BUSA 4800

1 Cournot Duopoly Model

1.1 The case of Monopoly

1.1.1 Linear Demand Curve:

A linear demand function, written in "slope-intercept" form (i.e. $Y = mX + b$) is expressed as

$$p = a - bQ$$

where a is the vertical intercept and b is the slope and Q is the market (total) output. This type of demand function suggests that market price p is determined by the total amount of output available. Therefore, this would be a commodity market; one where the product is considered "generic" (i.e. oil, pulp, wheat, lumber, water).

1.1.2 Marginal Revenue

If the demand curve is linear, then the marginal revenue function is always twice as steep as the demand curve[1]

$$p = a - bQ$$
$$MR = a - 2bQ$$

If the the market is supplied by a single firm, or monopolist, then the market marginal revenue curve is also the firm's marginal revenue.

Cost function

Every firm faces the same cost function of the form

$$TC = K + cq$$

where K is the firm's capital cost (fixed cost or setup cost) and c is the per-unit cost of production. Note that is $q = 0$ then total cost still equals K. This type of firm has a constant marginal cost equal to c ($MC = c$).

Remembering from economics that the profit maximization rule for all firms is to set

$$MR = MC$$

to determine the optimal output. By subsituting the parameter of our model

$$\begin{aligned} MR &= MC \\ a - 2bQ &= c \\ 2bQ &= a - c \\ Q^* &= \frac{a-c}{2b} \end{aligned}$$

[1] This relationship is well explained in any microeconomics text in the chapter on Monopoly. It is also a result of the quadratic total revenue function. Students with a bit of calculus should understand the following:

$$\begin{aligned} TR &= P \times Q \\ &= (a - bQ) \times Q \\ TR &= aQ - bQ^2 \end{aligned}$$

Marginal revenue, by definition, is the derivative of total revenue:

$$\frac{dTR}{dQ} = a - 2bQ = MR$$

from the power-function rule that says if $y = ax^n$, then $dy/dx = anx^{n-1}$.

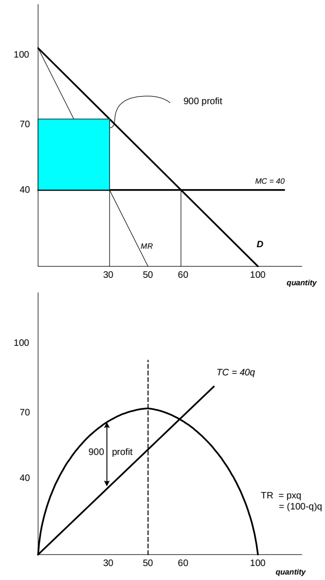

Figure 1:

1.1.3 Numerical Example:

Suppose our demand function is $P = 100 - Q$. then the marginal revenue is $MR = 100 - 2Q$. Further, let our marginal cost (c) be equal to 40. (and let $K = 0$ for now)

Applying the profit maximization rule

$$\begin{aligned} MR &= MC \\ 100 - 2Q &= 40 \\ 2Q &= 60 \\ Q &= 30 \end{aligned}$$

Subsituting Q into the demand function, we get the monopoly price

$$\begin{aligned} P &= 100 - Q = 100 - 30 \\ P &= 70 \end{aligned}$$

Finally, the firm's profit (π) is found by

$$\begin{aligned} \pi &= (P - c) \times Q - K \\ &= (70 - 40)30 - 0 \\ &= 900 \end{aligned}$$

This example is illustrated graphically in figure 1.

1.2 Duopoly

Now suppose we have two firms. The market output, Q, is now the sum of the output of each firm ($Q = q_1 + q_2$)

$$\begin{aligned} P &= a - b(q_1 + q_2) \\ P &= a - bq_1 - bq_2 \end{aligned}$$

where q_i is firm i's output ($i = 1, 2$).

1.2.1 Finding each firm's marginal revenue (MR_i)

Just like the case of monopoly, a firm's marginal revenue is twice as steep as the demand curve. However, in the case of more than one firm, each will derive its marginal revenue from the "residual" demand. The residual demand is that part of the market that is left over after the other firms have supplied their output into the market. Lets suppose firm 2's output is q_2. When firm 1 produces zero output, the market price would be

$$P = a - bq_2$$

as firm 1 adds output to the market, the price would then be

$$P = [a - bq_2] - bq_1$$

This is firm 1's **residual demand.** Firm 1's marginal revenue will be twice as steep, or

$$MR_1 = [a - bq_2] - 2bq_1$$

Similarly, firm 2's marginal revenue is given by

$$MR_2 = [a - bq_1] - 2bq_2$$

Each firm faces the same cost function

$$\begin{aligned} TC &= K + cq_i \\ (i &= 1, 2) \end{aligned}$$

and, as in the case of monopoly, $MC_i = c$

Each firm's profit function is

$$\begin{aligned} \pi_i &= Pq_i - cq_i - K \\ &= (p - c)q_i - K \end{aligned}$$

Max $\pi_{1,}$ treating q_2 as constant

$$\begin{array}{ll} MR_1 = MC_1 & \\ a - bq_2 - 2bq_1 = c & \text{solve for } q_1 \\ 2bq_1 = a - c - bq_2 & \\ q_1 = \frac{a-c}{2b} - \frac{1}{2}q_2 & \Rightarrow \text{"Best Response Function"} \end{array}$$

Best Response Function tells Firm 1 the profit maximizing q_1 for any level of q_2.

