12

UNDIFFERENTIATED OLIGOPOLY

Chapter Summary

Interdependence of Oligopolists

The "theory" of **oligopoly** does not have quite the elegant simplicity of the theories of monopoly and perfect competition. When one considers the definition of oligopoly—competition among the few—the reason for the relative complexity of oligopoly models becomes evident. In monopoly, only one firm, unthreatened by entry, produces in the market; in competition, many small firms produce, and no one firm perceives itself as capable of affecting market conditions. An oligopoly involves more than one firm, but not so many that a firm's actions can go unnoticed by its rivals. It is this **interdependence** among rivals' actions that distinguishes oligopoly from the other two market structures.

How should we model that interdependence? How do firms think in the real world? Does a firm believe that its rival will not react to its pricing strategy? At the other extreme, does it think that its rival will follow its every action? Does that behavior change if one firm is established in the market and the other firm is a new entrant in the market? And what exactly do firms choose: price, quantity, advertising, or something else?

Short-Run Models

These questions suggest some of the many aspects, absent from monopoly and competition, that one must consider when modeling an oligopolistic market. Two short-run models of **noncooperative** behavior—the **Cournot** and **Bertrand** models—differ in the choice variable of the firms. Cournot firms choose quantity; Bertrand firms choose price. In both cases, a **Nash equilibrium** is found in which each firm's strategy is profit-maximizing given the other firm's strategy, that is, firms are **individually rational.** As the number of firms increases in the Cournot model, the competitive solution is reached; however, in the Bertrand model, the equilibrium, characterized by price equals marginal cost, is invariant to the number of firms.

If firms were to behave **cooperatively** and collude on their choice of quantity and price, the best they could achieve would be the monopoly solution. If the profits were equally divided, all firms in the market would be better off than in either the Cournot or Bertrand noncooperative solution (assuming identical cost conditions for the firms). This collusive arrangement is **collectively rational** but individually irrational in that each firm has the incentive to cheat by producing more output than allotted under collusion or by shaving its price.

A well-known example of this conflict between individual and collective rationality is the **prisoners' dilemma game.** If the games played by noncooperative agents were repeated, and the agents were to devise **credible punishments** providing sufficient disincentives to cheat, then the collusive solution could be both individually and collectively rational.

Endogenous Market Structure

When market structure becomes endogenous—that is, when the number of firms is no longer fixed—the models of oligopoly become more complicated and fascinating. Under the assumptions of Cournot behavior and of knowledge of the "rules of the game" by all existing and potential firms in the industry, market structure is easily endogenized in this model through the introduction of **setup costs**, costs required to enter the market. From the perspective of an **established firm** in the market, set-up costs are **sunk** and have no effect on the firms' profit-making decisions. However, from the point of view of the **potential entrant**, the setup costs are avoidable by not entering and hence are important to the firm's entry decision. Moreover, the nature of the oligopoly behavior, if entry occurs, affects the decision to enter: The more cooperative the oligopoly is after entry, the more attractive entry will be. For example, in the case of Bertrand behavior, a "second" firm will never find entry profitable for positive setup costs; hence, monopoly is always predicted to arise.

Strategic Behavior

The established firm can be more aggressive and can behave strategically in such a way that a potential rival will not find entry profitable. Sylos and Labini suggest one way: An established firm can set a **limit output**; that is, the smallest value of industry output such that there is no profitable output level that the entrant can produce. The limit output, though possibly exceeding the monopoly output, may be profitable for the established firm to produce if it is entry-deterring. Although the Sylos postulate is instructive in strategic behavior by established firms, it is based on the unattractive assumption that the potential entrants take as given the current output of the established firm as an indication of the future output that will be produced by the established firm. This threat is not necessarily **credible**; that is, if the entrant *actually* enters despite the established firm's **threat**, the established firm may find a different output choice more profitable.

This brings us to recent developments in the economics literature. If an established firm makes high profits over a period of time, it follows that some barriers to entry must be in place protecting these profits. Structural features such as economies of scale and large research costs provide **natural barriers**; alternatively, incumbent firms can take deliberate actions to prohibit entry. Spence distinguishes between two types of actions. First, a firm may want to **position** itself prior to the entry of a rival in a way that will make its threats credible. For example, to indicate that it will produce more output if entry occurs, the firm may purchase additional equipment or carry excess inventories; or to show it will fight a price war, the firm may incur some expenditures on advertising specific to a price war before the war is actually fought. Second, firms can also **react** to rivals after entry occurs.

Key Words

Bertrand model
Best response function
Collusion
Cournot model
Credible threats
Dominant strategy
Duopoly
Established firm
Individual versus collective rationality
Limit-output model
No-entry condition
Nash equilibrium
Noncooperative behavior
Oligopoly
Potential entrant
Prisoners' dilemma game
Profit matrix
Reacting versus positioning
Setup costs
Sunk costs

Case Study: The Stability of OPEC

The oil cartel OPEC sells oil in the world oil market along with many smaller firms and oil-producing countries. A reasonable approximation of actual behavior by the smaller and fringe firms in this market is that they take the price set by OPEC and do not believe that their production of oil can have a large effect on the price. Figure 12.1 shows the costs and demand conditions for this oil market. MC_f is the horizontal summation of the fringe firms' supply curves, MC_c is the marginal cost curve of the cartel, and $D(y)$ is the market demand.

