Part 3 of 3

lines are competing for shares of a market having \$450,000 in total revenue. The cost for each additional daily departure is \$20,000. Let's derive an expression for airline A's profit. We denote the airlines' numbers of departures by a, b, and c, respectively. Then airline A's profit (in thousands of dollars) can be expressed as

$$\pi_a = 450(a/a + b + c) - 20a.$$

Here, A's share of total revenue is $a/(a + b + c)$. For instance, if all airlines fly identical numbers of flights, they obtain one-third market shares, or \$150,000 in revenue each. If airline A provides half the total flights, it claims half the total revenue, and so on. The second term in the profit expression is the total cost of providing this number of departures. (Analogous expressions apply for the other airlines.) Inspection of this equation reveals the airline's basic trade-off: By flying more flights, it claims a greater share of revenue, at the same time incurring additional costs. Moreover, the larger the number of competitors' flights, the smaller the airline's revenue share. If all airlines fly "too many" flights, they all incur large costs, but the result remains a revenue standoff.

Table 13.7 lists the typical airline's payoff table. Note two things about the table. First, it has condensed the decisions of the other two airlines into one variable: the total number of competitor flights. (The columns list numbers of these flights ranging from 5 to 17.) From the profit equation, we observe that an airline's profit depends only on the number of its own flights (a) and the total number of competitor flights (b + c). For example, if airline A mounts five flights and B and C have a total of five flights, A's profit is (5/10)(450 thousand) − (5)(20 thousand) = \$125 thousand, as shown in the table. Second, only the airline's own profit is listed (to save space). As we would expect, each firm's profit is highly sensitive to the number of flights flown by its competitors. By reading across the payoffs in any row, we see that an airline's profit falls drastically as the number of competing flights increases.

**TABLE 13.7**

An Airline's Payoff Table

The airline's best responses are shown in blue. In equilibrium, each of the three airlines flies five daily departures.

| Own Number of Flights | Total Number of Competitors' Flights: 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 50.0 | 50.0 | 50.0 | 50.0 | 41.8 | 35.0 | 29.2 | 24.3 | 20.0 | 16.3 | 12.9 | 10.0 | 7.4 |
| 3 | 75.0 | 75.0 | 75.0 | 62.7 | 52.5 | 43.8 | 36.4 | 30.0 | 24.4 | 19.4 | 15.0 | 11.1 | 7.5 |
| 4 | 100.0 | 100.0 | 83.6 | 70.0 | 58.5 | 48.6 | 40.0 | 32.5 | 25.9 | 20.0 | 14.7 | 10.0 | 5.7 |
| 5 | 125.0 | 104.5 | 87.5 | 73.1 | 60.7 | 50.0 | 40.6 | 32.4 | 25.0 | 18.4 | 12.5 | 7.1 | 2.3 |
| 6 | 125.5 | 105.0 | 87.7 | 72.9 | 60.0 | 48.8 | 38.8 | 30.0 | 22.1 | 15.0 | 8.6 | 2.7 | −2.6 |
| 7 | 122.5 | 102.3 | 85.0 | 70.0 | 56.9 | 45.3 | 35.0 | 25.8 | 17.5 | 10.0 | 3.2 | −3.0 | −8.8 |
| 8 | 116.9 | 97.1 | 80.0 | 65.0 | 51.8 | 40.0 | 29.5 | 20.0 | 11.4 | 3.6 | −3.5 | −10.0 | −16.0 |

Supposing the three airlines are going to compete month after month, how might they set their number of departures each period? In answering this question, we consider two possible benchmarks: equilibrium behavior and collusive behavior.

To help in identifying equilibrium behavior, best-response payoffs are in blue in Table 13.7. For example, if its competitors schedule only five total flights, the airline's best response is six flights, earning it $125,500 (the highest payoff in column 1); against 13 flights, the airline's best response is four flights; and so on. The table shows that no airline has a dominant strategy. (The more numerous the competitors' flights, the fewer flights the airline should fly.) However, it is striking that the best responses congregate closely around five flights (ranging from 3 to 6). In fact, the unique equilibrium has each of the airlines mounting exactly five flights. To confirm this, note that if the competitors fly five each (or ten total), the airline's best response also is to fly five. (You might want to check by trial and error that no other combination of flights is an equilibrium.) In equilibrium, each airline's profit is $50,000; total flights number 15, and industry profit is $150,000.