For Firm 2

$$\begin{array}{ll} MR_2 = MC_2 & \\ a - bq_1 - 2bq_2 = c & \\ q_2 = \frac{a-c}{2b} - \frac{q_1}{2} & \text{Firm 2's "Best Response Function"} \end{array}$$

The two "Best Response Functions"

$$\begin{array}{ll} \text{Firm 1} & q_1 = \frac{a-c}{2b} - \frac{q_2}{2} \\ \text{Firm 2} & q_2 = \frac{a-c}{2b} - \frac{q_1}{2} \end{array}$$

gives us two equations and two unknowns.

The solution to this system of equations is the equilibrium to the "Cournot Duopoly Game."

$$\begin{array}{ll} \text{1.} & q_1^* = \frac{a-c}{3b} \\ \text{2.} & q_2^* = \frac{a-c}{3b} \\ \text{Market Output} & q_1^* + q_2^* = \frac{2(a-c)}{3b} \end{array}$$

The two best response functions and the nash equilibrium can be seen in figure 2

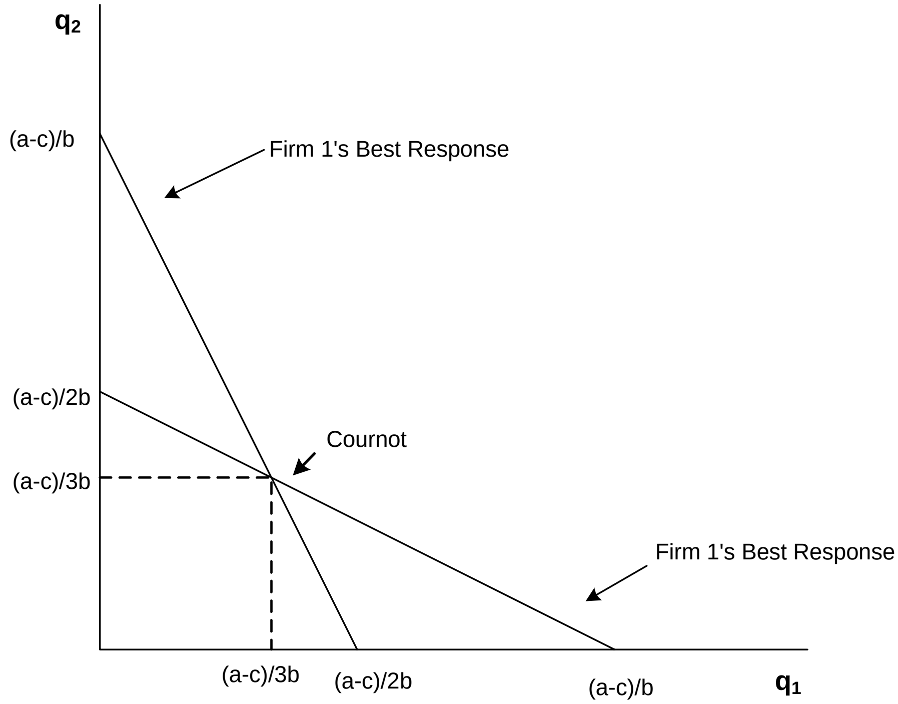

Figure 2:

1.3 Numerical Example

Using the example from the monopoly case,

$$\begin{aligned} P &= 100 - (q_1 + q_2) \\ P &= 100 - q_1 - q_2 \end{aligned}$$

Finding each firm's marginal revenue (MR_i)
Firm 1's marginal revenue will be twice as steep, or

$$MR_1 = 100 - q_2 - 2bq_1$$

Similarly, firm 2's marginal revenue is given by

$$MR_2 = 100 - q_1 - 2q_2$$

Each firm faces the same cost function

$$\begin{aligned} TC &= 40q_i \\ (i &= 1, 2) \end{aligned}$$

and, as in the case of monopoly, $MC_i = 40$. In this case fixed costs are zero ($K = 0$)
Max $\pi_{1,}$ treating q_2 as constant

$$\begin{array}{ll} MR_1 = MC_1 & \\ 100 - q_2 - 2q_1 = 40 & \text{solve for } q_1 \\ 2q_1 = 60 - q_2 & \\ q_1 = 30 - \frac{1}{2}q_2 & \Rightarrow \text{"Best Response Function"} \end{array}$$

For Firm 2

$$\begin{array}{ll} MR_2 = MC_2 & \\ 100 - q_1 - 2q_2 = 40 & \\ q_2 = 30 - \frac{q_1}{2} & \text{Firm 2's "Best Response Function"} \end{array}$$

The two "Best Response Functions"