A Under the assumption that OPEC is a profit-maximizing cartel, indicate the price that OPEC will set and the resulting output shares by the

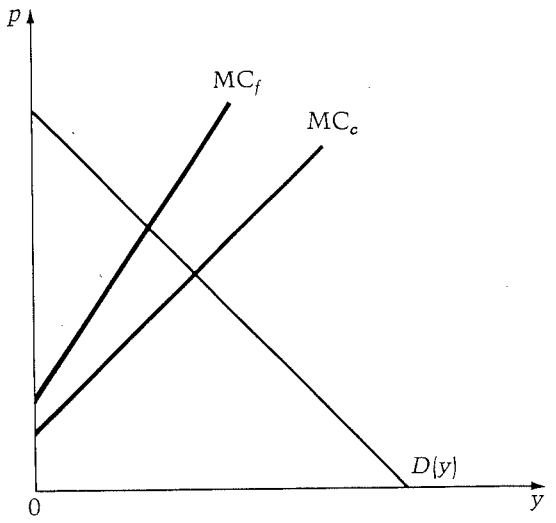

FIGURE 12.1

cartel and the fringe firms. (*Hint*: OPEC will want to take the fringe's reactions to its price announcements into account in choosing price. To do this, OPEC will subtract the output that the fringe firms will be willing to produce at every price and then will set its price using the "residual" demand that remains.)

B Given your solution in **A**, what might have been the reason for OPEC's surprise in 1976 when it discovered that the demand for oil was not as price-inelastic as had been thought?

C If the cost curves in the figure are short-run curves, what would you expect to happen in the long run if OPEC continued to set price at the short-run profit-maximizing level you found in **A**?

D In some industries, collusive aggreements among firms in the industry are stable; that is, they last for a long time. OPEC, for example, was formed in the early 1960s, dominated the oil market in the 1970s, and persisted as a strong cartel, until its recent problems began in 1985. Other collusive arrangements — for example, in the copper market — have had histories of instability. What factors contribute to the stability (or instability) of a cartel?

Exercises

Multiple-Choice

Choose the correct answer to each question. There is only one correct answer to each question.

1 The payoff matrix in Figure 12.2 gives the profits from spending either \$1000 or \$2000 on advertising to firms A and B, net of costs. The first number in each cell is the payoff to firm A, and the second number is the payoff to firm B, in thousands of dollars. The Nash equilibrium to this game is

a Firm A will spend \$1000 and firm B will spend \$2000 on advertising.

b Both firms will spend \$2000 on advertising.

c Both firms will spend \$1000 on advertising.

d Neither firm will advertise at all.

e None of the above.

2 Two firms behave as Cournot duopolists in the market for soft drinks. The firms face identical constant marginal costs of c and a market demand $p = a - by$. In equilibrium,

a The two firms will produce indentical quantities equal to $a/3b$.

		Firm B: \$1000	Firm B: \$2000
Firm A	\$1000	10, 10	1, 15
	\$2000	15, 1	3, 3

FIGURE 12.2

b The market price in this case will exceed the market price if the two firms colluded.

c The industry output will equal two-thirds the market demand at $p = c$.

d The market price will be $a/3$.

e None of the above.

3 As the number of firms in a Cournot oligopoly gets large,

a Output produced by each firm increases.

b Industry output increases, and price falls to the competitive level.

c The market price approaches the collusive price.

d The oligopolists behave in a more collectively rational manner.

e None of the above.

4 In the limit-output model,

a An entrant can be deterred from entering the market even if it faces the identical costs faced by the established firm.

b An entrant can be deterred from entering the market if output by the established firm is sufficiently high.

c An entrant responds to the residual demand, which is the market demand remaining after subtracting the established firm's output.

d All the above.

e Only **b** and **c**.

5 For a Bertrand oligopoly,

a The equilibrium price is equal to the monopoly price.

b The equilibrium price exceeds the price set by Cournot duopolists, assuming identical market conditions.

c The equilibrium concept is based on collectively rational behavior.

d The equilibrium price equals the competitive price.

e None of the above.

6 Figure 12.3 shows an entrant firm's long-run average cost curve and the market demand. Under the Sylos postulate, if the established firm sets an output level equal to $\bar{y}$,

a The potential entrant will enter the market between output levels y_1 and y_2.

b The potential entrant will not enter the market because the long-run average cost exceeds the residual demand at every quantity.

c The potential entrant will enter the market because the entrant does not believe the established firm will continue to produce $\bar{y}$.