What if the airlines could tacitly collude in determining the total number of daily departures? Because market revenue is fixed at $450,000, the best the industry can do is carry the 2,000 passengers at least cost, that is, by using the fewest flights. This requires ten daily departures (fully loaded), since capacity per plane is 200 passengers. Total industry profit is 450,000 − (10)(20,000) = $250,000. If the airlines fly three, three, and four flights, their respective profits are $75,000, $75,000, and $100,000. One possibility is a tacit agreement among the airlines limiting the number of departures—ostensibly to achieve efficient loadings—perhaps alternating delivery of the tenth flight.

Remember, however, that such a tacit understanding is very fragile. If the other airlines limit themselves to six total flights, the last airline's best response is six flights, not four. Although it maximizes industry profit, collusive behavior does not constitute an equilibrium. Any airline can profit by unilaterally increasing its number of departures. Indeed, until the late 1980s, airlines competed vigorously for passengers, by offering the convenience of frequent departures—but on flights that were far from filled. With the emergence of the airline hub systems in recent years, the number of departures has stabilized.

# SUMMARY

## Decision-Making Principles

1. The formal study of competitive behavior by self-interested players is the subject of game theory. In competitive settings, determining one's own optimal action depends on correctly anticipating the actions and reactions of one's rivals.

2. A dominant strategy is a best response (i.e., maximizes the player's profit) with respect to any strategy that a competitor takes. If a dominant strategy exists, a rational individual should play it.
3. In a (Nash) equilibrium, each player employs a strategy that maximizes his or her expected payoff, given the strategies chosen by the others. Game theory predicts that the outcome of any competitive situation will be an equilibrium: a set of strategies from which no player can profitably deviate.
4. In sequential competition, the manager must think ahead. His or her best course of action depends on anticipating the subsequent actions of competitors.

## Nuts and Bolts

1. Payoff tables are essential for analyzing competitive situations. A payoff table lists the profit outcomes of all firms as these outcomes depend on the firms' own actions and those of competitors.
2. In a zero-sum game, the interests of the players are strictly opposed; one player's gain is the other's loss. By contrast, a non-zero-sum game combines elements of competition and cooperation.
3. When players take independent actions (play noncooperatively), the solution of the game involves the play of equilibrium strategies.
4. When there are multiple equilibria, it is often advantageous to claim the first move.
5. If players can freely communicate and reach a binding agreement, they typically will try to maximize their total payoff.
6. A game tree lists the sequence of player actions and their resulting payoffs. It is possible to solve any game with perfect information by backward induction.
7. In repeated games, the use of contingent strategies and the formation of reputations serves to broaden the range of equilibrium behavior.

## Questions and Problems

1. Give a careful explanation of a Nash equilibrium. How is it different from a dominant-strategy equilibrium?
2. Is it ever an advantage to move first in a zero-sum game? When is it an advantage to have the first move in a non-zero-sum game? Provide an example in which it is advantageous to have the second move.

3. Consider the payoff table below.

| | | Firm Z | | |
|---|---|---|---|---|
| | | C1 | C2 | C3 |
| Firm Y | R1 | −1 | −2 | 4 |
| | R2 | 0 | 2 | 2 |
| | R3 | −2 | 4 | 0 |

   a. Does either player have a dominant strategy? Does either have a dominated strategy? Explain.
   b. Once you have eliminated one dominated strategy, see if some other strategy is dominated. Solve the payoff table by iteratively eliminating dominated strategies. What strategies will the players use?

4. a. Identify the equilibrium outcome(s) in each of the following payoff tables.
   b. In each table, predict the exact outcome that will occur and explain your reasoning.
   c. In table c, suppose the column player is worried that the row player might choose R2 (perhaps a one-in-ten chance). Given this risk, how should the column player act? Anticipating the column player's thinking, how should the row player act?