Firm 1	$q_1 = 30 - \frac{q_2}{2}$
Firm 2	$q_2 = 30 - \frac{q_1}{2}$

gives us two equations and two unknowns. By substituting one equation into the other

$$\begin{aligned}
q_1 &= 30 - \frac{1}{2}q_2 \\
q_1 &= 30 - \frac{1}{2}\left(30 - \frac{q_1}{2}\right) \\
q_1 &= 30 - 15 + \frac{1}{4}q_1 \\
q_1 - \frac{1}{4}q_1 &= 15 \\
\frac{3}{4}q_1 &= 15 \\
q_1^* &= \left(\frac{4}{3}\right)15 = 20
\end{aligned}$$

then, by substituting the solution to q_1 into firm 2's best response, we get

1.	$q_1^* = 20$
2.	$q_2^* = 20$
Market Output	$q_1^* + q_2^* = 40$
Price	$P = 60$

Finally, each firm's profits are $\pi_1 = \pi_2 = 400$.

The two firm's best response functions and the nash equilibrium are illustrated in figure 3

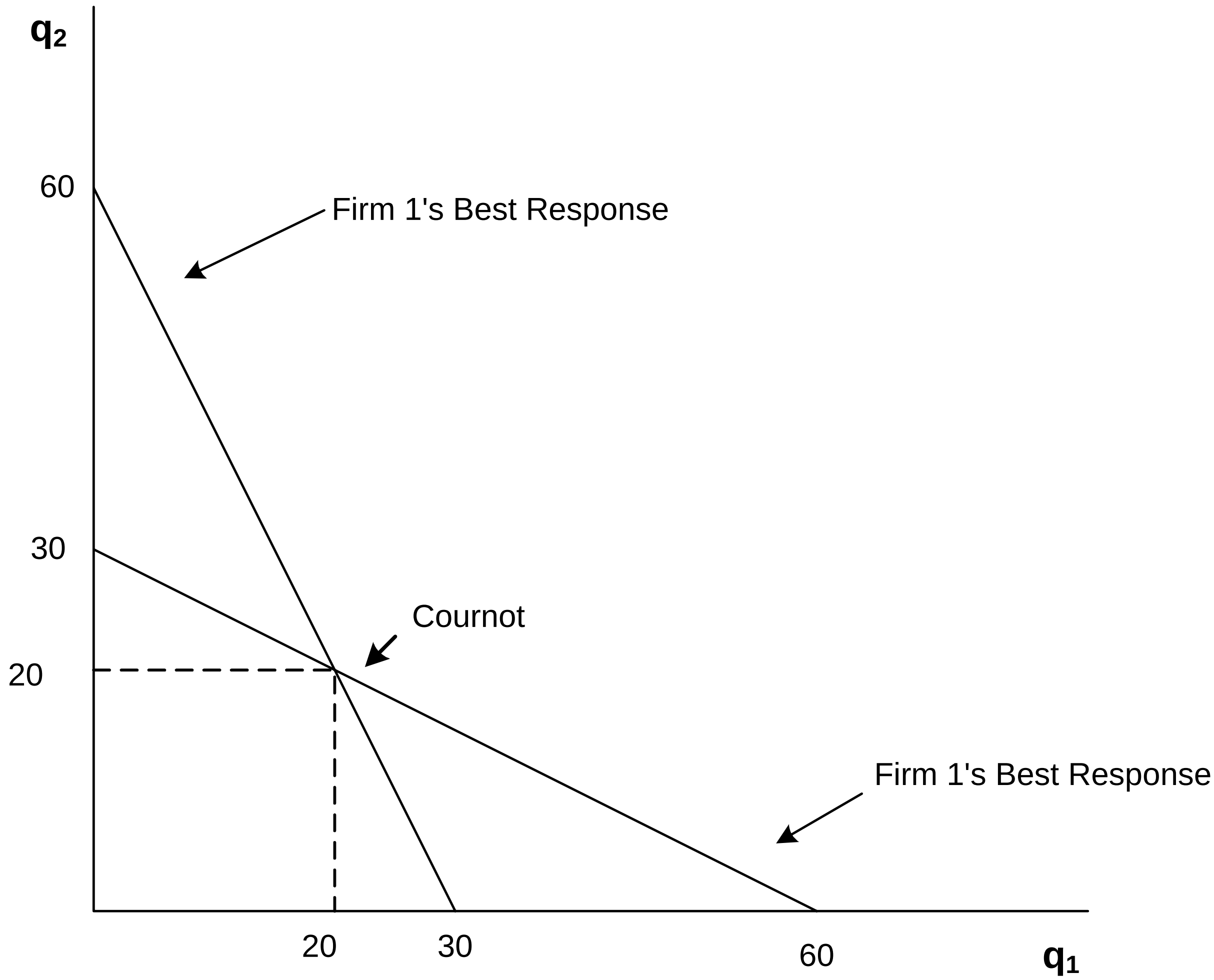
q2
60
Firm 1's Best Response
30
Cournot
20
Firm 1's Best Response
20
30
60
q1

Figure 3: