6

CHOICE MAKING UNDER IMPERFECT INFORMATION

Chapter Summary

Risky Choices

In this chapter, we allow the consumer to enter the interesting and unpredictable world of **risk** and **uncertainty.** A **risky choice** has several possible outcomes, each of which might occur with some **probability.**

An example of a risky choice is a lottery. An individual, Martha, is a sweepstakes winner. Her prize is a choice between *A* dollars for certain and a gamble that pays \$1 million with probability of 1/1,000,000 and 0 otherwise. Martha will choose the gamble if her reservation price for the gamble is at least as large as *A.* If Martha is risk-neutral, then her reservation price will equal the **expected value** of the gamble—the sum of the possible payoffs weighted by the probabilities of those payoffs being realized. In this case, Martha will take the gamble if *A* is not larger than \$1.

Martha's reservation price may not equal the expected value of the lottery. Suppose that Martha has the opportunity to pocket \$500,000 for certain or take a gamble that will make her a millionaire with a probability of $\frac{1}{2}$. For Martha *not* to take the gamble would seem perfectly reasonable. In this case, Martha's reservation price is less than \$500,000, the expected value of the lottery.

Expected-Utility Hypothesis

This example suggests that the decision to undertake risk depends on something more than simply a comparison between the certain outcome (the opportunity cost of the gamble) and the expected payoff. Bernoulli argued that the reservation price of a risky choice is equal to the **expected utility** of that choice rather than to the expected monetary payoffs, where the expected utility is a weighted average of the utilities associated with the payoffs. The weights are probabilities of outcomes, where the probabilities are either objectively known or **subjective.**

In recent years, the expected-utility hypothesis has been theoretically justified, proving that an expected-utility function exists for a set of reasonable assumptions on preferences over risky prospects. The completeness, two-term consistency and three-term consistency assumptions discussed in Chapter 3 must all hold for these risky prospects. Nonsatiation implies that larger assured outcomes are always preferred to smaller ones. Three additional assumptions are also necessary. First, **continuity** guarantees that individuals are willing to make some trade-offs between risky and riskless prospects. Second, the **substitution assumption** says that for any given prospect, an equivalent compound prospect can be created. Finally, the **ordering assumption** states that if two prospects involve the same outcomes, then the most preferred prospect is the one with the highest probability of winning the better outcome. Together, these assumptions imply the **expected utility theorem:** If an individual prefers one prospect to another, then the preferred prospect has greater expected utility; if an individual is indifferent between prospects, then their expected utilities must be the same. Although the expected utility hypothesis can be used to analyze a variety of economic problems with risky prospects, it is not appropriate when preferences depend on the **state of the world.**

Attitudes Toward Risk

Under the expected-utility hypothesis, a taxonomy can be defined over an individual's attitudes toward risk. Define a **riskless asset** as one that offers the expected payoff from some lottery, A, with probability 1. Then, an individual is said to be **risk-neutral** if his utility from that **riskless prospect** equals the expected utility from the risky prospect A. Such individuals have a **constant marginal utility of wealth.** If the utility of the riskless prospect exceeds the expected utility from the risky prospect A, then the individual is **risk-averse** and has a **diminishing marginal utility of wealth.** In this case, the additional utility that is possible from undertaking the risky prospect is not large enough to offset the loss in utility that is possible from that prospect. Finally, an individual who is **risk-inclined** will receive more utility from the risky prospect than from the riskless option. We generally expect most individuals to be risk-averse.

Risk Pooling and Insurance

Risk-averse individuals attempt to shed risk. When two individuals face independent risks, they may agree to enter into a **risk pooling** arrangement, in which they share in any losses that either or both individuals incur. This risk-pooling arrangement is a type of **insurance.** To find out when an individual would **fully insure** herself against some risk (that is, pay for insurance that would fully compensate her in the event of a disaster), we need to determine the **certainty equivalent** of the prospect (that is, the amount of money that would leave the individual with a utility level equal to the expected utility of the risky asset). An individual will fully insure herself against risk if the cost of the insurance is less than the difference between the initial wealth and the certainty equivalent. If risk-neutral insurance firms are willing to supply the insurance, an individual can be left with a level of wealth equal to the expected wealth in the absence of insurance but without risk! A second mechanism by which risk is shed is **risk spreading.** In this case, an indivisible and risky asset is owned by several individuals or firms. Risk-averse individuals may prefer to own part of a risky asset rather than the entire asset.