**a.**

| | C1 | C2 |
|---|---|---|
| R1 | 12, 10 | 10, 4 |
| R2 | 4, 8 | 9, 6 |

**b.**

| | C1 | C2 |
|---|---|---|
| R1 | 12, 10 | 4, 4 |
| R2 | 4, 4 | 9, 6 |

**c.**

| | C1 | C2 |
|---|---|---|
| R1 | 12, 10 | 4, 4 |
| R2 | 4, −100 | 9, 6 |

5. Firms J and K produce compact-disc players and compete against one another. Each firm can develop either an economy player (E) or a deluxe player (D). According to the best available market research, the firms' resulting profits are given by the following payoff table:

| | | Firm K | |
|---|---|---|---|
| | | E | D |
| Firm J | E | 30, 55 | 50, 60 |
| | D | 40, 75 | 25, 50 |

   a. The firms make their decisions independently and each is seeking its own maximum profit. Is it possible to make a confident prediction concerning their actions and the outcome? Explain.

b. Suppose that firm J has a lead in development and so can move first. What action should J take and what will be K's response?
c. What will be the outcome if firm K can move first?

6. Two firms dominate the market for surgical sutures and compete aggressively with respect to research and development. The following payoff table depicts the profit implications of their different R&D strategies:

| | | Firm B's R&D Spending | | |
|---|---|---|---|---|
| | | **Low** | **Medium** | **High** |
| **Firm A's R&D Spending** | **Low** | 8, 11 | 6, 12 | 5, 14 |
| | **Medium** | 12, 9 | 8, 10 | 6, 8 |
| | **High** | 11, 6 | 10, 8 | 4, 6 |

a. Suppose that no communication is possible between the firms; each must choose its R&D strategy independent of the other. What actions will the firms take and what is the outcome?
b. If the firms can communicate before setting their R&D strategies, what outcome will occur? Explain.

7. One way to lower the rate of auto accidents is strict enforcement of motor vehicle laws (speeding, drunk driving, and so on). However, maximum enforcement is very costly. The payoff table below lists the payoffs of a typical motorist and a town government. The motorist can obey or disobey motor vehicle laws, which the town can enforce or not.

| | | Town | |
|---|---|---|---|
| | | **Enforce** | **Don't Enforce** |
| **Motorist** | **Obey** | 0, −15 | 0, 0 |
| | **Don't Obey** | −20, −20 | 5, −10 |

a. What is the town's optimal strategy? What is the typical motorist's behavior in response?
b. What if the town could commit to a strict enforcement policy and motorists believed that this policy would be used? Would the town wish to do so?
c. Now suppose the town could commit to enforcing the law part of the time. (The typical motorist cannot predict exactly when the town's traffic police will be monitoring the roadways.) What is the town's optimal degree (i.e., percentage) of enforcement? Explain.

8. Consider the following zero-sum game.
   a. Does either player have a dominant strategy? Does either have a dominated strategy? Explain.
   b. Find the players' equilibrium strategies.

|  |  | Player C |  |  |
|---|---|---|---|---|
|  |  | **C1** | **C2** | **C3** |
| **Player R** | **R1** | 13 | 12 | 10 |
|  | **R2** | 14 | 6 | 8 |
|  | **R3** | 3 | 16 | 7 |

9. In 1992, Saudi Arabia and Iran (both members of OPEC) produced an average of 5,000,000 and 2,000,000 barrels of oil a day. Production costs were about $10 per barrel and the price of oil averaged $20. Each country had the capacity to produce an additional 1,000,000 barrels per day. At that time, it was estimated that each 1,000,000–barrel increase in supply would depress the average price of oil by $2.
   a. Fill in the missing profit entries in the payoff table below.

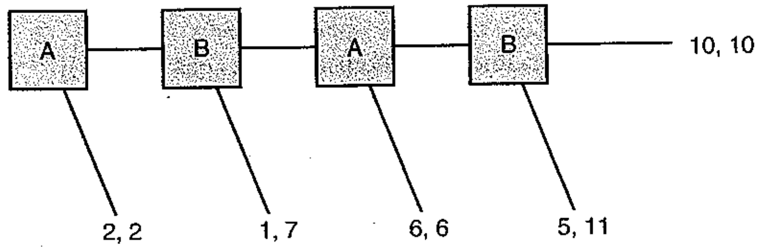
|  |  | Iran |  |
|---|---|---|---|
|  |  | **2 M barrels** | **3 M barrels** |
| **Saudi Arabia** | **5 M barrels** | —, — | —, — |
|  | **6 M barrels** | —, — | —, — |

   b. What actions should each country take and why?
   c. Does the asymmetry in the countries' sizes cause them to take different attitudes toward expanding output? Explain why or why not. Comment on whether or not a prisoner's dilemma is present.

