

# Two-Part Tariff with Maple Solution

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## Finding profit maximizing two-part tariff with different consumer types that cannot be separated.

Let  $Q_1$  be larger demand type and  $Q_2$  be smaller demand type. marginal cost is constant at 2

Three options:

Option 1: set  $P = MC$  and charge both types fee equal to smaller CS

Option 2: set  $P = MC$  and charge fee equal to larger type CS (This will cause smaller type to drop out of market)

Option 3: Hybrid. Set up profit function where fee and mark-up is a function of price. Maximize with respect to  $P$

From the three options, choose the one with the greatest profit

### Demand conditions:

>  $Q_1 := 18 - 3 \cdot P;$

$$Q_1 := 18 - 3 P \quad (1)$$

>  $Q_2 := 10 - 2 \cdot P;$

$$Q_2 := 10 - 2 P \quad (2)$$

### Total demand

>  $Q := Q_1 + Q_2;$

$$Q := 28 - 5 P \quad (3)$$

### Inverse of total demand:

>  $p := \text{solve}(Q=q, P);$

$$p := \frac{28}{5} - \frac{1}{5} q \quad (4)$$

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### Consumer Surplus functions for both types (as function of price) CS1 larger demand and CS2 smaller demand

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>  $CS1 := (6 - P) \cdot Q_1 \cdot 0.5;$

$$CS1 := 0.5 (6 - P) (18 - 3 P) \quad (5)$$

>  $CS2 := (5 - P) \cdot Q_2 \cdot 0.5;$

$$CS2 := 0.5 (5 - P) (10 - 2 P) \quad (6)$$

**Option 1:** set  $P = MC$  and charge both group a fee equal to smaller CS  
(**profit = 2 x CS2 when  $P = 2$** )

$$\begin{aligned} > \text{Option1profit} &:= 2 \cdot \text{subs}(P=2, \text{CS2}); \\ &\text{Option1profit} := 18.0 \end{aligned} \tag{7}$$

**Option 2:** set  $P = MC$  and charge fee equal to larger CS (**profit = CS1 when  $P = 2$** )

$$\begin{aligned} > \text{Option2profit} &:= \text{subs}(P=2, \text{CS1}); \\ &\text{Option2profit} := 24.0 \end{aligned} \tag{8}$$

**Option 3:** set up a Hybrid profit function:

$$\text{profit} = 2 \times \text{Fee} + (P - MC)(Q1 + Q2)$$

where the fee is equal to the formula for CS2

$$\begin{aligned} > \text{hybridprofit} &:= 2 \cdot \text{CS2} + (P - 2) \cdot Q; \\ &\text{hybridprofit} := 1.0 (5 - P) (10 - 2 P) + (P - 2) (28 - 5 P) \end{aligned} \tag{9}$$

$$\begin{aligned} > \text{hybridprime} &:= \text{diff}(\text{hybridprofit}, P); \\ &\text{hybridprime} := 18.0 - 6.0 P \end{aligned} \tag{10}$$

$$\begin{aligned} > Pstar &:= \text{solve}(\text{hybridprime} = 0, P); \\ &Pstar := 3. \end{aligned} \tag{11}$$

$$\begin{aligned} > \text{Option3profit} &:= \text{subs}(P=3, \text{hybridprofit}); \\ &\text{Option3profit} := 21.0 \end{aligned} \tag{12}$$

**Option 3 profit is 21 whereas option 2 profit is 24. (option 1 is the lowest profit at 18)**

**It is more profitable to use larger demand CS for fee and drive smaller demand types out of the market.**

**In this case, the DWL is the entire CS2 when  $P = 2$ .**

**Graphing the problem (requires inverse demands for both groups)**

>  $p1 := 6 - \left(\frac{1}{3}\right) \cdot q;$

$p1 := 6 - \frac{1}{3} q$  (13)

>  $p2 := 5 - \left(\frac{1}{2}\right) \cdot q;$

$p2 := 5 - \frac{1}{2} q$  (14)

>  $mc := 2;$

$mc := 2$  (15)

>  $plot(\{p1, p2, mc, Pstar\}, q = 0 .. 20);$