Adverse Selection and Signaling

Informational problems arise in markets for insurance. An insurance rate for a particular category of individuals is based on the expected cost of supplying insurance for all individuals in that category. Where there is **asymmetry of information**—an individual knows he is a good driver but the insurance company does not, for example—there can be a market failure of **adverse selection.** Because "low-risk" individuals cannot communicate convincingly that they are low-risk, the insurance company can offer only one average rate, which will be attractive to high-risk individuals. Low-risk individuals will choose not to take out the insurance, and insurance rates will climb; hence, the low-risk group may be driven out of the market and, therefore, be unable to shed any of its risk. Akerlof coined this problem of asymmetric information in the market for used cars as the **lemons principle.**

If an individual could acquire a **signal** that informs the insurance company of his low-risk characteristic, then he could be offered an acceptable insurance policy. The signal that enables the low-risk individuals to shed their risk is **productive** if the policy, in the absence of the signal, would be acquired only by high-risk individuals. Examples of signaling are pervasive, especially in hiring practices.

Key Words

Adverse selection
Assured outcome
Asymmetric information
Certainty equivalent
Continuity assumption
Expected utility
Expected value
Insurance
Lemons principle
Marginal utility of wealth
Ordering assumption
Outcomes
Probability
Productive versus unproductive signals
Prospects
Reservation price
Risk
Risk-averse
Risk-inclined
Riskless asset
Risk-neutral
Risk pooling
Risk spreading
State-dependent preferences
Subjective probabilities
Substitution assumption
Uncertainty

Case Study: Getting a Free Ride

One summer while living in Florence, Italy, I traveled frequently by bus, a very inexpensive and convenient mode of transportation in Italy. Tickets to ride the bus anywhere in the city can be purchased in any tobacco shop for a mere 400 lire, approximately $0.25 in U.S. dollars (1985). Passengers ascend the bus from the back door and, immediately upon entering the bus, are required to punch their tickets into a machine in the back of the bus. There is no transit official in the bus to check that all passengers validate their tickets. After riding the bus twice a day for two weeks without having my ticket checked, I began to wonder if my last-minute dashes to the tobacco shop to buy a ticket were worth the effort, especially since the buses were so crowded that I had to push through the crowds to reach the ticket machine.

So one day, after evaluating this problem in uncertainty, I boldly decided to break the law and did not punch my ticket. However, I was very nervous. Despite the seemingly relaxed attitude of the public transit authorities, the Italians seemed to make every effort to validate their tickets. What motivated the Italians to be such law-abiding citizens?

One day I found out. A public transit official entered the bus at one of the stops. At that point, everyone (including myself) got out their tickets to proudly indicate that, indeed, their tickets had been punched moments earlier, all except for one person. Her ticket apparently had been used twice, and after a long shouting match, she had to purchase a ticket from the official for 40,000 lire (100 times the regular price) or get off the bus.

A Does a model of legal institutions based on self-interest behavior by consumers seem to apply to this situation? Explain.

B Given the frequency of monitoring and the fine for disobeying the law (including the reprimand by an authority), do you think the Italian public transit authority has adopted a good scheme for enforcing the law? Explain.

C Is there a possibility that a culprit might successfully bribe the law enforcer? Why or why not?

Exercises

Multiple-Choice

Choose the correct answer to each question. There is only one correct answer to each question.

1 A person who places a smaller utility on gaining $1000 than on losing an equal amount

a Has an increasing marginal utility of money associated with larger amounts of money

b Would pay $1000 for a lottery ticket that paid either $2000 with probability 1/2 or $0 with probability 1/2

c Has a declining marginal utility of money associated with larger amounts of money

d Is correctly described by both **b** and **c**

e None of the above

Questions **2** and **3** pertain to a consumer with a utility function given by

$$U = \left(\frac{W}{500}\right)^{1/2}$$

where U is utility and w is wealth. Ordinarily, the consumer expects his income to be \$40,500. However, he faces the possibility that his house will burn down, reducing his income to \$4,500. This unfortunate event occurs with probability 1/3.