10. Firm A and firm B are battling for market share in two separate markets. Market I is worth $30 million in revenue; market II is worth $18 million. Firm A must decide how to allocate its three salespersons between the markets; firm B has only two salespersons to allocate. Each firm's revenue share in each market is *proportional* to the number of salespeople the firm assigns there. For example, if firm A puts two salespersons and firm B puts one salesperson in market I, A's revenue from this market is [2/(2 + 1)] $30 = $20 million and B's revenue is the remaining $10 million. (The firms split a market equally if neither assigns a salesperson to it.) Each firm is solely interested in maximizing the *total* revenue it obtains from the two markets.

a. Compute the complete payoff table. (Firm A has four possible allocations: 3-0, 2-1, 1-2, and 0-3. Firm B has three allocations: 2-0, 1-1, 0-2.) Is this a constant-sum game?
b. Does either firm have a dominant strategy (or dominated strategies)? What is the predicted outcome?

11. In the following game tree, players A and B alternate moves. At each turn, a player can terminate the game or pass the move to the next player. By passing, the player increases the rival's potential payoff by five units and reduces her own by one unit. Thus, as long as both players pass the move on to one another, their payoffs increase.




a. Suppose you are paired with another student with whom you will play the game. Just based on your judgment (no analysis), how would you play?
b. Now analyze the game tree by averaging backwards. What actions should the players take? What is the outcome? Briefly explain this result.

12. Four large used-car dealers compete for customers in a city where demand for used automobiles is constant at about 800 cars per year. By an implicit agreement, the dealers set comparable prices on their cars, with the result that price wars and competitive discounting are extremely rare. All dealers claim to have the lowest prices, but the facts say otherwise. The average (variable) cost of a used car to the dealer (procuring and readying it for sale) is \$2,400. The average sale price per car is \$4,000.

The dealers do compete with respect to the number and types of cars in their showrooms. The typical prospective buyer visits a number of dealers looking for the "right" car. The greater the number of cars a dealer has available, the better is its chance of making a sale. In fact, a particular dealer's share of the total market is proportional to the number of cars it holds in its showroom. Thus, dealer 1's profit can be expressed as $\pi_1 = 3{,}200{,}000\,[x_1/(x_1 + x_2 + x_3 + x_4)] - 2{,}400x_1$. The profit expressions for the other dealers are analogous.

The following table lists the profit of a typical dealer (for various inventories) when it faces competitors with different average inventories. For instance, stocking 175 cars when the other three dealers average 275 cars *each* (a total of 825) yields a profit of \$140,000. Dealer 1's inventory is 175/1,000 = 17.5 percent of the total. Thus, it sells (.175)(800) = 140 cars at \$4,000 each while paying for 175 cars at \$2,400 each. The net profit is \$140,000. The other entries follow from straightforward calculations.

**Average Auto Inventory of the Other Three Dealers**

| | | 150 | 175 | 200 | 225 | 250 | 275 | 300 | 325 |
|---|---|---|---|---|---|---|---|---|---|
| | **150** | 240 | 240 | 240 | 221.8 | 173.3 | 132.3 | 97.1 | 66.6 |
| | **175** | 280 | 280 | 280 | 238.8 | 185.4 | 140 | 100.9 | 67.0 |
| | **200** | 320 | 320 | 320 | 251.4 | 193.7 | 144.4 | 101.8 | 64.7 |
| **Dealer 1's** | **225** | 360 | 360 | 332.7 | 260 | 198.5 | 145.7 | 100 | 60 |
| **Inventory** | **250** | 400 | 400 | 341.2 | 264.9 | 200 | 144.2 | 95.6 | 53.0 |
| | **275** | 440 | 440 | 345.7 | 266.3 | 198.5 | 140 | 88.9 | 44 |
| | **300** | 480 | 443.6 | 346.7 | 264.6 | 194.3 | 133.3 | 80 | 32.9 |
| | **325** | 520 | 443.5 | 344.3 | 260 | 187.4 | 124.3 | 69.0 | 20 |
| | **350** | 560 | 440 | 339.0 | 252.7 | 178.2 | 113.2 | 56 | 5.3 |

a. Find dealer 1's best inventory response to the various inventory actions of the other dealers. (Circle the greatest profit entry in each column of the table.) Does inventory competition more closely resemble quantity or price competition?
b. What is the equilibrium inventory level for each of the four dealers?
c. If the dealers colluded to limit inventories, what is the maximum monopoly profit they could earn collectively? Would individual dealers have an incentive to cheat on their inventories? Explain.
d. What would be the effect of free entry into the used-car business?

