tive to defer some of their extraction to the second period. It follows that the resource price must, in equilibrium, rise at a rate which exceeds the interest rate.

A3.23 **TRUE.** The first part of the proposition was demonstrated in the answer to question 3.21 where from equation (1) it follows that the owner will extract in the second or first period according to whether C exceeds some threshold. But, when price is uncertain, for at least some owners, period 2 extraction must raise their expected returns (answer 3.22). At the same time it increases the risk of their return. In fact the problem of deciding how to allocate extraction between the two periods is closely analogous to the problem of an investor deciding how to allocate a given wealth portfolio between a safe and a risky asset. Just as in that case a risk averse investor typically holds a diversified portfolio (see Chapter 12), so in this case this result is achieved by extracting some of the resource in both periods. Owners with widely differing extraction costs will be spreading their risks by operating in both periods.

A3.24 **TRUE.** We have seen that in the case of a competitive industry with zero extraction costs price rises at the rate of interest. The monopolist will choose the price path such that the present value of marginal revenue is the same in each period, for otherwise a re-allocation of extraction could increase the present value of his income. Thus the price path must satisfy

$$MR_0 = MR_1/(1+r)$$

or

$$P_0(1-\frac{1}{\eta}) = P_1\,(1-\frac{1}{\eta})/(1+r)$$

where η is the constant elasticity of demand. From this last equation $P_0(1+r) = P_1$. Price therefore grows at the rate of interest. In the case of the two-period model, in which price elasticity exceeds unity and so it never pays to leave any of the resource in the ground at the end of the second period, this immediately implies that price in the two periods is the same under competition and monopoly and this conclusion also follows in a many period model. For further elaboration of this point see Stiglitz, J.E., 'Monopoly and the rate of extraction of exhaustible resources', *American Economic Review*, Vol. 66, 1976, pp. 655–61 and subsequent comments in Vol. 69, 1979, pp. 227–32.

4

Choice under Uncertainty

References

Becker Chapter 4. Ferguson p. 80–89. Friedman Chapter 4. Laidler Chapter 9. Layard Chapter 13.

Questions

Q4.1 A person who is averse to risk at all levels of income would never buy a share in a company which offered an uncertain return.

Q4.2 Three individuals have initial wealth W and are offered the opportunity, at a cost of \$5, to participate in a gamble with pay-offs of \$10 or nothing, each with probability one half. Suppose individual A accepts the gamble, B rejects it and C is indifferent. It follows that A has diminishing marginal utility of wealth, B has increasing and C constant utility of wealth (assume none of the individuals enjoy the experience of gambling as such).

Q4.3 If most people are risk averse then risky occupations will command higher wages than safe jobs.

Q4.4 A man has an income of £1000. He is offered the chance to invest in a project which has a 50% chance of making £200 and a 50% chance of losing £100. The utility he derives from various income levels is shown below.

Income	*Utils*
900	200
950	210
1000	214
1010	214.5
1100	218.5
1200	220

(i) He will invest in the project.
(ii) The cost of the risk associated with a project is defined as the difference between the expected monetary return it offers and the return which, if promised with certainty, would yield the individual the same utility. In the present case the cost of risk is £90.
(iii) Suppose the project in the question is to be equally shared between two investors both with the preferences above. They will wish to invest on this basis.
(iv) The total cost (i.e. summed over the two individuals) of risk is now zero.

Q4.5 The cost of risk for a given project will be greater for a risk averter than for a risk lover.

Q4.6 Suppose a family firm can borrow as much capital as it wants at the ruling market rate of interest. It would then have no incentive to go public and issue stock market traded equity in itself.

Q4.7 A risk averter will save more if the asset he invests in has no risk than if it offers the same expected return but is risky.

Further Questions

Q4.8 A risk-neutral consumer has the choice between:
(i) ordering supplies of coffee for delivery next period at a price to be paid then of $1 per jar.
(ii) buying coffee next period at the then current price which, with equal probability, may be either $0.90 or $1.10 per jar.

Bearing in mind that quantity demanded depends on price, which of the two options will he choose?

Q4.9 In 1732 Bernoulli composed the following game. A coin is tossed until it falls tails. If tails occurs on the first toss the player receives £2 and the game stops. Should the first tail appear on the second throw, £4 is paid and the game stops. If tails appears for the first time after n tosses, £2^n is paid.
(i) What is the expected monetary value of the pay-off to such a game?
(ii) Why are people not prepared to pay this much in order to play?
(iii) Show that if the individual has the utility function below and an initial income of £99, the most he will pay to play the game is £3.

Income	*Utility*
96	195
98	198
99	199.25
100	200
102	201
104	201
.	.
.	.
.	.
∞	201

Q4.10 A ship is overdue in port and a shortage of water develops. The limited supplies available are divided amongst all those on board. One of the crew receives 225 pints which is his supply of water from today, day 1, until the ship docks. His utility function is $U = 600P - 2.5P^2$, where U is utility for the day and P is daily consumption of water (in pints). For simplicity, but not very realistically, today's utility from water is therefore assumed to be independent of yesterday's consumption. Given this utility function, marginal utility is $MU = 600 - 5P$. The probability of making landfall at the end of day 1 is 0.6, at the end of day 2 is 0.3 and at the end of day 3 is 0.1. How many pints of water does he allocate to consumption on each of the three days? [Question based on problem in Jevons *The Theory of Political Economy*, first published in 1871, Penguin edition, pp 123–125.]