2 The consumer's expected utility, if uninsured, is
a 57
b 14
c 114,000
d 7
e None of the above

3 The most the consumer would be willing to pay for full insurance is
a \$32
b \$98,000
c \$24,500
d \$16,000
e None of the above

4 A lottery with a $\frac{1}{2}$ chance of winning \$100,000 and $\frac{1}{2}$ chance of losing \$80,000
a May well be accepted by a risk averter
b Will never be accepted by a risk averter
c Will never be accepted by a risk-neutral person
d Will always be accepted by a risk-averse individual
e None of the above

To answer questions **5** to **7**, refer to Figure 6.1, which illustrates the utility function of two risk-averse individuals. Both individuals have initial wealth w_0. Each individual faces the uncertain outcome that her house will be robbed, in which case she will incur a loss of L. The probability of this event is p_1 for individual 1 and p_2 for individual 2. The proportion of low-risk individuals is $\frac{1}{2}$.

5 If the market for insurance is competitive and if insurance companies cannot distinguish between high-risk and low-risk individuals, then
a Only individual 1 will purchase insurance.
b Only individual 2 will purchase insurance.
c Both individuals 1 and 2 will purchase insurance.
d Neither individual will purchase insurance.

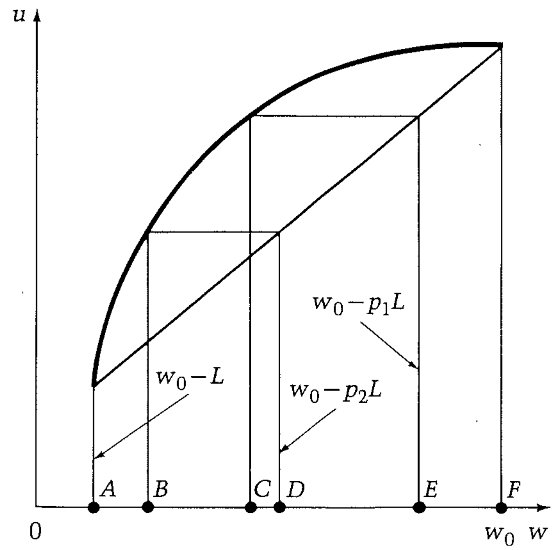

FIGURE 6.1

e No firm will be willing to supply an insurance policy.

6 If the insurance company cannot distinguish between high-risk and low-risk individuals, then
a The maximum premium that individual 1 is willing to pay for the insurance is *CE*.
b The maximum premium that individual 2 is willing to pay for insurance is *BF*.
c If both low- and high-risk individuals purchase insurance, the minimum price that the insurance company will be willing to accept for the insurance policy exceeds *DF*.
d Since only low-risk individuals will purchase the insurance, the price of the insurance will be *CF*.
e None of the above.

7 If low-risk and high-risk individuals can be identified and the market is competitive,
a Insurance will be sold at two prices, *BD* for the high-risk individuals and *CE* for the low-risk individuals.
b Insurance will be sold at two prices, *DF* for the high-risk individuals and *EF* for the low-risk individuals.
c Only the high-risk individuals will buy insurance at a price of *DF*.
d Insurance will be sold at one price, less than *DF* and greater than *EF*.
e None of the above.

8 Suppose that the utility of wealth is given by $U(w) = w^{1/2}$. If there are two states of the world, each occurring with probability 1/2, and a prospect pays $100 in state 1 and $36 in state 2, then the certainty equivalent of the prospect

a Is $68

b Is $8

c Is $64

d Cannot be calculated without more information

e None of the above

9 For two prospects with the same expected value,

a A risk-neutral individual will be indifferent between the two prospects.

b A risk-averse individual will prefer the prospect with the larger spread in the outcomes.

c A risk-inclined individual will prefer the prospect with the smaller spread in the outcomes.

d A risk-averse individual will be indifferent between the two prospects.

e None of the above.

10 An individual is risk-averse if

a The certainty equivalent of a prospect is greater than the expected value of the prospect.

b The expected utility of the prospect is larger than the utility of the expected value of the prospect.

c The utility of the certainty equivalent is greater than the utility of the expected value of the prospect.

d He is not willing to take a gamble at a price equal to the expected value of the gamble.

e None of the above.

11 A signal that distinguishes between high- and low-productivity workers is productive if

a The acquisition of the signal increases the individual's productivity.

b Both high- and low-productivity individuals acquire the signal.

c High-productivity individuals who would not accept a job at the wage offered without signaling are willing to work at the wage received after acquiring the signal.

d The high-productivity individuals are able to recover some of the "subsidy" that their presence provides for the low-productivity individuals.

e None of the above.