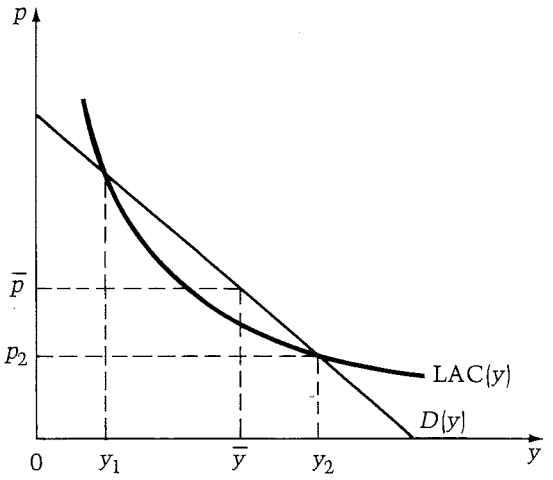

FIGURE 12.3

d The potential entrant will enter the market and price will fall to p_2.

e None of the above.

To answer questions **7** and **8**, consider Figure 12.4, which shows the reaction functions of two Cournot firms. Each firm has marginal costs equal to 0; market demand is $p = 1 - y$.

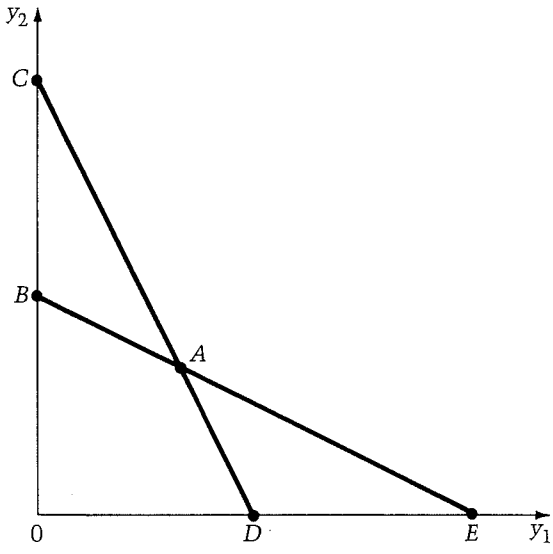

FIGURE 12.4

7 In Figure 12.4,

a Firm 1's reaction function is CD; firm 2's reaction function is BE.

b Firm 1's reaction function is BE; firm 2's reaction function is CD.

c Distances $0B$ and $0D$ equal the monopoly output, $y = \frac{1}{2}$.

d Both **a** and **c**.

e Both **b** and **c**.

8 Suppose that only firm 2's marginal costs increase to $\frac{1}{4}$. Then, which of the following will occur?

a Firm 2's reaction curve will shift downward.

b Firm 1's reaction curve will shift upward.

c The new equilibrium set of outputs (y_1^*, y_2^*) will lie along CD, with $y_2^* < y_1^*$.

d Both **a** and **c**.

e Both **b** and **c**.

*9 For a Cournot oligopoly with n firms facing constant marginal costs c and market demand $p = a - by$, the equilibrium price will be

a $(a + cn)/(n + 1)$

b $(a - c)n/b(n + 1)$

c c

d $a/2$

e None of the above

True-False

10 Industry output in a duopoly model of Cournot behavior exceeds the industry output under collusive behavior.

11 The larger the setup costs faced by the entrant, the larger will be the limit output chosen by the established firm in the Sylos model.

12 The more firms that produce in a Cournot oligopoly, the larger is the industry output.

13 If firms in a Cournot oligopoly collude, the incentives to cheat increase as the number of cartel members increases.

14 The equilibrium price in a Bertrand oligopoly falls as the number of firms increases.

15 The larger the setup costs of entering a Cournot oligopoly, the larger will be the number of firms in long-run equilibrium.

16 If the setup costs of entering a Cournot oligopoly must be incurred by new entrants, then established firms may make economic profits in long-run equilibrium.

17 The incentive to collude is greater among Bertrand oligopolists than among Cournot oligopolists, everything else held constant.

18 Bertrand pricing behavior and positive setup costs are sufficient to deter potential entrants from a market in which only one firm currently produces and earns monopoly profits.

19 The Nash equilibrium concept is a collectively rational concept.

Short Problems

20 Until 1924, the delivered price of steel in the United States was the base price in Pittsburgh plus freight costs from Pittsburgh to the customer's location. The Pittsburgh-plus-freight price would be quoted by any steel manufacturer, regardless of that particular manufacturer's location. This is called *basing-point pricing*; that is, the strategy of setting delivered prices by firms with plants in different locations equal to some common "base" price (f.o.b.) plus transport costs from *one* location. Why would firms be willing to adopt such a pricing practice?

21 Suppose that a ban on advertising is imposed in the cigarette industry, which we assume to consist of two firms. Show, using a simple game-theoretical model, how this policy may result in higher equilibrium profits for the two firms.

22 Suppose that two firms in the same market tacitly agree to share the market. Show that this agreement results in the collusive outcome when the firms' cost curves are identical.