Q4.11 A man has the utility function $U = \log Y - C/M$ where Y is expenditure on consumption goods and M is expenditure on medical insurance. C is 1 if the man is ill and 0 if he is well. This utility function thus has the form that the more insurance that is taken out the better the medical care and the less onerous the illness. The probability he will fall ill is ½. If he has an income of $10, how much medical insurance will he take out?

Q4.12 Local government has the problem of attempting to reduce the number of people who park illegally. A perennial question is whether it should increase the probability that illegal parking will result in a conviction, or whether it should raise the fine which is imposed once convicted. If law breakers are risk averse it follows that a 10% rise in the fine will have a greater disincentive effect than a 10% increase in the probability of conviction.

Q4.13 (i) Suppose a manufacturer introduces a new good. If the product lives up to the claims made by the seller a consumer's utility is given by $U = X^{1/2} Y^{1/2}$ where X is the number of units of the new good consumed and Y the number of units of the other good he buys. There is, however, a half chance that the new good X does not live up to expectations and delivers only ¼ of the expected flow of services, in which case utility is given by $U = (\frac{1}{4}X)^{1/2} Y^{1/2}$. Assuming both goods cost £1 per unit, that it is impossible to test X before purchase and that no comeback is possible if it performs badly, how much X will be bought if income is £16?

(ii) If the individual believes that each unit of X bought yields a unit of service with certainty how much will he buy? How much does the reduction in uncertainty raise his welfare?

(iii) Suppose the probability that a unit of X will yield only ¼ of a unit of X services remains at ½. To compensate for this, the price of X is reduced to $\frac{1}{2} \times 1 + \frac{1}{2} \times \frac{1}{4} = 5/8$ so that £1 buys an expected quantity of one unit of X services. What will be the quantity of X purchased and will utility be as high as in (ii)?

√ Q4.14 The utility an individual derives from various income levels next period is shown below.

$U(260) = 1000$
$U(285) = 1500$
$U(290) = 1595$
$U(310) = 1856$
$U(315) = 1926$
$U(320) = 1980$
$U(330) = 2080$
$U(340) = 2170$

This period he decides to save £200 and next period will receive income from other sources of £100. The £200 of saving can be invested either in a bond which yields 10% with certainty or a company share which yields 20% capital gain with a probability of 0.8 and a 20% loss with probability 0.2. Both shares and bonds are sold in units of £100.

(i) With no tax on investment income he puts all the £200 in bonds.

(ii) Let us introduce a tax system which does not allow losses to be offset against gains. Thus if an individual loses £20 on a share and gains £20 on a bond he is not allowed to claim that his income is zero. That is, the tax authorities do not allow pooling and therefore tax must be paid on the gain. If a 50% tax is introduced on all investment income whether in the form of capital gains or interest and no loss offset is permitted this will not change the result of (i).

(iii) Let us now consider a tax system which does allow for full loss offset. Thus if one loses £20 on a share, for tax purposes this loss may be offset against any income derived from bonds. If again we assume that all sources of income are taxed at 50%, it follows that our individual will buy both a share and a bond.

√ Q4.15 The effort of carrying an umbrella reduces my utility by ½ a unit. If it rains and I have no umbrella, my utility falls by 3 units, whilst it only falls by 1 unit if I do have an umbrella. I consider that the probability it will rain is ½. Therefore I carry an umbrella.

√ Q4.16 (i) A farmer can grow wheat or potatoes or both. If the weather is good, an acre of land yields a profit of \$2000 if devoted to wheat, and a profit of \$1000 if devoted to potatoes. Should the weather be bad, an acre of wheat yields \$1000, and of potatoes \$1750. Good and bad weather are equally likely. Assuming the farmer has utility function $U = \log Y$, where Y is income, what proportion of his land should he turn over to wheat?

(ii) Suppose the farmer can buy an insurance policy which for every \$1 of premium pays \$2 if the weather is bad, and nothing if the weather is good. How much insurance will he take out and what proportion of his land will he devote to wheat?

(iii) What would the answer be if the policy paid only \$1.50 to compensate for bad weather?

Q4.17 Arrow defines a state of the world as 'a description of the world so complete that, if true and known, the consequences of every action would be known'. For example, in the previous two questions the two possible states of the world were either it rains or it is fine (Q4.15) or that the weather was good or bad (Q4.16), so states of the world are mutually exclusive. If we know the state of the world we know the utility associated with an income receipt in that particular state. Risk aversion implies that an individual's indifference curves between income in one state of the world and income in another will be convex to the origin.

√ Q4.18 If insurance is sold at a fair price (i.e. so that the cost of the policy equals its expected payout) then a risk averter will take out insurance such that his income is equal in all states. (For this question assume the utility of income function is the same in all states.)

Q4.19 Investment A offers a ½ probability of paying £10 and a ½ probability of £6. Investment B, for the same cost, has a ½ chance of returning £9 and a ½ chance of £5. An individual would definitely prefer investment A since $\frac{1}{2}U(10) + \frac{1}{2}U(6) > \frac{1}{2}U(9) + \frac{1}{2}U(5)$.

Answers

A4.1 **FALSE.** An individual who is risk averse would not stake $5 on a gamble which offers a 50% chance of winning $10 and a 50% chance of getting nothing. But obviously this does not mean that he would not accept a gamble with odds in favour of winning.