True-False

12 A risk-averse individual would never be willing to pay more than the expected loss from a risky project for full insurance.

13 Adverse selection is a market failure that occurs when individuals have information about themselves that they cannot communicate to the market.

14 A risk-averse individual prefers all certain prospects over a particular risky prospect.

15 Given two prospects with the same expected value, a risk-averse individual prefers the prospect with the smaller spread in the outcomes.

16 The certainty equivalent of a prospect for a risk-averse individual is less than the expected value of the prospect.

17 Given two prospects, (0.5, 0.5: 100, 0) and (0.7, 0.3: 30, 70), the risk-averse individual will definitely prefer the second prospect because the spread in the outcomes is smaller.

18 An expected utility maximizer with a utility function $U(w) = w^{1/2} + 0.1w$ would prefer to keep his initial wealth of $1000 rather than enter into a lottery (0.4, 0.6: $2500, 0). (The outcomes stated in the lottery include the initial wealth and the price of the lottery ticket.)

19 If high-risk and low-risk individuals cannot be distinguished by competitive insurance companies, then the price of insurance will be at least as large as an insurance company's expected costs of supplying insurance to a low-risk individual.

20 If the marginal utility of income is diminishing, then the expected utility of a prospect exceeds the utility of the expected value of the prospect.

*21 If a risk-averse individual with $500 income assigns a utility of 100 to $450 and a utility of 120 to $500, and if he is willing to pay $50 at maximum for a lottery ticket that pays $250 with probability 1/2 and $0 with probability 1/2, then the utility of $700 is 140.

Short Problems

22 Suppose that an individual is indifferent between $100 for certain and a 50/50 chance of

\$200 and \$50, and between \$140 for certain and a 70/30 chance between \$200 and \$50. If the individual satisfies the axioms of expected utility theory, what can you say about the individual's preference between a 50/50 chance of \$100 and \$140 and a 50/50 chance between the following two gambles: a 50/50 chance between \$200 and \$50 and a 70/30 chance between \$200 and \$50? Explain. What assumption must be satisfied to answer the question?

23 Suppose that, in the absence of medical insurance, individuals who break an arm pay, on average, \$500 for medical services. Assume the probability that an individual breaks her arm is 0.01 per year and the costs of providing insurance are zero. Will a competitive insurance industry provide insurance against the medical costs of broken arms at \$5 for risk-neutral individuals, greater than \$5 for risk-averse individuals, and less than \$5 for risk-inclined individuals? Why or why not?

24 Explain, using a diagram, why a risk-averse individual, choosing between two prospects with the same expected value, prefers the prospect with the smaller spread in the outcomes.

25 Winfred Whiz is playing on a game show. He must choose between two offers. The first offer is a payment of \$2000, which he can take for simply being on the show, or he can enter a gamble. In the gamble, he chooses one of two curtains that conceal two items. He makes a draw for curtain 1 or 2 from a hat; he receives the gift behind the curtain picked. He knows that behind one curtain is an automobile valued at \$4000 and behind the other curtain is a set of encyclopedias valued at \$500. If his initial wealth is \$1000 and his utility function can be described by $U(w) = 1 - 1000/w$, then what must be the probability of drawing the car for Winfred to be indifferent between the two choices?

26 Explain the relationship between the certainty equivalent of some gamble and the maximum price of insurance that a risk-averse individual would be willing to pay.

27 A risk-neutral individual's preference ordering over prospects can be based entirely on the expected values of the prospects. Explain why this is true.

28 Let A, B, C, and D represent four gambles available to an individual, where

$$A = (0.8, 0.2\colon 4000, 0)$$
$$B = (1, 0\colon 3000, 0)$$
$$C = (0.2, 0.8\colon 4000, 0)$$
$$D = (0.25, 0.75\colon 3000, 0)$$

If the individual chooses B over A and C over D, is his behavior consistent with the axioms of expected utility theory? Explain.

29 Ms. Gamble currently has an income of \$25,000. Assign the utility number 100 to this income level and the utility number 85 to the income level \$20,000. It is known that Ms. Gamble would be willing to pay a maximum of \$5000 for a lottery ticket that yields \$10,000 with a probability of 3/5 (and yields zero otherwise). What is the utility number appropriate to the income level \$30,000? Explain.