13. The payoff table below lists the profits of a buyer and a seller. The *seller acts first* by choosing a sale price (\$9, \$8, or \$6). The buyer then decides the quantity of the good to purchase (two units, four units, six units, or eight units).

**Buyer Quantities**

| | | 2 units | 4 units | 6 units | 8 units |
|---|---|---|---|---|---|
| | **P = \$9** | 10, 6 | 20, 5 | 30, 0 | 40, −8 |
| **Seller Prices** | **P = \$8** | 8, 8 | 16, 9 | 24, 6 | 32, 0 |
| | **P = \$6** | 4, 12 | 8, 17 | 12, 18 | 16, 16 |

a. Suppose the buyer and seller transact only once. Does the buyer have a dominant strategy? Depending on the price quoted, what is her best response? What price should the seller set? Explain carefully.
b. Suppose the seller and buyer are in a multi-year relationship. Each month, the buyer quotes a price and the seller selects her quantity. How might this change the players' strategies?
c. Now suppose the buyer and seller are in a position to negotiate an agreement specifying price and quantity. Can they improve upon the result in part a? Which quantity should they set? What price would be equitable? Explain.

14. In the 1990s, the Delta Shuttle and the U.S. Air Shuttle continue to battle for market share on the Boston/New York and Washington, D.C./New York routes. In addition to service quality and dependability (claimed or real), the airlines compete over price via periodic fare changes. The hypothetical payoff table below lists each airline's estimated profit (expressed on a per-seat basis) for various combinations of one-way fares.

| **Delta Shuttle** | **U.S. Air Shuttle Fares** | | |
|---|---|---|---|
| **Fares** | **$139** | **$119** | **$99** |
| **$139** | $34, $38 | 15, 42 | 6, 32 |
| **$119** | 42, 20 | 22, 22 | 10, 25 |
| **$ 99** | 35, 7 | 27, 9 | 18, 16 |

a. Suppose that the two airlines will select their fares independently and "once and for all." (The airline's fare cannot be changed.) What fares should the airlines set?
b. Suppose, instead, that the airlines will set fares over the next 18 months. In any month, each airline is free to change its fare if it wishes. What pattern of fares would you predict for the airlines over the 18 months?
c. Pair yourself with another student from the class. The two of you will play the roles of Delta and U.S. Air and set prices for the next 18 months. You will exchange written prices for each month. You then can determine your profit (and your partner's profit) from the payoff table. The competition continues in this way for 18 months, after which time you should compute your total profit (the sum of your monthly payoffs). Summarize the results of your competition. What lessons can you draw from it?

15. Consider the payoff tables on the following page.

In the first period, each firm determines its price: high or low. In the second period, each firm chooses a design standard (design 1

| Firm 1 \ Firm 2 | H | L |
|---|---|---|
| H | 4, 4 | 1, 6 |
| L | 6, 1 | 3, 3 |

| Firm 1 \ Firm 2 | $D_1$ | $D_2$ |
|---|---|---|
| $D_1$ | 5, 5 | 0, 0 |
| $D_2$ | 0, 0 | 2, 2 |

or design 2). They both gain if they choose the same, compatible standard. They receive no gain if they choose different, incompatible standards.

a. What are the equilibria of each payoff table considered separately?

b. In the two-stage competition, check whether the following strategies are or are not in equilibrium.

i. Both players use H followed by D1.

ii. Both players use H followed by D1 if the opponent used H originally and D2 otherwise.

What do your answers suggest about a way out of the prisoner's dilemma?