23 Suppose that a firm in an oligopolistic industry faces the kinked demand curve for its product shown in Figure 12.5, where p^* is the current equilibrium price in the industry.

a Why might the firm believe that its demand curve has this shape?

b Explain why the price of the commodity may stay constant even if input prices, and therefore costs, change.

24 The International Air Transport Associa-

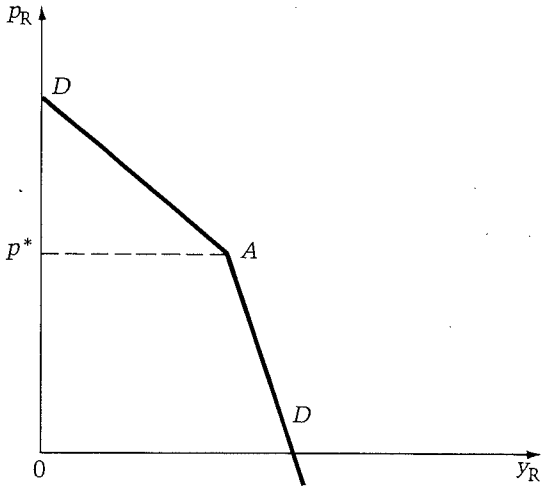

FIGURE 12.5

tion (IATA), a trade association consisting of most of the world's international airlines, sets the rates for international routes. Two basic fare levels are set by the IATA for its members to follow: regular and excursion fares. All member airlines of the IATA that fly the same route on a regularly scheduled basis are required to charge the same fares. Because IATA members cannot compete with each other in prices, many IATA member airlines offer tourist travel packages to attract customers. Offered at some price set by the airline, a travel package provides for all the needs of the tourist including the airline flight, hotel accommodation, vehicle rental, and sightseeing tours.

a Why is it in the interest of the airlines *as a group* to charge an identical price?

b Why do the IATA members offer the travel packages?

c Explain why IATA would want to check each package offered and have the right to veto any offerings.

Long Problems

25 Consider the following game. A firm, call it the established firm I, has a monopoly in the market for good Y. The demand curve for Y is given by $p = 50 - 0.5y$. The marginal costs of production from the current technology are constant and equal to 10. There are no fixed costs of production.

a Show that the monopoly profits that firm I will receive if there is no entry are equal to \$800.

b Now suppose that a substitute technology is available that can reduce the marginal production costs to \$0. The costs of developing the technology are fixed at \$550. (The firms cannot speed up development by spending more money.) Firm I and a potential entrant E are each deciding whether or not to develop the technology. If only firm I develops it, then the firm will discard the old technology and simply produce as a monopolist with the new technology. If only firm E develops it, then the firms will produce as Cournot duopolists, where firm I will have marginal costs $MC_I = 10$ and firm E will have marginal costs $MC_E = 0$. If both develop the technology, they will produce as Cournot duopolists, where both firms I and E will have marginal costs equal to 0. If no one develops it, the status quo discussed in **a** will occur. Show the following:

(1) The monopoly profits to firm I if *only* firm I develops the new technology are \$700.

(2) The Cournot profits to firms I and E if *only* E develops the technology are $\pi_I = \$200$ and $\pi_E = \$250$.

(3) The Cournot profits to firms I and E if *both* firms develop the technology are $\pi_I = \pi_E = \$5.56$.

c Given the profits in **a** and **b**, construct a payoff matrix for firms I and E with strategies "develop" (D) and "do not develop" (N). Show that if both firms move simultaneously in this game, there is only one Nash equilibrium, in which only firm E develops the technology. Give an intuitive explanation for that result.

d Would your answer in **c** change if firm I could move first in the research game? Explain. Would your answer change if firm E could move first? Explain.

26 Two firms produce two products that are perfect substitutes for each other, but the costs of production are different for the two firms. The demand and cost conditions are described by

$$p = 100 - \frac{(y_1 + y_2)}{2}$$

and

$$MC_1 = 19 \qquad MC_2 = y_2$$

where p is the market price, y_1 is the quantity

produced by firm 1, y_2 is the quantity produced by firm 2, MC_1 is the marginal cost of production by firm 1, and MC_2 is the marginal cost of production by firm 2.

a Derive the quantity reaction function for each firm on the assumption of Cournot behavior (that is, each firm maximizes its profits with respect to quantity, given its rival's output). Determine the equilibrium quantities for each firm and the market price.

b What will happen to industry output and price if the two firms maximize joint profits rather than behave as Cournot duopolists? Determine the equilibrium price and quantity in the collusive case.

27 Suppose that an industry is characterized by the inverse demand function

$$p = 1 - y$$

and zero variable costs. Firms in the industry reach a Cournot–Nash equilibrium in quantities.

a Find the equilibrium price, profits per firm, if there are

(1) Two firms

(2) Three firms

(3) N firms

b Suppose that variable costs are still zero, but each firm incurs a setup cost of 0.05. Repeat (1), (2), and (3) from **a**. If there is free entry into the market, what will be the long-run equilibrium number of firms?

c Suppose each firm believes that if it produces in this market, it will sell $1/N$ of total output. Under this assumption, what will be the long-run equilibrium number of firms? Compare this answer with the one in **b**.