Long Problems

30 a Consider the following model provided by Spence in his 1973 paper.[1] Suppose that there are two types of people: high- and low-productivity workers. If a worker is known to be highly productive, then she would receive a wage of 2; if she is known to have low productivity, then she would receive a wage of 1. Assume that education does not increase an individual's productivity but simply awards the individual with a certificate. Assume that high-productivity individuals incur education costs $C^H = x/2$, where x = units of education, and that low-productivity individuals incur education costs $C^L = x$. Using this model of job market signaling, describe how education can have a value to certain individuals even if it does not increase their productivity.

b Are some signaling equilibria Pareto-improving over others? That is, would an increase or a decrease in the critical level of the signal, set by the employer to separate the good from the poor workers, make some employees better off without making others worse off? Explain.

[1] Spence, M. (1973) "Job Market Signalling," *Quarterly Journal of Economics.* 87:355–374.

31 Many spokespersons of the women's movement have argued that women must be twice as good as men to be hired for many management positions. Explain how this argument could be valid, using the model of job market signaling in problem **30**.

32 Betty Bat loves the New York Yankees. She has followed their exploits since she was 5 years old. This year they made it to the World Series. Betty has just thought up a clever plan. She has \$1000 in savings that she has hidden under her bed. She could spend \$600 of the \$1000 in making Yankees championship paraphernalia: buttons, cups, pens, and so on. Then, if the Yankees win, she estimates that she would earn \$1500. If the Yankees lose, she won't be able to sell any of her stock. Betty figures that the Yankees have a 0.6 chance of winning the World Series. Betty's utility function is given by $U(w) = w^{1/2}$.

a If Betty is an expected utility maximizer, will she make the \$600 investment into Yankees championship gadgets?

b Calculate the certainty equivalent of Betty's clever prospect.

c Suppose that a friend offers her insurance. He says to Betty, "If you pay me *F* dollars whether or not the Yankees win, then, in the event that the Yankees lose, I will pay you \$1500, the amount that you would have earned had the Yankees won the World Series. If the Yankees win, I will pay you nothing." What is the maximum value of *F* that Betty is willing to pay for the insurance policy? If Betty's friend is risk-neutral, will he gain by this venture? Explain.

*33 An individual is about to place a bet on her favorite racehorse, Lightning Speed. She can make one of two bets. Each bet costs \$100. In the first option, she bets that Lightning Speed will win the next race. If Lightning Speed wins, the individual will receive \$1000 (not including the ticket price); otherwise, she receives \$0. There is a 1/5 chance that the favorite horse will win. For the second option, she places a bet on Lightning Speed in the second and third races. If Lightning Speed wins in the second race, she receives a chance to play in the third race. There is a 1/4 chance that the favorite horse will win in the second race. If Lightning Speed wins in the third race, the individual receives \$1000; if Lightning Speed loses in either the second or third race, she receives \$0.

a Assuming that the individual satisfies the assumptions of expected-utility theory, what must be the probability of Lightning Speed winning the third race for the individual to be indifferent between the two bets?

b Which assumption of expected-utility theory has to be satisfied to answer **a**?

*34 Suppose $w_1 > w_2 > w_3 > w_4$, and $U(w_1) + U(w_4) = U(w_2) + U(w_3)$, where w is wealth. Show that any individual (regardless of preferences toward risk) would prefer $(p, 1-p: w_1, w_4)$ over $(p, 1-p: w_2, w_3)$ if $p > 1-p$. Give intuition for this answer.

Answers to Chapter 6

Case Study

A The legal institution in question here is the method by which the Italian transit authorities have chosen to collect bus fares. Although the self-interested behavior of consumers suggests that they would like to save themselves the amount of the bus fare by cheating, they do not want to have to pay a fine if caught cheating. Thus, a model of legal institutions based on self-interested behavior does apply because the high fine compensates for the low probability of getting caught.

B If people are risk-averse and self-interested, they will obey the law if the expected utility from cheating is less than the utility from being honest. Given the costs of having frequent inspections, it seems likely that the Italian public transit authority has found an effective way to reduce cheating. The fine plus the humiliation of a public reprimand should lower the expected return from cheating to a point at which most passengers would obey the law.

C For many "crimes" with large fines, a culprit might attempt to bribe a law enforcer. However, in this case, the risk of receiving a more severe punishment for attempting to bribe a law enforcer, especially on a public vehicle, would deter such behavior.