In figure 4.1 we can see that the expected utility from a gamble which involves giving up a certain income of C for B with probability p and A with probability $1 - p$ is less than the utility of C with certainty. But if the probability of B occurring rises to q, then the gamble is accepted in preference to the certain income C. So, if our individual is given the chance of investing in the company's share, he would accept the offer if he believes the probability of terminal wealth B was q and not if it was p.

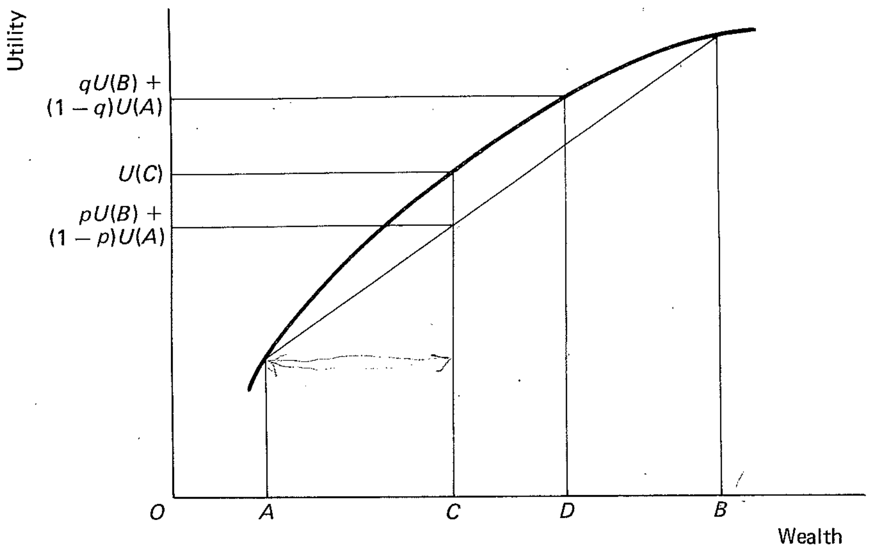

Figure 4.1

A4.2 **FALSE.** In the case of individual A

$$\tfrac{1}{2}U(W+5) + \tfrac{1}{2}U(W-5) > U(W)$$

Rearranging,

$$\tfrac{1}{2}U(W+5) - \tfrac{1}{2}U(W) > \tfrac{1}{2}U(W) - \tfrac{1}{2}U(W-5)$$

or

$$U(W+5) - U(W) > U(W) - U(W-5)$$

This last inequality shows that the addition of $5 to an initial wealth of W adds more to utility than does the addition of $5 to a wealth of $W - 5$. Hence for this individual, increments to wealth are more valuable the more wealth he has. The marginal utility of wealth is increasing, not decreasing as stated in the question. By similar reasoning, individual B must have constant marginal utility of income and C decreasing.

A4.3 **FALSE.** It is true that risk averters will require higher wages if they are to be induced to work in risky jobs and risk lovers will be the reverse. But although there may be fewer risk lovers than risk averters, it is possible that there are even fewer risky jobs. If so, it is the risk-free jobs that will have to command the premium in a competitive market to induce the risk lovers to work in them.

A4.4 (i) **FALSE.** If the man rejects the project, he has a certain income of £1000 and so a utility of 214. Investing yields an expected utility of (½ × 200) + (½ × 220) = 210. Thus he rejects the project.

(ii) **FALSE.** Undertaking the project yields a utility level of 210 which is equal to the utility of £950 with certainty. In other words, the project in question which has an expected return of £150 is equivalent in utility terms to one which is sure to lose £50. Thus the cost of risk, which is the difference between these two amounts, is £200.

(iii) **TRUE.** By undertaking the project on a shared basis each has an income of £1100 with probability ½ and £950 with probability ½. This yields an expected utility of (½ × 218.5) + (½ × 210) = 214.25 and so they would both wish to invest on a shared basis.

? (iv) **FALSE.** Each investor would be indifferent between £10 with certainty and the shared project. The expected value of the shared project is (½ × 100) + (½ × 50) = £75 per investor. Thus the cost of risk is £65 each and so £130 in total.

A4.5 **TRUE** – as it must be if the definition of risk aversion is to make sense. The cost of risk is the amount of expected income an individual is willing to give up to exchange an uncertain for a certain prospect. To see how to measure this cost, let us assume that two individuals, A and B, face a gamble which involves a 50/50 chance of losing or gaining some given sum. A is a risk averter, while B is a risk lover. Now, as can be seen from figure 4.2, A would be willing to pay up to π_0 to remove the risk. On the other hand, B would in fact have

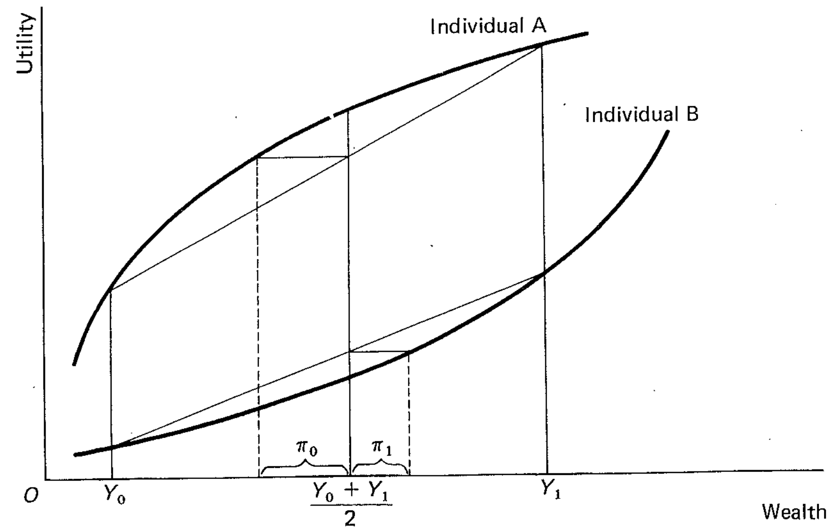

Figure 4.2

to be paid π_1 not to accept the risk. Thus, the cost of risk is positive for a risk-lover and it is negative for a risk averter.