***28** An established firm in the market for widgets faces the following demand and cost conditions.

$$P = 100 - \frac{y}{2}$$

$$MC_1 = 40$$

where p is the price and y is the quantity demanded. A second firm is considering entering this market. If this firm enters, it can produce only with a production technology that has constant marginal costs,

$$MC_2 = 40$$

If both firms produce in the market, they will behave as Cournot duopolists.

a Find the Sylos limit output when setup costs of entry equal $450.

b Will the incumbent have the incentive to set the limit output to deter entry?

***29** Mac N. Ro, Inc., has a monopoly in the production of tennis shoes. The marginal cost of producing tennis shoes is $18. Mac Ltd., however, does not have the facilities for selling the shoes to consumers, so it sells them wholesale to a firm that, in turn, sells the shoes to consumers in a monopoly retail market. The retail firm faces marginal costs equal to the wholesale price of the shoes, p_w, plus marginal retail costs of $10. The consumer demand for tennis shoes is given by $p = 100 - y$.

a Determine the wholesale and retail prices and the market output.

b Show that Mac Ltd. could increase profits by vertically integrating forward into the retail business. That is, it would produce and sell the shoes, incurring a total marginal cost of $28.

c Can you think of an alternative to vertical integration that would yield the same profits?

Answers to Chapter 12

Case Study

A Figure A12.1 shows the market demand curve $D(y)$ and the marginal cost curves of the fringe MC_f, and of the cartel MC_c. At p_1 MC_f equals the market demand, and the fringe will supply the entire market; the cartel supplies 0. At p_2, the fringe supplies 0, so the cartel will satisfy the entire market. The residual demand for the cartel is dd'. The marginal revenue of the residual demand is set equal to the marginal costs of the cartel, MC_c. The profit-maximizing quantity of the cartel is y_d^*, and the price is set from the residual demand at p^*. The fringe equates p^* with MC_f and produces y_f^* output.

B OPEC may have calculated the elasticity from the market demand rather than the residual demand.

C If the fringe firms were making a profit at p^*, more firms would try to enter, through increased production, exploration of new reserves, or de-

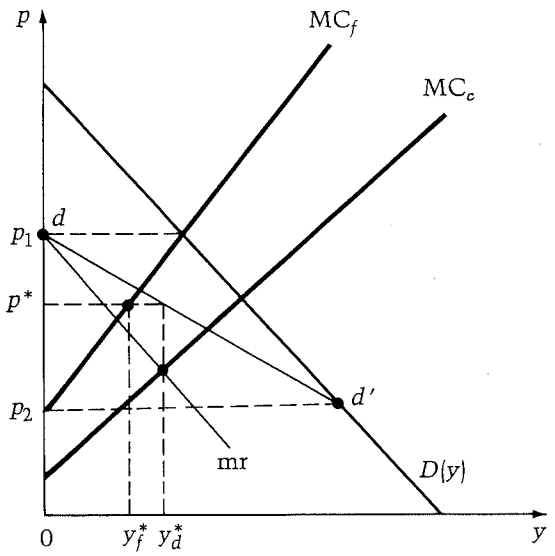

FIGURE A12.1

velopment of substitutes for oil. If this entry is successful, MC_f will shift to the right, and the residual demand of the cartel will shift inward, pushing the price down.

D A cartel will be more stable the fewer the firms in the cartel because cheaters can be more easily detected. Stability of the cartel is also more likely if entry barriers into the market are high. If supply from firms not in the cartel can respond quickly to increases in the cartel price, the cartel will be unstable. Where there are fixed supplies of resources, for example, fewer entrants will be able to enter the market and put downward pressure on price. The more there is to lose by cheating on the cartel, the less likely there will be cheating. The fewer substitutes that exist for the good, the less elastic the demand and the more stable the cartel.

Multiple Choice

1 b **2** c **3** b **4** d **5** d
6 b **7** d **8** d ***9** a

True-False

10 T **11** F **12** T **13** T **14** F
15 F **16** T **17** T **18** T **19** F

Short Problems

20 This practice is a collusive agreement by firms. Without this agreement, firms would compete in prices. Joint profits are higher under this agreement. This strategy may not be individually rational, however, because firms will have an incentive to lower the price below the collusive price for nearby customers.

21 Assume two things: (1) Profits to both firms using the strategy of no advertising are higher than those using an advertising strategy when both firms adopt the same strategy. (2) Each firm has the incentive to advertise, regardless of the rival's action. The payoff matrix in Figure A12.2 satisfies these assumptions. Thus, in the absence of a ban, the Nash equilibrium will be for both firms to advertise. Hence, a ban can raise profits from (100, 100) to (200, 200).