Multiple-Choice

1 c	2 d	3 d	4 a	5 c	6 b
7 b	8 c	9 a	10 d	11 c	

True-False

12 F	13 T	14 F	15 T	16 T
17 F	18 T	19 T	20 F	*21 T

Short Problems

22 The individual is indifferent between a 50/50 chance of \$100 and \$140 and a 50/50 chance of (0.5, 0.5: 200, 50) and (0.7, 0.3: 200, 50). The substitution assumption requires that these relationships be true, that is, an equivalent compound prospect is found by substituting the \$100 and \$140 outcomes in the first prospect with their respective equivalent lotteries.

23 No. If the insurance company is risk-neutral, it will provide insurance at a price equal to the expected cost from an accident. The expected costs are $500(0.01) = 5$ for all individuals, regardless of their preferences toward risk.

24 Figure A6.1 shows an individual's utility function and two prospects $A = (1/2, 1/2: w_1, w_2)$ and $B = (1/2, 1/2: w_3, w_4)$. The expected values of the two prospects are equal: $(w_1 + w_2)/2 = (w_3 + w_4)/2 = w_e$. The expected utility of A, the prospect with the larger spread, is less than the expected utility of B. Since a risk-averse individual has diminishing marginal utility of income, the individual prefers a prospect that has a smaller down-side risk (that is, the one with the chance of *losing* a smaller amount of money) than the one that has a chance of *winning* more money.

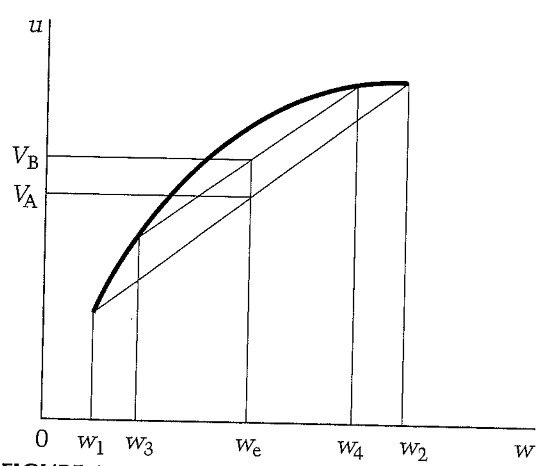

FIGURE A6.1

25 For Winfred to be indifferent between the two choices, the probability of drawing the car must satisfy

$$\begin{aligned} U(2000 + 1000) &= p[U(4000 + 1000)] \\ &\quad + (1 - p)[U(500 + 1000)] \\ &= U(1500) \\ &\quad + p[U(5000) - U(1500)] \end{aligned}$$

Substituting the utility values into this expression gives

$$1 - \frac{1000}{3000} = 1 - \frac{1000}{1500} + p\left(\frac{1000}{1500} - \frac{1000}{5000}\right)$$

or

$$p = \frac{5}{7}$$

26 The certainty equivalent is the amount of money that with certainty would yield the same utility as the expected utility from a risky prospect. Full insurance would leave the individual at a certain level of income in all states of the world. The maximum price an individual would pay for insurance is the amount of money that would leave him at a utility level no less than the utility from the certainty equivalent wealth. Let I_r be the maximum demand price for insurance. Then $w_o - I_r = w_{ce}$, where w_{ce} is the certainty equivalent wealth. I_r and w_{ce} are illustrated in Figure A6.2.

27 For a risk-neutral individual, $U(w_e) = p[U(w_1)] + (1 - p)[U(w_2)]$, where w_1 and w_2 are two possible outcomes from a risky prospect and w_e is the expected value. Hence, $w_e = w_{ce}$, where w_{ce} is the certainty equivalent. Since a preference ordering over prospects is the same as a preference ordering over certainty equivalent wealth values, it can be defined over expected values for risk-neutral individuals.

