ECON 331

ASSIGNMENT 5

Questions 1, 2, and 3

QUESTION 1

```
> restart:
```

'Inverse' production function:

```
> z:=q^3+4*q:
```

Average revenue function:

```
> p:=10-2*q:
```

Total revenue function:

```
> R:=p*q;
```

$$R := (10 - 2\,q)\,q$$

Find by the chain rule and the inverse function rule:

```
> dR_dz:=diff(R,q)*(1/diff(z,q));
```

$$dR\_dz := \frac{-4\,q + 10}{3\,q^2 + 4}$$

The point elasticity of total sales revenue with respect to the amount of labour used is:

```
> epsilon:=dR_dz*z/R;
```

$$\varepsilon := \frac{(-4\,q + 10)\,(q^3 + 4\,q)}{(3\,q^2 + 4)\,(10 - 2\,q)\,q}$$

When , the elasticity is:

```
> q:=2:
> 'epsilon'=epsilon;
```

$$\varepsilon = \frac{1}{6}$$

QUESTION 2

```
> restart;
> with(linalg):
Warning, the protected names norm and trace have been redefined and unprotected
```

Enter the system of equations as a vector:

```
> A:= vector([x*y-w,y-w^3-3*z,w^3+z^3-2*w*z]):
```

The Jacobian for the system is:

```
> J:=jacobian(A,[x,y,w]);
```

$$J := \begin{bmatrix} y & x & -1 \\ 0 & 1 & -3\,w^2 \\ 0 & 0 & 3\,w^2 - 2\,z \end{bmatrix}$$

Since z is treated as exogenous, the right hand side column vector can be generated as follows:

```
> b:=jacobian(A,[z]);
```

$$b := \begin{bmatrix} 0 \\ -3 \\ 3\,z^2 - 2\,w \end{bmatrix}$$

Use the **map** command to change the signs of the elements:

```
> d:=map(x->-x,b);
```

$$d := \begin{bmatrix} 0 \\ 3 \\ -3\,z^2 + 2\,w \end{bmatrix}$$

We can find expressions for the following derivatives by solving the system:

```
> matrix(3,1,[Diff(x,z),Diff(y,z),Diff(w,z)]) = multiply(inverse(J),d);
```

$$\begin{bmatrix} \frac{\partial}{\partial z} x \\ \frac{\partial}{\partial z} y \\ \frac{\partial}{\partial z} w \end{bmatrix} = \begin{bmatrix} -\frac{3\,x}{y} + \frac{(3\,x\,w^2 - 1)\,(-3\,z^2 + 2\,w)}{y\,(-3\,w^2 + 2\,z)} \\ 3 - \frac{3\,w^2\,(-3\,z^2 + 2\,w)}{-3\,w^2 + 2\,z} \\ -\frac{-3\,z^2 + 2\,w}{-3\,w^2 + 2\,z} \end{bmatrix}$$

Assign values to:

```
> w:=1:
> z:=1:
```

Solve for :

```
> sols:=solve({x*y-w,y-w^3-3*z},{x,y});
```

$$sols := \left\{ y = 4, x = \frac{1}{4} \right\}$$

Assign these values to :

```
> x:=1/4:
> y:=4:
```

Now the vector of derivatives is:

```
> multiply(inverse(J),d);
```

$$\begin{bmatrix} \frac{-1}{4} \\ 0 \\ -1 \end{bmatrix}$$

So, .

QUESTION 3

```
> restart;
```

Equation:

```
> F:=x^2+y^2+z^2+x*y+x*z+y*z+x+y+z-1;
```

$$F := x^2 + y^2 + z^2 + x\,y + x\,z + y\,z + x + y + z - 1$$

Partial derivatives:

```
> dz_dx:=implicitdiff(F,z,x);
>
```

$$dz\_dx := -\frac{2\,x + y + z + 1}{2\,z + x + y + 1}$$

```
> dz_dy:=implicitdiff(F,z,y);
```

$$dz\_dy := -\frac{2\,y + x + z + 1}{2\,z + x + y + 1}$$

At the function has continuous partials. We can check to see if   is nonzero:

```
> dF_dz:=diff(F,z);
```

$$dF\_dz := 2\,z + x + y + 1$$

```
> x:=1:
> y:=-1:
> z:=-1:
```

```
> is(dF_dz=0);
```

false

So the implicit function exists, and the partial derivatives are:

```
> dz_dx;
```

1

```
> dz_dy;
```

-1

```
>
```