22 In Figure A12.3, market demand is given by $D(y)$ and the corresponding marginal revenue is $MR(y)$; $mc(y)$ is the marginal cost of one firm, and $MC(y)$ is the horizontal summation of the two marginal cost curves. The collusive outcome is $MR(y) = MC(y)$, at market output y^* and price p^*. Each firm produces $\frac{1}{2}y^*$.

Under the market-sharing rule, each firm faces one-half the market demand or $d(y)$, which is identical to $MR(y)$. Each firm equates the marginal revenue of its demand $mr(y)$ to $mc(y)$ and sets the price p^* and output $\frac{1}{2}y^*$.

23 a The firm believes that any price decrease will be matched by rivals; hence, demand is more

		Firm 2: Advertise	Firm 2: Do not advertise
Firm 1	Advertise	100, 100	300, 50
Firm 1	Do not advertise	50, 300	200, 200

FIGURE A12.2

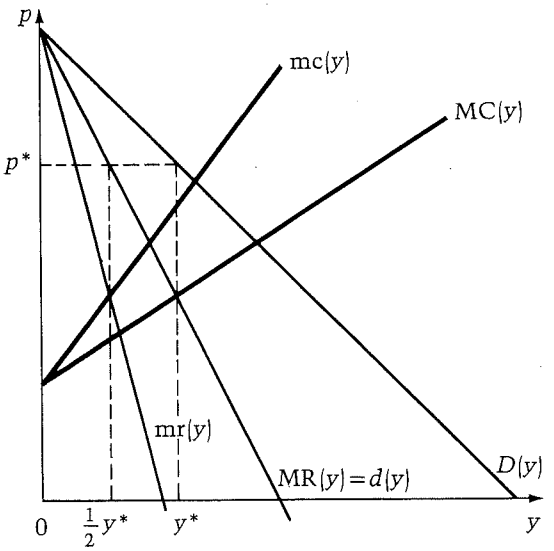

FIGURE A12.3

inelastic for price decreases. Furthermore, any price increases will not be matched; hence, demand is more elastic at higher prices.

b Because of the kink in the demand, the marginal revenue is discontinuous. Hence, MC(y) can increase over a large range and MR(y) = MC(y) at the same output level y^* and price p^*, as shown in Figure A12.4.

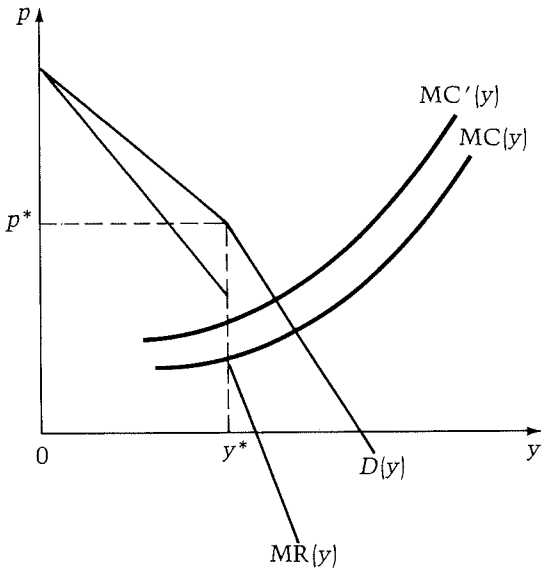

FIGURE A12.4

24 a Collusion results in larger profits for the group as a whole.

b The travel packages may be a way of disguising price cuts, or they may represent nonprice competition.

c Since the travel packages may be a way of cheating on the collusive agreement, IATA's right of veto is a way to police the agreement.

Long Problems

25 a Monopoly price and quantity are found by equating MR_I and MC_I.

$$MR_I = 50 - y = 10 = MC_I \quad \text{so} \quad y = 40$$
$$p = 50 - 0.5 \times 40 = 30$$
$$\pi = (30 - 10)40 = \$800$$

b (1) If only firm I develops the technology,

$$MR_I = 50 - y = 0$$
$$y = 50$$
$$p = 50 - 0.5 \times 50 = 25$$
$$\pi = 50 \times 25 - 550 = 700$$

(2) Reaction functions are found for $MC_I = 10$ and $MC_E = 0$:

$$MR_I = 50 - 0.5y_E - y_I = 10$$

which implies that $y_I = 40 - 0.5y_E$ and

$$MR_E = 50 - 0.5y_I - y_E = 0$$

so

$$y_E = 50 - 0.5y_I$$

Substitute the second equation into the first:

$$y_I = 40 - 0.5(50 - 0.5y_I)$$

so

$$y_I = 20 \qquad y_E = 40$$
$$p = 50 - 0.5 \times 60 = 20$$
$$\pi_I = (20 - 10) \times 20 = 200$$
$$\pi_E = 20 \times 40 - 550 = 250$$