28 If B **P** A, then

$$U(3000) > 0.8[U(4000)] + 0.2[U(0)] \qquad (1)$$

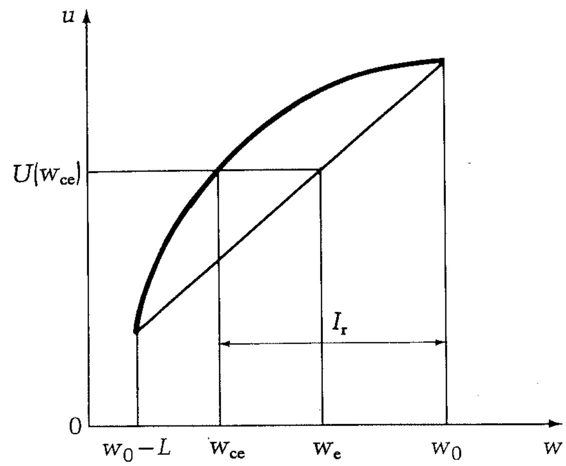

FIGURE A6.2

If *C* **P** *D*, then

$$0.2[U(4000)] + 0.8[U(0)] > 0.25[U(3000)] + 0.75[U(0)] \quad (2)$$

Both sides of equation (2) are multiplied by 4 and the equation is then rearranged to give

$$U(3000) < 0.8[U(4000)] + 0.2[U(0)] \quad (3)$$

Since inequalities (1) and (3) cannot occur simultaneously, the behavior is not consistent with expected-utility theory.

29 Ms. Gamble has the option of taking the assured prospect of \$25,000 if she chooses not to buy a lottery ticket. Alternatively, she can purchase a ticket and face the risky prospect: (3/5, 2/5: w_1, w_2), where w_1 and w_2 are given by

$$w_1 = 25{,}000 + 10{,}000 - 5000 = 30{,}000$$
$$w_2 = 25{,}000 + 0 - 5000 = 20{,}000$$

Since \$5000 is Ms. Gamble's reservation price for the lottery ticket, then it must be the case that

$$U(25{,}000) = 3/5[U(30{,}000)] + 2/5[U(20{,}000)]$$

Substituting $U(25{,}000) = 100$ and $U(20{,}000) = 85$ implies that $U(30{,}000) = 110$.

Long Problems

30 a If the employer does not know the productivity of workers, she can separate the high- and low-productivity workers through an education requirement. For example, she can set an education level x^* to separate the high- from the low-productivity workers. If $x \geq x^*$, she will pay 2; if $x < x^*$, she will pay 1.

As indicated in Figure A6.3, high-productivity workers earn more from getting x^* units of education because $2 - x^*/2 > 1$. Low-productivity workers earn more from not getting educated because $1 > 2 - x^*$, hence, there is a signaling equilibrium.

b If the level of the signal is raised above x^*, the low-productivity workers will be no worse off because they will continue not to get educated; however, for larger x^*, high-productivity workers will have to expend wasteful resources to acquire the additional units of the signal.

31 Use problem **30** to answer this question. Suppose that men and women face the same costs $C = x/2$ for acquiring x units of education. However, because employers have less information on women's abilities, they require the cutoff education level between high- and low-productivity women to be higher than the cutoff for men. For example, if women have education $x \geq x_w^*$ education, then they will be paid 2; if men have education $x \geq x_m^*$, where $x_w^* > x_m^*$, they will be paid 2. Figure A6.4 shows the cost curves and the signaling cutoff level for men and women. Note that $2 - x_m^*/2 > 1$, so men will get x_m^* units of education. However, $2 - x_w^*/2 < 1$, so women do not get the required amount of the signal to get the higher wage (even though they are equally productive). For women to receive the same wage as men in this model, they must have lower costs of getting educated (that is, they must be more productive).

