Cournot

March 1, 2014
10:29 PM

market Demand

$P = 100 - Q \qquad Q = \sum_{1}^{n} q_i$

where q_i = output for firm i

n = # of firms

Cost function for each firm

$C(q_i) = K + 40 q_i$

$MC = C'(q_i) = 40 \qquad K$ = fixed costs

$n = 1$ monopoly

$TR = PQ = 100Q - Q^2$

$Q = q_1$ (market Q = firm q for monopoly)

$\Pi = TR - TC$

$\pi = 100Q - Q^2 - K - 40Q$

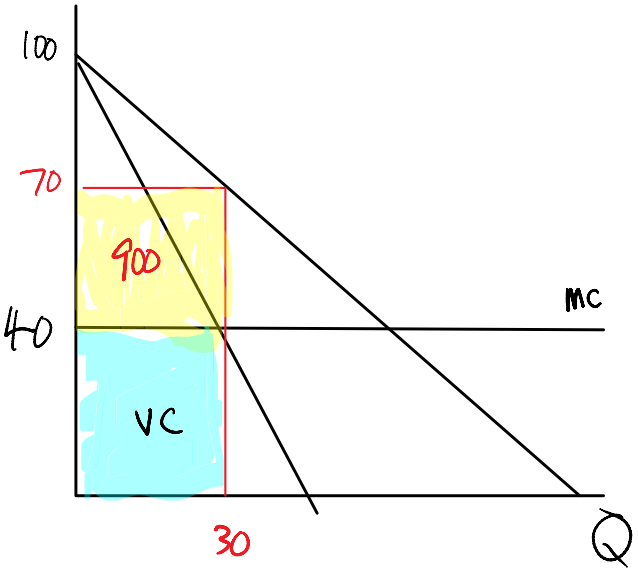
$\frac{d\pi}{dQ} = 100 - 2Q - 40 = 0$

$Q = 30$

$P = 100 - Q = 70$

$\pi = 900 - K$

if $K < 900$ $\pi > 0$



$n = 2$ Duopoly

$Q = q_1 + q_2$

$P = 100 - Q \rightarrow P = 100 - q_1 - q_2$

Profit per firm

$\pi_1 = Pq_1 - K - 40q_1$

$\pi_2 = Pq_2 - K - 40q_2$

where $P = 100 - q - q$

Where $P = 100 - q_1 - q_2$

substitute into π_1 & π_2

Firm 1

$\pi_1 = (100 - q_1 - q_2)q_1 - K - 40q_1$

simplify

$\pi_1 = 100q_1 - q_1^2 - q_1q_2 - K - 40q_1$

$\pi_1 = 60q_1 - q_1^2 - q_1q_2 - K$

Firm 2

$\pi_2 = (100 - q_1 - q_2)q_2 - K - 40q_2$

simplify

$\pi_2 = 100q_2 - q_1q_2 - q_2^2 - K - 40q_2$

$\pi_2 = 60q_2 - q_2^2 - q_1q_2 - K$

Finding Equilibrium

First, we make the "Nash" assumption that each firm will Maximize Profit, treating the other firm's output as a constant

Firm 1

$$\pi_1 = 60q_1 - q_1^2 - q_1q_2 - K$$

$$\frac{d\pi}{dq_1} = 60 - 2q_1 - q_2 = 0$$

$$2q_1 = 60 - q_2$$

$$q_1 = 30 - \tfrac{1}{2}q_2$$

This is firm 1's Best response function

Best Response

Tells firm 1 its profit Max output for ANY q_2 that firm 2 chooses

i.e.

if $q_2 = 0$ $q_1 = 30$ (monopoly output)

if $q_2 = 60$ $q_1 = 0$ (60 drives Price to 40)

Firm 2

$$\pi_2 = 60q_2 - q_2^2 - q_1 q_2 - K$$

$$\frac{d\pi}{dq_2} = 60 - 2q_2 - q_1 = 0$$

$$q_2 = 30 - \tfrac{1}{2} q_1$$ Firm 2's Best Response

Equilibrium

Best Response functions give us 2 equations and 2 unknowns

(1) $q_1 = 30 - \tfrac{1}{2} q_2$

(2) $q_2 = 30 - \tfrac{1}{2} q_1$

sub (2) into (1)

$$q_1 = 30 - \tfrac{1}{2}\left[30 - \tfrac{1}{2} q_1\right]$$ (q_2)

$$q_1 = 30 - 15 + \tfrac{1}{4} q_1$$

$$q_1 = 15 + \frac{1}{4}q_1$$

$$q_1 - \tfrac{1}{4}q_1 = 15$$

$$\frac{3}{4}q_1 = 15 \qquad q_1 = \left(\frac{4}{3}\right)(15) = 20$$

sub $q_1 = 20$ into 2's Best Response

$$q_2 = 30 - \tfrac{1}{2}q_1 = 30 - \tfrac{1}{2}(20) = 20$$

Therefore

$$q_1 = 20, \; q_2 = 20 \qquad Q = q_1 + q_2 = 40$$

Price is

$$P = 100 - Q \qquad P = 100 - (40)$$

$$P = 60$$

$$\pi_1 = 400 - K$$

$$\pi_2 = 400 - K$$

Note if $K < 300$
Both firms profits > 0

but if $400 < K < 900$, then monopolist makes profit but Duopoly makes losses

monopoly $\pi < 100 - K$ monopoly

monopoly vs Duopoly equilibrium Graphically