As you can easily check, this result generalises to the proposition in the question which holds for any given prospect faced by both A and B.

A4.6 **FALSE.** If the company sells equity in itself it has a commitment to share profits with the equity owners in proportion to their holding. On the other hand, borrowing by means of an overdraft, the issue of debentures, or other non-equity debt issue involves undertaking to pay a certain sum at the end of the period no matter what the circumstances of the company. In this case the debt holder does not share in the risks of trade.

If expansion involves projects with uncertain returns a risk averse owner may only be prepared to expand if by means of issuing shares he can share his risks with others. The rationale is as in Q4.4(iii). Hence, even with no capital market constraints the firm's owners may wish to sell equity. Of course, expansion by selling equity does mean sharing ownership and loss of control, which may for various reasons be considered undesirable. Nevertheless, the benefits in terms of risk reduction may exceed the costs.

A4.7 **FALSE.** Suppose investing £100 in the safe asset yields £110 with certainty next period. However, the risky asset may return £100 or £120 with equal probability. Now if the investor would

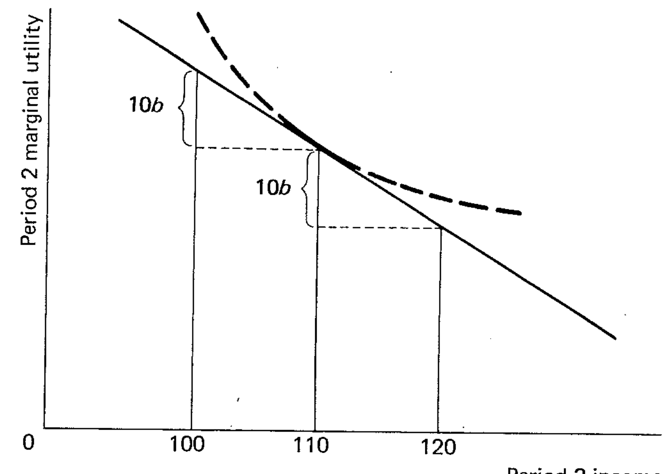

Figure 4.3

save £100 with the certain asset it follows that the last pound saved involves the loss of as much utility this period as it yields expected utility next period. If, with the uncertain asset, £100 were also to be saved then the loss of current utility from the last pound saved would be the same. But what of the expected gain next period? Suppose the marginal utility of income is declining in linear fashion, as in the solid curve in figure 4.3 (linear marginal utility implies a quadratic utility function; i.e. $U = aY - bY^2/2 + c \Leftrightarrow dU/dY = a - bY$). As compared to the marginal utility when income in period 2 is £110 with certainty, under uncertainty marginal utility when the asset yields £100 is $10b$ utils higher and when the asset yields £120 marginal utility is $10b$ utils lower. Thus the expected marginal utility yielded by the last pound saved changes by $\frac{1}{2}(10b - 1.2 \times 10b) = -0.1 \times 10b$ utils. Since expected marginal utility is lower, the increase in risk will reduce saving. However, if the marginal utility schedule were of the dashed variety which yields higher marginal utility than the linear, both when income is £100 and when it is £120, then expected marginal utility would be higher with uncertainty and hence saving would rise.

A4.8 **Option (ii) chosen.** It might seem that the two options offer the same expected return and differ only in risk. A risk-neutral consumer knowing his next period preferences with certainty would therefore be indifferent between the two alternatives. This is false. Given the demand curve of figure 4.4, the certain return from alternative (i) equals area $A + B$, which is consumer surplus when price is $1. If alternative (ii) is chosen and price turns out to be