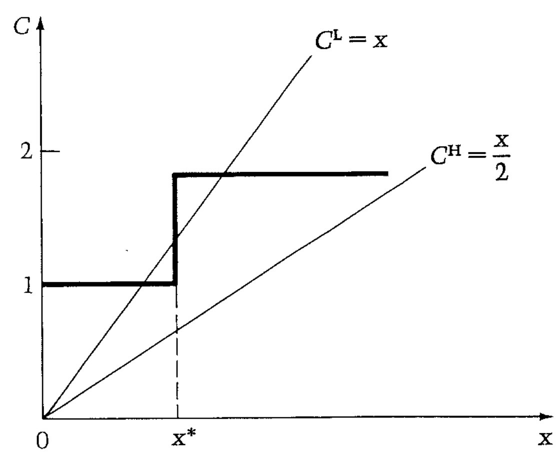

FIGURE A6.3

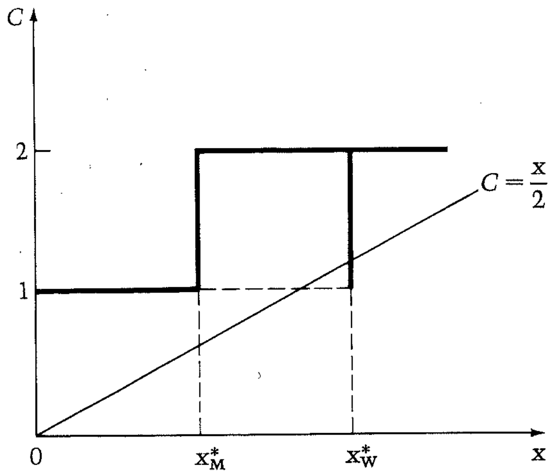

FIGURE A6.4

32 a If the Yankees win (state 1), Betty will earn \$1500 in revenues less the \$600 from the investment, or net revenues of \$900. Given her initial income of \$1000, $w_1 = \$1900$. If the Yankees lose, she will lose the \$600 and have a total wealth of $w_2 = \$400$. Betty will undertake the investment if the expected utility from the prospect exceeds the utility from keeping the \$1000 under her bed. That is,

$$U(1000) < 0.6[U(1900)] + 0.4[U(400)]$$

Substituting in the expression for the utility function gives

$$1000^{1/2} < 0.6(1900^{1/2}) + 0.4(400^{1/2})$$
$$31.6 < 26.15 + 8 = 34.15$$

Yes, Betty will invest the \$600 in the risky prospect.

b The certainty equivalent is that level of wealth w_{ce} that yields the same utility as the prospect; that is,

$$w_{ce}^{1/2} = 34.15, \quad \text{so} \quad w_{ce} = \$1166.22$$

c Betty receives \$1500 if the Yankees win or lose (or a net revenue of \$900 in addition to the initial income) under the insurance plan. At maximum, she is willing to pay the amount of money from her total certain income of \$1900 that will leave her at the same level of utility as with the certainty equivalent. That is,

$$(1900 - F)^{1/2} = 34.15$$

and so

$$F = \$733.78$$

Betty is willing to pay no more than \$733.78 for the insurance policy. The friend will offer the insurance policy because the expected cost of the policy, 0.4(1500) = \$600, is less than F.

***33 a** The two prospects can be written as

$$A = (1/5, 4/5: w_o + 900, w_o - 100)$$
$$B = (1/4, 3/4: [p, 1 - p; w_o + 900, w_0 - 100], w_o - 100)$$

The expected utility from each prospect is

$$V_A = (1/5)[U(w_o + 900)] + (4/5)[U(w_o - 100)]$$
$$V_B = (1/4)\{p[U(w_0 + 900)] + (1\text{-}p)[U(w_o - 100)]\} + 3/4\,[U(w_o - 100)]$$
$$= (1/4)\,p[U(w_o + 900)] + (1 - 1/4p)[U(w_o - 100)]$$
$$= \hat{p}[U(w_o + 900)] + (1 - \hat{p})[U(w - 100)]$$

Then, $V_A = V_B$ if $\hat{p} = (1/4)p = 1/5$, which implies $p = 4/5$.

b To answer part **a**, the substitution assumption of compound prospects must be satisfied, that is, the simple prospect $(1/5, 4/5: w_o + 900, w_o - 100)$ must be equivalent to the compound prospect B.

***34** An individual prefers $A = (p, 1 - p: w_1, w_4)$ over $B = (p, 1 - p: w_2, w_3)$ if

$$p[U(w_1)] + (1 - p)[U(w_4)] > p[U(w_2)] + (1 - p)[U(w_3)]$$
$$p[(U(w_1) - U(w_2)] > (1 - p)[U(w_3) - U(w_4)]$$

Then, since $U(w_1) + U(w_4) = U(w_2) + U(w_3)$, $U(w_1) - U(w_2) = U(w_3) - U(w_4)$, and substitution yields

$$p[U(w_1) - U(w_2)] > (1 - p)[U(w_1) - U(w_2)]$$
$$\text{or} \quad p > 1 - p$$

When $p = 1/2$, the lotteries are identical; hence, if $p \neq 1/2$, the lottery with the larger payoff in the more likely state of the world is preferred. For example, if $p > 1/2$, then because $w_1 > w_2$, lottery A is preferred. If $p < 1/2$, lottery B is preferred because the payoff in the more likely state of the world is higher for lottery B than for lottery A.