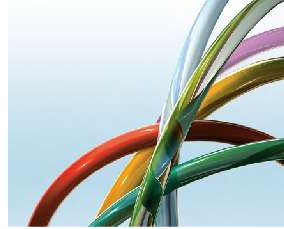
$$\mu = \frac{MP_K(K, L)}{r}$$

$$\mu = \frac{MP_L(K, L)}{w}$$

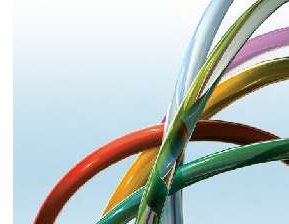
(A7.14)

$$\Rightarrow MP_K(K, L) / r = MP_L(K, L) / w$$

This is the same result as (A7.5)—that is, the necessary condition for cost minimization.



# The Cobb-Douglas Cost and Production Functions



- **Cobb-Douglas production function** Production function of the form  $q = AKL^\beta$ , where  $q$  is the rate of output,  $K$  is the quantity of capital, and  $L$  is the quantity of labor, and where  $A$ ,  $\alpha$ , and  $\beta$  are positive constants.

$$F(K, L) = AK^\alpha L^\beta$$

We assume that  $\alpha < 1$  and  $\beta < 1$ , so that the firm has decreasing marginal products of labor and capital.<sup>2</sup> If  $\alpha + \beta = 1$ , the firm has *constant returns to scale*, because doubling  $K$  and  $L$  doubles  $F$ . If  $\alpha + \beta > 1$ , the firm has *increasing returns to scale*, and if  $\alpha + \beta < 1$ , it has *decreasing returns to scale*.

To find the amounts of capital and labor that the firm should utilize to minimize the cost of producing an output  $q_0$ , we first write the Lagrangian

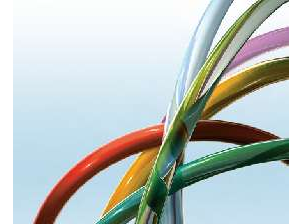
$$\Phi = wL + rK - \lambda(AK^\alpha L^\beta - q_0) \quad (\text{A7.15})$$

Differentiating with respect to  $L$ ,  $K$ , and  $\lambda$ , and setting those derivatives equal to 0, we obtain

$$\partial\Phi/\partial L = w - \lambda(\beta AK^\alpha L^{\beta-1}) = 0 \quad (\text{A7.16})$$

$$\partial\Phi/\partial K = r - \lambda(\alpha AK^{\alpha-1} L^\beta) = 0 \quad (\text{A7.17})$$

$$\partial\Phi/\partial\lambda = AK^\alpha L^\beta - q_0 = 0 \quad (\text{A7.18})$$



From equation (A7.16) we have

$$\lambda = w/A\beta K^\alpha L^{\beta-1} \quad (\text{A7.19})$$

Substituting this formula into equation (A7.17) gives us

$$r\beta AK^\alpha L^{\beta-1} = w\alpha AK^{\alpha-1} L^\beta \quad (\text{A7.20})$$

or

$$L = \frac{\beta r}{\alpha w} K \quad (\text{A7.21})$$

A7.21 is the expansion path. Now use Equation (A7.21) to substitute for  $L$  in equation (A7.18):

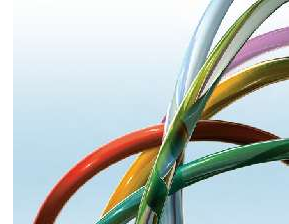
$$AK^\alpha \left( \frac{\beta r}{\alpha w} K \right)^\beta - q_0 = 0 \quad (\text{A7.22})$$

We can rewrite the new equation as:

$$K^{\alpha+\beta} = \left( \frac{\alpha w}{\beta r} \right)^\beta \frac{q_0}{A} \quad (\text{A7.23})$$

or

$$K = \left( \frac{\alpha w}{\beta r} \right)^{\frac{\beta}{\alpha+\beta}} \left( \frac{q_0}{A} \right)^{\frac{1}{\alpha+\beta}} \quad (\text{A7.24})$$



(A7.24) is the factor demand for capital. To determine the cost-minimizing quantity of labor, we simply substitute equation (A7.24) into equation (A7.21):

$$L = \frac{\beta r}{\alpha w} K = \frac{\beta r}{\alpha w} \left[ \left( \frac{\alpha w}{\beta r} \right)^{\frac{\beta}{\alpha+\beta}} \left( \frac{q_0}{A} \right)^{\frac{1}{\alpha+\beta}} \right] \quad (\text{A7.25})$$

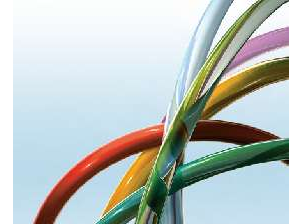
$$L = \left( \frac{\beta r}{\alpha w} \right)^{\frac{\alpha}{\alpha+\beta}} \left( \frac{q_0}{A} \right)^{\frac{1}{\alpha+\beta}}$$

The total cost of producing *any output*  $q$  can be obtained by substituting equations (A7.24) for  $K$  and (A7.25) for  $L$  into the equation  $C = wL + rK$ . After some algebraic manipulation we find that

$$C = w^{\beta/(\alpha+\beta)} r^{\alpha/(\alpha+\beta)} \left[ \left( \frac{\alpha}{\beta} \right)^{\beta/(\alpha+\beta)} + \left( \frac{\alpha}{\beta} \right)^{-\alpha/(\alpha+\beta)} \right] \frac{q^{1/(\alpha+\beta)}}{A} \quad (\text{A7.26})$$

This *cost function* tells us (1) how the total cost of production increases as the level of output  $q$  increases, and (2) how cost changes as input prices change. When  $\alpha + \beta$  equals 1, equation (A7.26) simplifies to

$$C = w^{\beta} r^{\alpha} [(\alpha/\beta)^{\beta} + (\alpha/\beta)^{-\alpha}] (1/A) q \quad (\text{A7.27})$$



The firm's cost function contains many desirable features. To appreciate this fact, consider the special constant returns to scale cost function (A7.27). Suppose that we wish to produce  $q_0$  in output but are faced with a doubling of the wage. How should we expect our costs to change? New costs are given by

$$C_1 = (2w)^\beta r^\alpha \left[ \left( \frac{\alpha}{\beta} \right)^\beta + \left( \frac{\alpha}{\beta} \right)^{-\alpha} \right] \left( \frac{1}{A} \right) q_0 = 2^\beta \underbrace{w^\beta r^\alpha \left[ \left( \frac{\alpha}{\beta} \right)^\beta + \left( \frac{\alpha}{\beta} \right)^{-\alpha} \right] \left( \frac{1}{A} \right) q_0}_{C_0} = 2^\beta C_0$$

If a firm suddenly had to pay more for labor, it would substitute away from labor and employ more of the relatively cheaper capital, thereby keeping the increase in total cost in check.