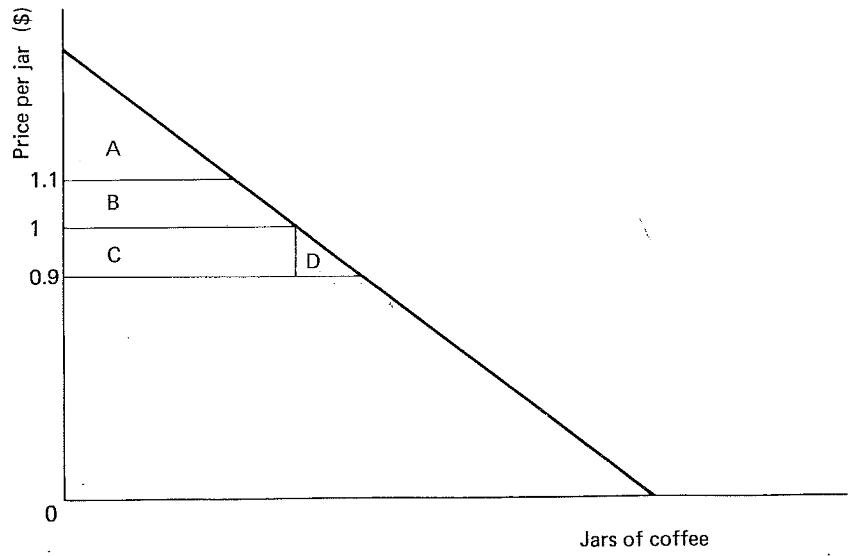

Figure 4.4

$1.10 then consumer surplus is $A + B$ and if price is $0.90 it is $A + B + C + D$. Since the two prices are equally likely, expected consumer surplus is $A + ½B + ½C + ½D$. As $C > B$, it is clear that expected consumer surplus in this latter case is greater than with the certain price of $1. Being risk neutral (i.e. having a constant marginal utility of income) the consumer will choose the alternative which yields the highest expected monetary value of consumer surplus, which is option (ii).

A4.9 (i) **INFINITE.** The probability that tails will occur on the first toss of the coin is ½. The probability of obtaining tails for the first time on the nth toss is $(½)^n$. Since there is no finite number of throws within which we can guarantee that a tail will occur we have for the expected payoff of the game, EP,

$$EP = \sum_{n=1}^{\infty} (½)^n\, 2^n = 1 + 1 + 1 + \ldots$$

An individual whose objective is to maximise expected money income should be willing to forgo all his wordly goods in exchange for an offer to play the game.

(ii) Bernoulli suggested that the reason why people would not be prepared to pay their entire income to play such a game is that the

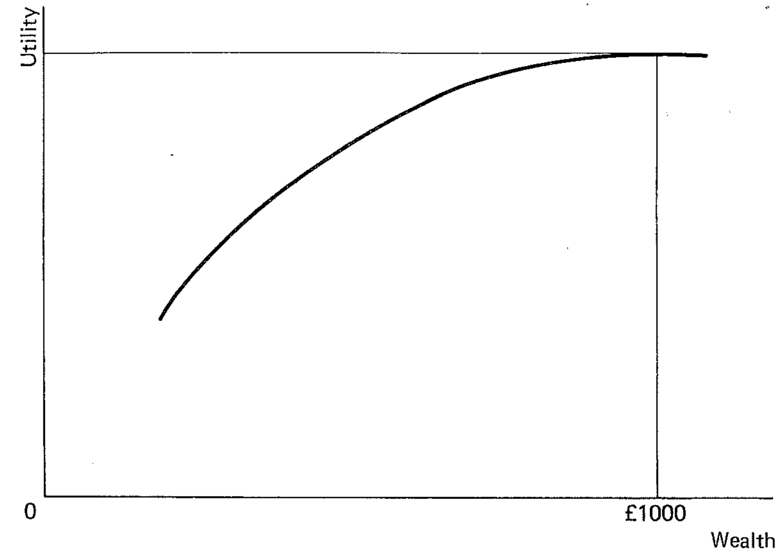

Figure 4.5

marginal utility of income is a declining function of the level of income. Suppose, for example, that marginal utility is zero when income equals £1000 (figure 4.5). It follows that no game can be more valuable to an individual than £1000 with certainty. Thus the maximum amount that our individual would possibly pay to enter the game must be less than £1000. Also, no one is in a position to offer such a game

(iii) This numerical example is illustrative of (ii). As we can see from the table the individual's utility function is bounded at a utility level of 201. Any increase in income beyond £102 adds nothing to the individual's welfare, so beyond this point the marginal utility of income is zero. If the individual does not play the game his utility will be 199.25, given he has an income of £99. But to see whether the game is in fact worth playing we must calculate the expected utility of entering. As it costs £3 to enter, a tail on the first toss will result in an income of 98, while no head on the first toss and one on the second will result in an income of £100 and so on. As we argued in (i), the probability of a tail first occuring on the nth toss is $(½)^n$, so the expected utility (EU) from playing the game will be

$$EU = \frac{1}{2}(198) + \frac{1}{4}(200) + \frac{1}{8}(201) + \frac{1}{16}(201) + \frac{1}{32}(201) + \ldots$$

$$= \frac{1}{2}(198) + \frac{1}{4}(200) + 201\left[\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots\right]$$

$(½)^i$

$1/8\left[1 + \frac{1}{2} + \frac{1}{4} + \ldots\right] = 1/8 \sum_{i=0}^{\infty} (½)^i$

The expression in square brackets is the probability that a head appears after the second throw. Since there is a ¾ chance that the game terminates before the third throw the square-bracketed term must be ¼, as you can easily check by summing the geometric series. Hence

$$EU = \frac{1}{2}(198) + \frac{1}{4}(200) + \frac{1}{4}(201) = 199.25$$

which equals expected utility if the opportunity to play the game is forgone. Thus at an entrance fee of £3 the individual is indifferent as to whether he plays the game or not. In other words, if the fee rose above £3 he would definitely not play and hence, £3 is the maximum he will pay.

