ECON 331
Assignment 5: Answer Key to Question 4

4. A simple form of the $IS - LM$ model is

$$Y = C(Y, \tfrac{M_0}{P}) + I(r) + G_0 \qquad M_0/P = L(Y, r)$$

Note that the term, $\frac{M}{P}$ appears in the consumption function. This what is sometimes referred to as the *Real Balances Effect.*

(a) Make a sensible assumption about the sign of $\partial C/\partial(\frac{M}{P})$? Justifying your assumption (only your first sentence will be read).

(b) Setup and sign the Jacobian of this system.

(c) Determine the comparitive static results about how changes in M_0 affect Y and r. Use the normal economic assumptions about the derivatives of, $I(r)$ and $L(Y, r)$.

(d) Redo (c) except this time let P be the exogenous variable that influences Y and r (remember to use the chain rule).

Answers:

(a)

$$\begin{aligned} (1 - C_y)dY - I'dr &= dG_0 - C_{\frac{M}{P}}\left(\frac{M_0}{P^2}\right)dP + C_{\frac{M}{P}}\left(\frac{1}{P}\right)dM \\ L_Y dY + L_r dr &= \frac{1}{p}dM_0 - \frac{M_0}{P^2}dP \end{aligned}$$

Matrix form

$$\begin{bmatrix} (1 - C_y) & -I' \\ L_Y & L_r \end{bmatrix} \begin{bmatrix} dY \\ dr \end{bmatrix} = \begin{bmatrix} dG_0 - C_{\frac{M}{P}}\left(\frac{M_0}{P^2}\right)dP + C_{\frac{M}{P}}\left(\frac{1}{P}\right)dM \\ \frac{1}{p}dM_0 - \frac{M_0}{P^2}dP \end{bmatrix}$$

The Jacobian is

$$|J| = (1 - \overset{(+)}{C_y})\overset{(-)}{L_r} + \overset{(+)}{L_Y}\overset{(-)}{I'} < 0$$

(b) $C_{\frac{M}{P}} > 0$ the real balance effect is usually assumed to be positive.

(c) Find $\frac{\partial Y}{\partial M}$ and $\frac{\partial r}{\partial M}$

$$\begin{bmatrix} (1 - C_y) & -I' \\ L_Y & L_r \end{bmatrix} \begin{bmatrix} dY/dM \\ dr/dM \end{bmatrix} = \begin{bmatrix} C_{\frac{M}{P}}\left(\frac{1}{P}\right) \\ \frac{1}{p} \end{bmatrix}$$

$$\frac{\partial Y}{\partial M} = \frac{\begin{vmatrix} C_{\frac{M}{P}}\left(\frac{1}{P}\right) & -I' \\ \frac{1}{p} & L_r \end{vmatrix}}{|J|} = \frac{\overset{(+)}{C_{\frac{M}{P}}}\overset{(+)}{\left(\frac{1}{P}\right)}\overset{(-)}{L_r} + \overset{(-)}{I'}\overset{(+)}{\frac{1}{p}}}{|J|} = \frac{(-)}{(-)} > 0$$

and

$$\frac{\partial r}{\partial M} = \frac{\begin{vmatrix} 1 - C_y & C_{\frac{M}{P}}\left(\frac{1}{P}\right) \\ L_Y & \frac{1}{p} \end{vmatrix}}{|J|} = \frac{\overset{(+)}{(1 - C_Y)}\overset{(+)}{\frac{1}{p}} - \overset{(+)}{C_{\frac{M}{P}}}\overset{(+)}{\left(\frac{1}{P}\right)}\overset{(+)}{L_Y}}{|J|} = \frac{(?)}{(-)} \gtrless 0$$

Find $\frac{\partial Y}{\partial P}$ and $\frac{\partial r}{\partial P}$

$$\begin{bmatrix} (1-C_y) & -I' \\ L_Y & L_r \end{bmatrix} \begin{bmatrix} dY/dP \\ dr/dP \end{bmatrix} = \begin{bmatrix} -C_{\frac{M}{P}}\left(\frac{M}{P^2}\right) \\ -\frac{M}{p^2} \end{bmatrix}$$

$$\frac{\partial Y}{\partial P} = \frac{\begin{vmatrix} -C_{\frac{M}{P}}\left(\frac{M}{P^2}\right) & -I' \\ -\frac{M}{p^2} & L_r \end{vmatrix}}{|J|} = \frac{-\overset{(-)}{C}_{\frac{M}{P}}\overset{(+)}{\left(\frac{M}{P^2}\right)}\overset{(-)}{L_r} - \overset{(-)}{I'}\overset{(+)}{\frac{1}{p}}}{|J|} = \frac{(+)}{(-)} < 0$$

and

$$\frac{\partial r}{\partial P} = \frac{\begin{vmatrix} 1-C_y & -C_{\frac{M}{P}}\left(\frac{M}{P^2}\right) \\ L_Y & -\frac{M}{p^2} \end{vmatrix}}{|J|} = \frac{(1-\overset{(+)}{C_Y})\overset{(-)}{\left(-\frac{M}{p^2}\right)} + \overset{(+)}{C}_{\frac{M}{P}}\overset{(+)}{\left(\frac{M}{P^2}\right)}\overset{(+)}{L_Y}}{|J|} = \frac{(?)}{(-)} \gtreqless 0$$