ECON 6500, Summer 2008

BCIT

Practice Questions: *Cost Minimization and Profit Maximization.*

Problem 1.
A firm has Cobb-Douglas production function $y = KL$. Input prices are as follows: rental rate on capital $r = 4$, wage is $w = 1$.

a) Suppose in SR capital is fixed at 5 units, find short run TC function.

b) Use Lagrangean to derive firm's LR conditional factor demands for capital and labor *(You can substitute input prices into the total cost to make calculations easier.)*

From FOC you should obtain that in cost-minimizing combination of K and L TRS= input price ratio, $\frac{MP_L}{MP_K} = \frac{w}{r}$, *or* $\frac{K}{L} = \frac{1}{4}$; *After substituting into the constraint get* $K^* = \frac{1}{2}\sqrt{y}$, $L^* = 2\sqrt{y}$. *Reality check: K and L enter production function symmetrically, they are relatively equally productive, capital is more expensive, given the inpit prices firm optimally demands less capital.*
TC function:
TC=wL+rK, substitute input prices and factor demands:
$TC = 1 \cdot 2\sqrt{y} + 4 \cdot .5\sqrt{y} = 4\sqrt{y}$

c) What combination of inputs minimizes total cost of producing 100 units of output? What is the total cost of producing 100 units of output?

(Find how much labor and capital the firm needs: L=20 and K=5, then find how much it will cost to buy it. You can calculate $TC = 1 \cdot 20 + 4 \cdot 5 = 40$, *or plug 100 into the TC function to get the same number.)*

d) Use diagram to show the results from part (b): plot an isoquant for $y = 100$, show the cost-minimizing combination of inputs and respective isocost (be careful with the slopes).

d) Use your diagram to show the impact of wage going up to 2: what is the slope of the new isocost? Will firm demand more or less labor? capital?

Problem 1. *Short and Long Run Profit Maximization.*

A firm has the following production function: $y = L^{\frac{1}{3}}K^{\frac{1}{2}}$.

a) Does this production function exhibit increasing, decreasing, or constant returns to scale? *(decreasing)*

b) Suppose in SR capital is fixed at $\overline{K} = 100$. Find firm's unconditional factor demand for labor.

'Unconditional factor demands' means that you have to solve profit maximization problem. At first set up profit function as a function of output and input prices, ***fixed*** *level of capital, and the amount of labor:*

$$\pi = p10L^{\frac{1}{3}} - wL - 100r$$

FOC:

$$\frac{\partial \pi}{\partial L} = p10\frac{1}{3}L^{-\frac{2}{3}} - w = 0$$

Solve for $L^* = \left(\frac{10p}{3w}\right)^{\frac{3}{2}}$. *Check whether this makes sense. If price of output goes up, firm will demand more labor, probably this is because now it can get more revenue per unit of output so it will produce more and will need more labor. As wage increases demand for labor decreases, this is also reasonable, as labor becomes more expensive, costs per unit of output increase, firm is likely to decrease production (because of DRS, decrease in output will decrease average costs) and employ less labor hours.*

c) Find firm's unconditional factor demands in the long run.

In the LR both factors are variable, then firm's profits as a function of amount of K and L are:

$$\pi = pK^{\frac{1}{2}}L^{\frac{1}{3}} - wL - rK$$

The firm will maximize profits with respect to both factors. FOC:

$$\frac{\partial \pi}{\partial L} = p\frac{1}{3}L^{-\frac{2}{3}}K^{\frac{1}{2}} - w = 0$$

$$\frac{\partial \pi}{\partial K} = p\frac{1}{2}L^{\frac{1}{3}}K^{-\frac{1}{2}} - r = 0$$

From FOC you should be able to obtain optimal (profit maximizing and cost minimizing) ratio of capital to labor:

$$\frac{K}{L} = \frac{3w}{2r}$$

Which solves for $K = \frac{3w}{2r}L$. *At this point you can see that if for example* $w = r = 1$, *or both factors cost the same, firm will demand more capital than labor. If you look at the production function you can see that mathematically capital enters the function with higher power, which implies that other things equal capital is more productive, so this capital to labor ratio makes sense. Substitute for K into either FOC to find that:*

$$L^* = \frac{p^6}{(6rw)^3}$$

$$K^* = \frac{p^6}{2^4 3^6 r^4 w^2}$$

d) Think about what you can do with these input demand functions: if you know all prices you can calculate how much labor and capital the firm will demand. Knowing capital and labor you can calculate how much the firm will produce. You can also calculate total revenue, total costs, and profits. Make sure that you understand how to do it.