A4.10 **Day 1 consumption 110 pints, day 2 consumption 95 pints, day 3 consumption 20 pints.** If the man maximises his expected utility it must be impossible for him to increase his expected utility by transferring a pint of water from planned consumption on one day to that on another. Thus, the marginal utility of consumption on each day, times the probability the water allocated to that day will actually be consumed, must be equal for all three days. It is certain that the man will be at sea for the first day and so consume the water allocated to day 1. But the chance he will still be at sea during day 2 (the probability of not making landfall on day 1) is only 0.4. The chance of still being at sea and so needing water on day 3 is 0.1. Writing the quantity of water allocation to each of the three days as P_1, P_2 and P_3, the man's consumption programme must satisfy

$$1(600 - 5P_1) = 0.4(600 - 5P_2) \quad [E(MU \text{ day } 1) = E(MU \text{ day } 2)]$$

$$0.4(600 - 5P_2) = 0.1(600 - 5P_3) \quad [E(MU \text{ day } 2) = E(MU \text{ day } 3)]$$

$$P_1 + P_2 + P_3 = 225 \quad [\text{total allocated water equals supply available}]$$

These three linear equations may be solved by simple substitution to give $P_1 = 100, P_2 = 95, P_3 = 20$.

A4.11 **\$2 is spent on medical insurance.** Substituting the budget constraint in the expression for expected utility yields

$$E = ½ \log Y + ½ (\log Y - 1/M)$$

Y+M=10
M=10-Y

Maximising E with respect to Y requires

$$\frac{dE}{dY} = \frac{1}{Y} - \frac{0.5}{(10-Y)^2} = 0$$

This is solved if $Y = 8$ and hence $M = 2$. The solution involves the expected marginal utility of expenditure on consumption goods and medical services being equal.

Incidentally, you should be able to check that a risk averter, such as the individual here, will indeed cover all medical costs by insurance so long as the policy is offered on actuarily fair terms and the policy avoids moral hazard problems (see Q14.3) perhaps by the company directly providing medical care of the contracted quality, or else paying a lump sum in the event of illness rather than paying a fixed percentage of all medical costs incurred.

A4.12 **TRUE.** If the individual parks illegally the expected monetary cost changes by the same amount whether the fine rises by 10% or the probability of having to pay it rises by 10%. However, with the increase in the fine, parking illegally is now more of a gamble. There is a smaller chance of a bigger loss. Naturally we should expect a risk averter to be more deterred by this prospect. This is illustrated in figure 4.6 where the individual has an income of Y_0, but if he pays the lower fine his disposable income is reduced to Y_f. When he parks illegally his expected income is $\bar{Y}$ and so his utility is U_0. With the higher fine, if the parker is caught, his disposable income is Y'_f but the lower probability of being caught leaves expected income at $\bar{Y}$. Expected utility now falls to U'_0 and so the individual is discouraged more by the higher fine than the lower probability of being caught.

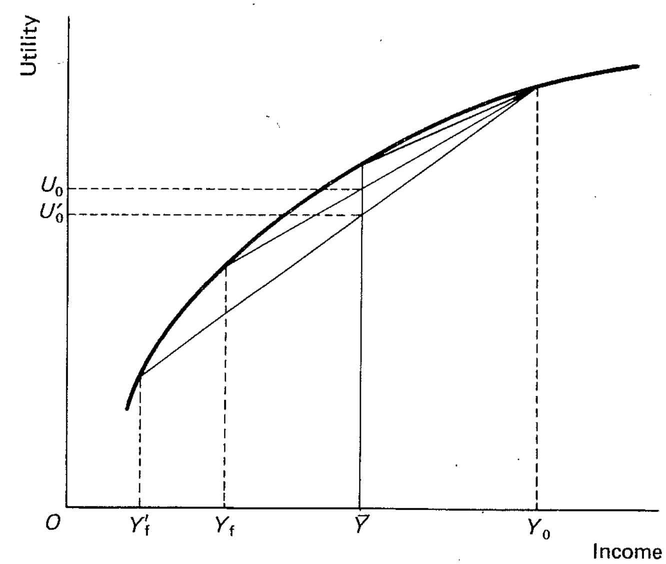

Figure 4.6

A4.13 (i) Let $\overline{X}$ and $\overline{Y}$ be the number of units of the two goods bought. For good Y units consumed, Y, equal units bought so $Y = \overline{Y}$, but for good X there is a 50% probability that $X = \frac{1}{4}\overline{X}$. Hence if $\overline{X}$, $\overline{Y}$ units are bought, expected utility is

$$EU = \tfrac{1}{2}\overline{X}^{1/2}\ \overline{Y}^{1/2} + \tfrac{1}{2}(\tfrac{1}{4}\overline{X})^{1/2}\ \overline{Y}^{1/2} = \tfrac{3}{4}\ \overline{X}^{1/2}\ \overline{Y}^{1/2}$$

The budget constraint is $\overline{X} + \overline{Y} = 16$. Maximising expected utility therefore amounts to a standard problem of maximising a Cobb–Douglas utility function subject to a conventional budget constraint. However, immediate appeal to symmetry reveals $\overline{X} = \overline{Y}$ and hence $2\overline{X} = 16$ and $\overline{X} = 8$.

(ii) Now utility is $\overline{X}^{1/2}\ \overline{Y}^{1/2}$ with certainty. The budget constraint is as before and again by symmetry $\overline{X} = \overline{Y} = 8$. Thus purchases of $\overline{X}$ remain the same. However, in the presence of uncertainty expected utility was $\frac{3}{4}\overline{X}^{1/2}\,\overline{Y}^{1/2}$ and is now $\overline{X}^{1/2}\,\overline{Y}^{1/2}$ and so has increased by a third.