(3) If both develop the technology, then —from (2)—the reaction functions for the two firms are

$$y_I = 50 - 0.5y_E \quad \text{and} \quad y_E = 50 - 0.5y_I$$

Substitution of firm E's reaction function into firm I's yields

$$y_I = y_E = \frac{100}{3}, \qquad p = 50 - \frac{200}{6} = \frac{50}{3}$$

and

$$\pi_I = \pi_E = \left(\frac{50}{3}\right)\left(\frac{100}{3}\right) - 550 = 5.56$$

c The Nash equilibrium set of strategies is "develop" (D) for the entrant and "do not develop" (N) for the incumbent. As indicated in Figure A12.5, N is a dominant strategy for firm I: Whether firm E chooses D or N, firm I can do better by choosing N rather than D. For the entrant, D is a dominant strategy.

d If firm I could move first or firm E could move first, the answer in **c** would not change. Let firm I move first. It knows that whatever it does, firm E will choose D. Hence, firm I chooses the strategy with the larger payoff, N, and firm E follows with D. If firm E moves first, it knows that whatever it does, firm I will react by choosing N. Hence, firm E chooses the strategy with the largest payoff, D, and firm I follows with N.

26 a Each firm equates marginal revenue with its marginal cost.

Firm 1: $100 - \frac{y_2}{2} - y_1 = 19$, so

$$y_1 = 81 - \frac{1}{2}y_2$$

Firm 2: $100 - \frac{y_1}{2} - y_2 = y_2$, so

$$y_2 = 50 - \frac{1}{4}y_1$$

Substitute firm 2's reaction function into firm 1's reaction function:

$$y_1 = 81 - \left(\frac{1}{2}\right)\left(50 - \left(\frac{1}{4}\right)y_1\right)$$

$$= 56 + \frac{y_1}{8}$$

Solving for y_1 gives

$$y_1 = 64$$

Substitute $y_1 = 64$ into firm 2's reaction function:

$$y_2 = 50 - \frac{64}{4} = 34$$

Then, total output is $y = y_1 + y_2 = 98$, and market price is $p = 51$.

b If the two firms collude and maximize joint profits, they will allocate output to each firm so as to minimize total production costs. The horizontal summation of the marginal costs is shown in Figure A12.6. This problem is similar to a multiplant monopolist. That is, the two firms will choose output in each "plant" where $MC_1 = MC_2$ or

$$y_2 = 19 \qquad (1)$$

Total output is found by equating marginal revenue to the marginal cost of each firm. Since $MC_1 = MC_2 = 19$, total output is given by

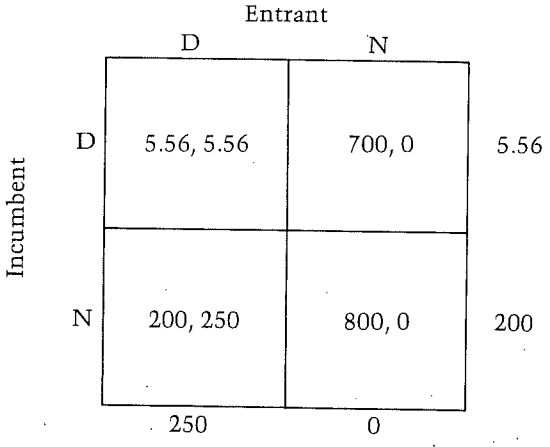

FIGURE A12.5

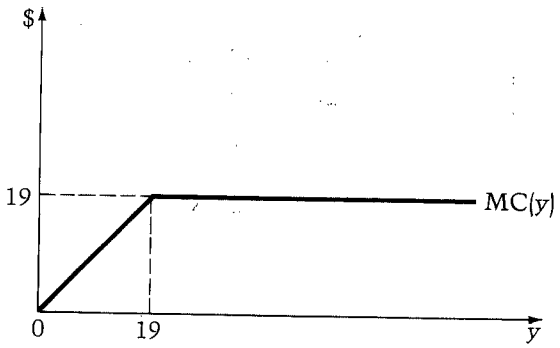

FIGURE A12.6

$$100 - y = 19$$
$$y = 81 \qquad (2)$$

Equations (1) and (2) imply that $y_1 = 62$, $y_2 = 19$, and $p = 59.5$.

27 a (1) For two firms, the profit function for each firm (for example, firm 1) is $(1 - y_1 - y_2)y_1$. Firm 1's reaction function is given by

$$1 - y_2 - 2y_1 = 0 \qquad \text{so} \qquad y_1 = \frac{1}{2} - \frac{y_2}{2}$$

Similarly, firm 2's reaction function is

$$y_2 = \frac{1}{2} - \frac{y_1}{2}$$

Solving the two equations simultaneously for y_1 and y_2 gives

$$y_1 = y_2 = \frac{1}{3}$$
$$p = 1 - \frac{2}{3} = \frac{1}{3}$$
$$\pi_1 = \pi_2 = \frac{1}{9}$$