**CHECK STATION ANSWERS**

1. Store 1 does not have a dominant strategy, but store 2 does. Regardless of store 1's action, store 2's optimal choice is to promote women's clothing. Anticipating this behavior, store 1's best response is to promote girls' clothing. Despite the seeming symmetry of the example, store 1 fares much better than store 2.
2. This competition is a constant-sum game because the players' payoffs in each cell add up to the same sum. (Here the market shares always add up to 100.) The method of circles and squares pinpoints the equilibrium at R2 and C2. This is exactly the same outcome as in the zero-sum version in Table 13.2. There is no strategic difference between a zero-sum game and its constant-sum counterpart.
3. If its rival produces six units, the firm's best response is either eight or ten units. (Actually, an amount not shown, nine units, would be absolutely best.) If its rival produces eight units, the firm's best response is eight units. If its rival produces ten units, the firm's best response is either six or eight units (actually, seven is best). This shows that neither firm has a dominant strategy. The firms might hope to produce six units each, but this is not an equilibrium. Either could gain at the other's expense by increasing output (ideally, to nine units). The sole equilibrium has each firm producing eight units—the same answer as found in Chapter 12.
4. Originally, there are two equilibria: Boeing alone produces or Airbus alone produces. With government subsidy, Airbus's dominant strategy is to produce. Knowing this, Boeing gives up the market.

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*Appendix to Chapter 13*

# Mixed Strategies

Whenever a player selects a particular course of action with certainty, we refer to this as a *pure* strategy. All of the applications in the main body of this chapter have involved pure-strategy equilibria, for instance, R2 versus C2 in the market-share competition. However, in other settings, optimal play frequently requires the use of *mixed (or randomized)* strategies. Here, a player randomizes between two or more pure strategies, selecting each with fixed probabilities. Consider a second version of the market-share competition.

**MARKET COMPETITION REVISITED** Suppose that the firms have only their first and third strategies available. The payoff table in Table 13A.1 is identical to that of Table 13.2 except that the second strategy of each player is omitted. Now, there is no pure-strategy equilibrium. Instead, the players' best responses "cycle" and never settle down to any pair of stategies. For example, beginning at R1,C1, firm 1 would gain by switching to R3. But R3,C1 is not stable since now firm 2 would gain by switching to C3. But R3,C3 will give way to R1,C3 (after firm 1 switches) and, in turn, this gives way to R1,C1 (after firm 2 switches). We are back to where we began.

Though there is no equilibrium in pure strategies, the payoff table does have a unique equilibrium when players use particular mixed strategies. To qualify as a mixed-strategy equilibrium:

> The player's chosen probabilities must ensure that the other player earns the same expected payoff from any of the pure strategies making up his or her mixture.

**TABLE 13A.1**

Mixed Strategies in a Zero-Sum Game

| | | Firm 2 C1 | Firm 2 C3 |
|---|---|---|---|
| Firm 1 | R1 | −2 | 4 |
| Firm 1 | R3 | 7 | −5 |

| | | | Firm 2 (1/2) C1 | Firm 2 (1/2) C3 | Firm 1's Expected Payoff |
|---|---|---|---|---|---|
| Firm 1 | (2/3) | R1 | −2 | 4 | 1 |
| Firm 1 | (1/3) | R3 | 7 | −5 | 1 |
| | | Firm 2's Expected Payoff | 1 | 1 | |

This statement is quite a mouthful and requires some explaining.

Why must the opponent's pure strategies earn the *same* expected payoff? To see this, let's turn back to the market-share competition. Suppose firm 1 decided to randomize between R1 and R3, each with probability .5. This is a plausible mixed strategy but, as we shall see, is not in equilibrium. Suppose firm 2 anticipates firm 1 using this 50–50 mixture. What is firm 2's best response? Suppose firm 2 considers C1. Because firm 1's actual action is uncertain, firm 2 must compute its expected payoff. From Table 13A.1, this is: $(.5)(-2) + (.5)(7) = 2.5$. Alternatively, using C3, firm 2's expected payoff is: $(.5)(4) + (.5)(-5) = -.5$. Clearly, firm 2 always prefers to play C3. (Remember, firm 2 is trying to *minimize* the expected market share increase of firm 1.) But, if firm 2 always is expected to play C3, then it would be foolish for firm 1 to persist in playing the 50–50 mixture. Firm 1 should respond to C3 by playing R1 all the time. But then firm 2 would not want to play C3, and we are back in a cycle of second guessing. In short, mixed strategies where one player's pure strategies have different expected payoffs cannot be in equilibrium.