(iii) Expected utility is again $\frac{3}{4}\overline{X}^{1/2}\,\overline{Y}^{1/2}$. With a Cobb–Douglas utility function it is well known and easily checked that expenditure shares equal the ratio of exponents. So in this case $P_X\overline{X} = P_Y\overline{Y} = 8$. Hence if $P_X = \frac{1}{4}$, $\overline{X} = 32$ and with $P_Y = 1$, $\overline{Y} = 8$. Expected utility $= \frac{3}{4} \times 32^{1/2} \times 8^{1/2} = 7.6$. Under certainty in (ii) $\overline{X} = \overline{Y} = 8$ and $U = \overline{X}^{1/2}\,\overline{Y}^{1/2} = 8$. Thus utility is lower in (iii) than (ii). Bearing in mind the utility function exhibits constant marginal utility of income can you work out why this result occurs?

A4.14 (i) **TRUE.** The choice open to the individual is to invest the whole sum in bonds, or in shares, or to divide it equally between the two assets. If two bonds are bought then, next period £100 of non-investment income is received plus £220 from the bonds. The utility achieved is

$$U(320) = 1980$$

If two shares are bought the expected utility is,

$$0.2U(260) + 0.8U(340) = (0.2 \times 1000) + (0.8 \times 2170) = 1936.$$

If one bond and one share, expected utility is

$$0.2U(290) + 0.8U(330) = (0.2 \times 1575) + 0.8 \times 2080) = 1979$$

Thus, as the objective of the individual is to maximise expected utility he will invest the entire sum in bonds.

(ii) **TRUE.** The expected utility from buying the two bonds will now be calculated as follows. The income yield will be £20 on which a tax of 50% is levied so net income is £10; thus the expected utility is

$$U(310) = 1856$$

Proceeding in the same way for two shares, expected utility is

$$0.2U(260) + 0.8U(320) = (0.2 \times 1000) + (0.8 \times 1980) = 1784.$$

For one share and one bond, expected utility is

$$0.2U(285) + 0.8U(315) = (0.2 \times 1500) + (0.8 \times 1926) = 1840.8$$

Thus with no tax loss-offset the individual will continue to invest in the two bonds.

(iii) **TRUE.** With full loss-offset utility is

$$U(310) = 1856 \text{ for the case of two bonds}$$

For the case of one bond and one share, expected utility is

$$0.2U(290) + 0.8U(315) = (0.2 \times 1575) + (0.8 \times 1926) = 1856.8$$

For the case of two shares, expected utility is

$$0.2U(260) + 0.8U(320) = (0.2 \times 1000) + (0.8 \times 1980) = 1784.$$

Thus in this case the individual will buy one share and one bond.

A4.15 **TRUE.** Suppose my utility if it does not rain and no umbrella is carried is $\overline{U}$. If I carry no umbrella my expected utility is therefore $\frac{1}{2}\overline{U} + \frac{1}{2}(\overline{U} - 3) = \overline{U} - 1\frac{1}{2}$. Carrying an umbrella yields expected utility of $\frac{1}{2}(\overline{U} - \frac{1}{2}) + \frac{1}{2}(\overline{U} - 1\frac{1}{2}) = \overline{U} - 1$. Thus, if I am an expected utility maximiser, I carry an umbrella.

A4.16 (i) $\frac{2}{3}$. Suppose the farmer owns L acres of land and devotes a fraction, α, to wheat. His expected utility will then be

$$E(U) = \tfrac{1}{2}\log\,[2000\alpha L + 1000\,(1-\alpha)] + \tfrac{1}{2}\log\,[1000\alpha L + 1750\,(1-\alpha)L]$$

where the first square-bracketed term shows income if the weather is fine and the second bracketed term is income if the weather is bad. The problem is to choose α so as to maximise $E(U)$. First order conditions are

$$\frac{dE(U)}{d\alpha} = \frac{1}{2}\,\frac{2000L - 1000L}{2000\alpha L + 1000(1-\alpha)L} + \frac{1}{2}\,\frac{1000L - 1750L}{1000\alpha L + 1750(1-\alpha)L} = 0$$

Cancelling L this equation solves to give $\alpha = \frac{2}{3}$.

(ii) **All land in wheat and \$500 of insurance.** Diverting an acre of land from wheat to potatoes reduces income by \$1000 if the weather is good and raises income by \$750 if the weather is bad. But if instead the land were left in wheat and \$1000 of insurance taken out the fall in income if the weather is good is unchanged at \$1000, but now the rise is \$1000 when the weather is bad. It is therefore better for the risk averse farmer to put all his land in wheat, which offers the higher expected return, and offset the risk by insuring, rather than growing both wheat and potatoes. Since the insurance policy allows the farmer to trade-off income dollar for dollar between the two equally likely states of the world he will equalise marginal utility in the two states, and hence income. To do this requires that he takes out \$500 (per acre) of insurance for then each acre of land will earn him \$1500 whatever the weather.

(iii) $\frac{2}{3}$ **to wheat and no insurance.** Potatoes are now clearly superior to insurance as a means of risk avoidance. Thus the farmer reverts to his original strategy.