(2) For three firms, the reaction function of firm 1 is given by the condition

$$1 - y_2 - y_3 - 2y_1 = 0$$

Then the equations that are solved simultaneously for y_1, y_2, and y_3 are

$$y_1 = \frac{1}{2} - \frac{y_2}{2} - \frac{y_3}{2}$$
$$y_2 = \frac{1}{2} - \frac{y_1}{2} - \frac{y_3}{2}$$
$$y_3 = \frac{1}{2} - \frac{y_1}{2} - \frac{y_2}{2}$$

Solving the three equations gives

$$y_1 = y_2 = y_3 = \tfrac{1}{4}$$
$$p = 1 - \tfrac{3}{4} = \tfrac{1}{4}$$
$$\pi_i = \tfrac{1}{16}, \qquad i = 1, 2, 3$$

(3) For N firms, the reaction function for each firm (consider firm 1 again) is given by

$$1 - y_2 - y_3 - \cdots - y_N - 2y_1 = 0$$

But since the firms have identical costs, $y_1 = y_2 = \cdots = y_N$ in equilibrium, the profit-maximization condition can be written as

$$1 - (N + 1)y_i = 0$$

which implies that $y_i = 1/(N + 1), i = 1, \ldots, N$

$$p = 1 - \frac{N}{N+1} = \frac{1}{N+1}$$
$$\pi_i = \left(\frac{1}{N+1}\right)^2, \qquad i = 1, \ldots, N$$

b With setup costs, the price and quantities will be the same. However, profits will be smaller by 0.05. In the long-run, the number of firms will be such that

$$\left(\frac{1}{N+1}\right)^2 - 0.05 = 0 \qquad \text{so} \qquad N = 3.47$$

Assuming that the number of firms must be an integer, $N = 3$.

c If each firm believes that it will sell $(1/N)$ of total output, then it will maximize

$$\pi = (1 - Ny)y - 0.05$$

where y is the output of a representative firm. Profit-maximization gives

$$1 - 2Ny = 0 \qquad p = 1 - \frac{1}{2} = \frac{1}{2}$$
$$y = \frac{1}{2N}$$

Note that the collusive output and price are reached. The reason is that each firm thinks rivals will do exactly the same thing, so they are able to collude tacitly and set the monopoly output. In the long run, firms will enter until

$$\pi = \frac{1}{4N} - 0.05 = 0 \qquad \text{so} \qquad N = 5$$

Note that more firms are attracted into the industry because monopoly profits rather than oligopoly profits are shared.

***28 a** If the incumbent has y_1 units of output, then the reaction function for the entrant is

$$\text{MR}_2 = 100 - \frac{y_1}{2} - y_2 = 40$$

so

$$y_2 = 60 - \frac{y_1}{2}$$

If the second firm entered, total output in the industry would be

$$y = y_1 + y_2 = y_1 + 60 - \frac{y_1}{2} = 60 + \frac{y_1}{2}$$

Price can be written in terms of y_1 as

$$p = 100 - 0.5\left(60 + \frac{y_1}{2}\right) = 70 - \frac{y_1}{4}$$

and the profits to the entrant would be

$$\pi_2 = \left(30 - \frac{y_1}{4}\right)\left(60 - \frac{y_1}{2}\right) - F$$

where F is setup costs. Then, if $F = 450$, the condition for no entry is

$$\left(30 - \frac{y_1}{4}\right)\left(60 - \frac{y_1}{2}\right) = 450$$

so

$$1800 + \frac{y_1^2}{8} - 30y_1 = 450$$

Using the quadratic formula, we get

$$y_1 = 60$$

b Yes. The limit output happens to be the output that would be chosen by a monopolist, unthreatened by entry. In this case, entry is said to be blockaded.

***29** Given that the monopolist sets a wholesale price p_w, the retailer maximizes

$$\pi = (100 - y)y - (p_w + 10)y$$

Profits are maximized where

$$100 - 2y = p_w + 10, \quad \text{so}$$

$$p_w = 90 - 2y$$

This relationship gives the demand by the retailers for the shoes. Mac N. Ro, Inc., now maximizes

$$\pi = p_w y - 18y$$
$$= (90 - 2y)y - 18y$$

Profit-maximizing output and prices are

$$y = 18 \qquad p_w = 90 - 36 = \$54$$
$$p = 100 - 18 = \$82$$

Note that there are two markups: The manufacturer sets a price \$36 higher than the marginal costs of \$18, and the retailer sets a price \$18 over its marginal costs of \$54 + \$10. The markup by the retailer lowers the manufacturer's profit. Manufacturer's profits are $\$36 \times 18 = \$648$.

b Mac N. Ro could do better integrating into the retail market. In this case, marginal costs would be \$28. Then profits would be

$$\pi = (100 - y)y - 28y$$

Profit-maximization would occur where $MR(y) = MC(y)$ or

$$100 - 2y = 28 \quad \text{so} \quad y = 36 \quad p = \$64$$

Profits would equal \$1296.

c Alternatively, Mac could impose a ceiling of \$64 on the retail price. By setting a wholesale price of \$54 (the vertical integration price minus the retailer's average cost), Mac could earn the same profits as under vertical integration, whereas the retailer would only be able to cover costs.