Now we are ready to compute the "correct" equilibrium probabilities for each firm's mixed strategy. Start with firm 1. Let x denote the probability it plays R1 and $1 - x$ the probability it plays R3. If firm 2 uses C1, its expected payoff is: $(x)(-2) + (1 - x)(7)$. If, instead, it uses C3, its expected payoff is: $(x)(4) + (1 - x)(-5)$. Firm 2 is indifferent between C1 and C3 when these expected payoffs are equal:

$$-2x + 7(1 - x) = 4x - 5(1 - x), \qquad [13A.1]$$

or 12 = 18x. Thus, x = 2/3. In equilibrium, firm 1 uses R1 and R3 with probabilities 2/3 and 1/3 respectively. Turning to firm 2, let y denote the probability it plays C1 and 1 − y the probability it plays C3. If firm 1 uses R1, its expected payoff is: (y)(−2) + (1 − y)(4). If, instead, it uses R3, its expected payoff is: (y)(7) + (1 − y)(−5). Equating these expected payoffs implies:

$$-2y + 4(1 - y) = 7y - 5(1 - y), \qquad [13A.2]$$

or 9 = 18y. Thus, y = 1/2. In equilibrium, firm 2 uses C1 and C3, each with probability 1/2. In the lower payoff table of Table 13A.1, we display these mixed strategies.[1] Finally, what is each firm's expected payoff when it uses its mixed strategy? If we substitute x = 2/3 into either side of Equation 13A.1, we find that firm 2's expected payoff is 1 from either of its pure strategies. Thus, the expected payoff for its mixed strategy is also 1. Similarly, firm 1's expected payoff is 1 from either of its strategies. These expected payoffs also are shown in the bottom section of Table 13A.1. In short, when both sides use their optimal mixed strategies, firm 1's expected gain in market share (and firm 2's expected loss) is one percent.

**REMARK** In this equilibrium, neither side can improve its expected payoff by switching from its mixed strategy. In fact, a player actually does not lose by switching to some other strategy proportion. For instance, as long as firm 2 uses its 50–50 mixed strategy, firm 1 earns the same expected payoff from *any* mixture of R1 and R3 (one-third/two-thirds, 50–50, etc.). The penalty for switching from equilibrium proportions comes in a different form: A smart opponent can take advantage of such a switch. Using its equilibrium strategy, firm 1 *guarantees* itself an expected payoff of 1 against the equilibrium play of firm 2 or against any other play. If firm 1 were to switch to non-equilibrium strategy proportions (let's say 60–40 proportions), it gives firm 2 the chance to gain at its expense by switching to C3. (Against a 60–40 mix by firm 1, the expected payoff of C3 is .4.) Firm 1's original advantage (an expected payoff of +1 in equilibrium) now would be eroded. In short, wandering from the original equilibrium play is ill-advised.

[1]There is a simple rule for finding the mixed strategies in a 2-by-2 payoff table like the one to the right. Firm 1's proportions are x = (d − c)/[(d − c) + (a − b)] and 1 − x = (a − b)/[(d − c) + (a − b)]. Firm 2's proportions are y = (d − b)/[(d − b) + (a − c)] and 1 − y = (a − c)/[(d − b) + (a − c)]. To find the R1 chance, take the difference between the entries in the *opposite row* (d − c) and then divide by the sum of the row differences, (d − c) + (a − b). The same opposite-row rule works for the R2's chances, and an *opposite-column* rule works for computing firm 2's mixed strategy proportions.

| | | y | 1 − y |
|---|---|---|---|
| | | **C1** | **C2** |
| x | **R1** | a | b |
| 1 − x | **R2** | c | d |

**TABLE 13A.2**

Mixed Strategies in a Non-Zero-Sum Game

| | | Player 2: Trusting | Player 2: Skeptical |
|---|---|---|---|
| Player 1 | Straightforward | 20, [20] | (10), 10 |
| | Bluff | (50), −10 | 0, [0] |

| | | | Player 2: (.25) Trusting | Player 2: (.75) Skeptical | Player 1's Expected Payoff |
|---|---|---|---|---|---|
| Player 1 | (.5) | Straightforward | 20, 20 | 10, 10 | 12.5 |
| | (.5) | Bluff | 50, −10 | 0, 0 | 12.5 |
| | | Player 2's Expected Payoff | 5.0 | 5.0 | |