A4.17 **TRUE.** Let $U_1(Y_1)$ be the utility of consuming an income of Y_1 in state of the world 1 and $U_2(Y_2)$ that of consuming Y_2 in state of the world 2. If p is the probability of state 1 occuring and $1-p$ the probability of state 2, the expected utility of the income combination Y_1, Y_2 is

$$E(U) = p\,U_1(Y_1) + (1-p)U_2(Y_2) \tag{1}$$

The indifference curves plot combinations of Y_1 and Y_2 which preserve a constant level of expected utility. Differentiation of (1) holding $E(U)$ constant yields

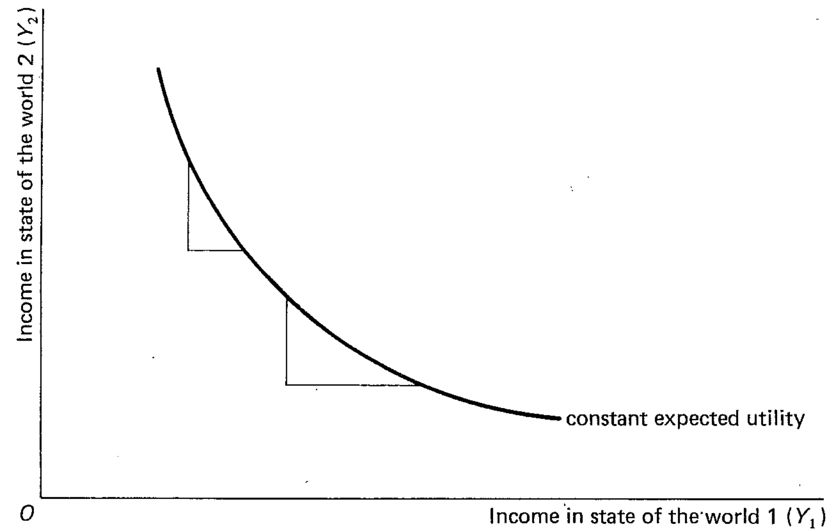

Figure 4.7

$$\frac{dY_2}{dY_1} = -\frac{p\,dU_1/dY_1}{(1-p)dU_2/dY_2} \tag{2}$$

which is the slope of the indifference curve between income in the two states. Suppose Y_1 is increased; it follows from the RHS of (2) being negative that Y_2 must be reduced. We have also assumed risk aversion which is equivalent to diminishing marginal utility. Thus with Y_1 higher, dU_1/dY_1 must be lower. Also Y_2 lower means that dU_2/dY_2 is higher. Hence with Y_1 higher (2) is less negative and the slope of the indifference curve flattens out as Y_1 increases. This is illustrated in figure 4.7.

A4.18 **TRUE.** An individual will be in equilibrium when the slope of his inter-state budget constraint is equal to that of his inter-state indifference curve. We know that the slope of the indifference curve is given by equation (2) of A4.17. (Note that since the utility function is now assumed to be the same in both states we drop the subscripts 1 and 2.)

Now let us find the slope of the budget constraint. If insurance is sold at a price such that the cost of the policy equals the expected payout, then it will be the case that the rate at which income can be transferred from state II to state I is the ratio of the probabilities of the two states occurring. To see this let us define π to be the insurance premium (cost) of a policy paying y if state I occurs and nothing otherwise. The expected net return from selling the policy is therefore

premium = π
pay out = y
net pay out : $y-\pi$

$$EP = (1-p)\pi + p(y-\pi)$$

If $EP = 0$

$$\frac{p}{1-p} = -\frac{\pi}{(y-\pi)}$$

$\pi/(y-\pi)$ is simply the rate at which we can transfer income out of state 2 into state 1. Therefore, we require for equilibrium that

$$\frac{p}{1-p} = \frac{dU/dY_1}{dU/dY_2}\,\frac{p}{1-p}$$

where the LHS of the equation is the slope of the indifference curve (see equation (2) of A4.17). This in turn requires

$$\frac{dU(Y_1)}{dY_1} = \frac{dU(Y_2)}{dY_2}$$

which can only hold if income is equalised in both states.

A4.19 **FALSE.** The answer suggested in the question implicitly assumes that the two investments yield their high and low returns in the same state of the world. But this does not follow. Investment B may yield its £9 payoff in the state of the world in which asset A yields £6. If the individual is short of income in this state of the world (say a state associated with crop failure) or if the utility of income in this state is particularly high, he may well prefer asset A. The mistake is that the inequality in the question does not specify in which states the various payoffs occur.

5

Production and Cost Theory

References

Becker Chapter 7. Ferguson Part II. Friedman Chapters 5 and 6. Hirshleifer Chapter 9. Laidler Chapters 11 and 12. Lancaster Chapters 4 and 5. Layard Chapters 9 and 10. Mansfield Chapters 5, 6 and 7. Stigler Chapters 6 to 9.

Questions

Q5.1 The production function shows how much output is obtained from given quantities of inputs.

Q5.2 To produce a given output at minimum cost the marginal rate of technical substitution should equal the wage–interest ratio.

Q5.3 If the production function exhibits increasing returns to scale everywhere, a firm's long-run average cost curve must be declining.

Q5.4 If capital and labour must be combined in fixed proportions to produce a good, marginal products are undefined.

Q5.5 A firm uses 10 units of labour and 20 units of capital to produce 10 units of output. The marginal product of labour is 0.5. If there are constant returns to scale the marginal product of capital must be 0.25.

Q5.6 Short-run average total cost is never less than long-run average total cost.

Q5.7 Short-run marginal cost is never less than long-run marginal cost.

Q5.8 If the average product of labour is at a maximum it must equal the marginal product.