**A GAME OF TRUST** Table 13A.2 depicts a non-zero-sum game that might be called a game of trust. Each player has two actions. The players' highest payoffs occur if player 1 is "straightforward" and player 2 is "trusting." The catch is that player 1 might try to take advantage of a trusting partner by playing the "bluff" strategy. In turn, player 2, recognizing this possibility, could take a "skeptical" position, and so on. The greater the incidence of bluffing and/or skepticism, the lower is the sum of the players' payoffs. Thus, this behavior is detrimental. One finds the basic features of this game in many economic settings. For instance, a contractor might be tempted to pass on unexpected cost overruns to a more or less trusting government agency. Alternatively, in an out-of-court settlement, party A might try to extract excessive monetary compensation from party B.

The circles and squares in Table 13A.2 show the best responses for the respective players. We see that there is no pure strategy equilibrium. To find the mixed-strategy equilibrium, we follow the approach used earlier. Player 1's proportions (x and 1 − x) must leave player 2 indifferent between being trusting or skeptical. It follows that:

$$20x - 10(1 - x) = 10x + 0(1 - x).$$

The left side is player 2's expected payoff from being trusting; the right side is her payoff from being skeptical. The solution is x = .5. Thus, player 1 is straightforward or bluffs with equal probability. In turn, player 2's proportions (y and 1 − y) must leave player 1 indifferent between being straightforward or bluffiing. It follows that:

$$20y + 10(1 - y) = 50y + 0(1 - y).$$

This reduces to $10 = 40y$ or $y = .25$. Thus, player 2 should be trusting 25 percent of the time and skeptical 75 percent of the time.

Notice that player 2 must be inclined toward skepticism precisely in order to keep player 1 honest. If player 2 were too trusting, player 1 always would bluff. The bottom portion of Table 13A.2 shows these mixed strategies and the players' resulting expected payoffs. Both players' expected payoffs fall well short of the 20 in profit each would enjoy in the upper-left cell. However, the straightforward and trusting strategies do not constitute a viable equilibrium.

A fundamental result in game theory holds that every game (having a finite number of players and actions) has at least one Nash equilibrium. Thus, if a payoff table lacks a pure-strategy equilibrium, there will always be a mixed-strategy equilibrium. Deliberately taking randomized actions might seem strange at first. But, as the examples indicate, mixed strategies are needed to sustain equilibrium. Indeed, in a zero-sum game lacking a pure strategy equilibrium, mixed strategies are required to protect oneself against an opponent's opportunistic play.

Finally, many games may have both pure-strategy and mixed-strategy equilibria. One example is the market-entry game in Table 13.5. We already have identified a pair of pure-strategy equilibria in which one firm enters and the other stays out. There is also a mixed-strategy equilibrium in which each firm enters with probability .5. When the competitor enters with this frequency, the firm's expected profit from entering is: $(.5)(4) + (.5)(-4) = 0$, the same as if it stayed out. Obviously, this equilibrium is not very desirable for the firms. If they compete for new markets repeatedly, the firms mutually would prefer to divide up the available markets by alternating between the two pure-strategy equilibria.

## Problems

1. A stranger in a bar challenges you to play the following zero-sum game. The table lists your payoffs in dollars.

| | | Him | |
|---|---|---|---|
| | | C1 | C2 |
| You | R1 | −16 | 24 |
| | R1 | 8 | −16 |

What is your optimal mixed strategy? What is your opponent's? How much should you expect to win or lose on average?

2. The payoff table below offers a simple depiction of the strategy choices of the Allies and Germany with respect to the 1944 D-Day invasion during World War II. The Allies can land at either Calais or

| | | Germany | |
|---|---|---|---|
| | | Calais | Normandy |
| Allies | Calais | .6 | .9 |
| | Normandy | .8 | .6 |

Normandy, and Germany can mount a defense at one, but not both, locations. Payoffs can be interpreted as the Allies' probability of ultimately winning the war.

Find the mixed-strategy equilibrium. Explain briefly these optimal strategies. What is the value of the game, i.e., the Allies' winning chances?