ECON 331

ASSIGNMENT #6

QUESTION 1

```
> restart:
```

Average revenue:

```
> AR:=-y^3+12*y^2-30*y+1000;
```

$$AR := -y^3 + 12\,y^2 - 30\,y + 1000$$

Average Cost:

```
> AC:=2*y+1000-100/y;
```

$$AC := 2\,y + 1000 - \frac{100}{y}$$

The profit function, :

```
> pi:=expand(AR*y-AC*y);
```

$$\pi := -y^4 + 12\,y^3 - 32\,y^2 + 100$$

The first-order condition for a maximum is satisfied at the following values for :

```
> sols:=solve({diff(pi,y)},{y});
```

$$sols := \{y = 0\}, \left\{ y = \frac{9}{2} + \frac{1}{2}\sqrt{17} \right\}, \left\{ y = \frac{9}{2} - \frac{1}{2}\sqrt{17} \right\}$$

```
> evalf(sols);
```

$$\{y = 0.\}, \{y = 6.561552813\}, \{y = 2.438447187\}$$

The second-order derivative function, "():

```
> soc:=expand(diff(pi,y$2));
```

$$soc := -12\,y^2 + 72\,y - 64$$

Substitute the solutions to the first-order condition into the second-order derivative function to check the second-order condition for a unique maximum, "() < 0. Evaluate the profit function for those solutions which satisfy the second-order condition:

```
> subs(sols[1],soc);
```

$$-64$$

```
> subs(sols[1],pi);
```

$$100$$

```
> evalf(subs(sols[2],soc));
```

$$-108.2159013$$

```
> evalf(subs(sols[2],pi));
```

$$258.639194$$

```
> evalf(subs(sols[3],soc));
```

$$40.2159013$$

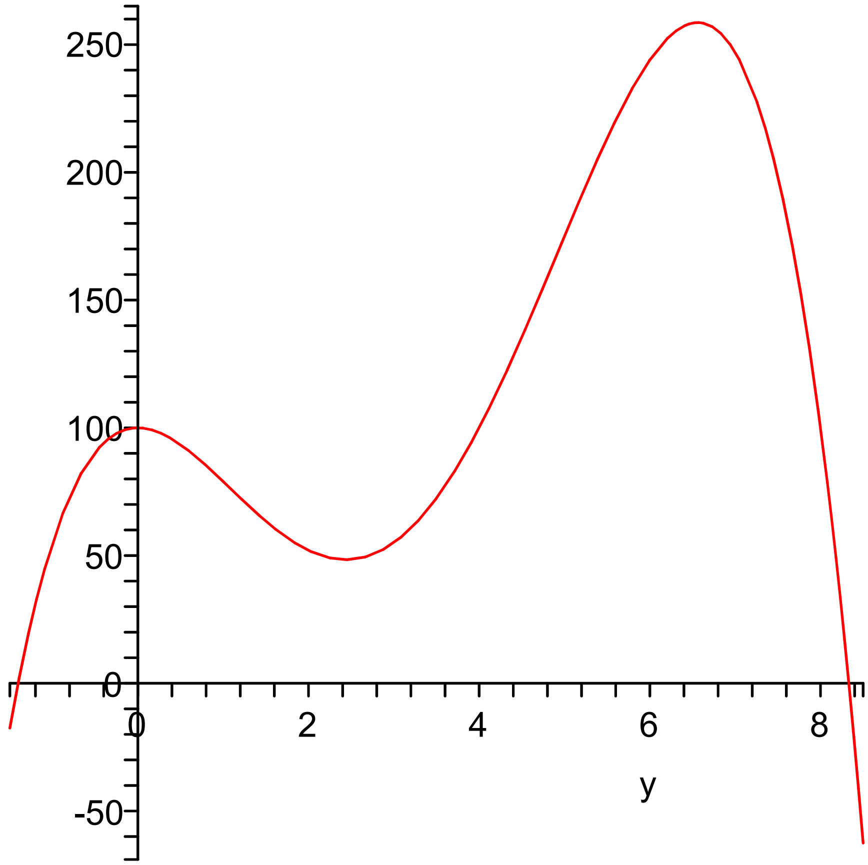
Solution (3) violates the second-order condition. Solution (2) generates the global maximum for . The profit maximising output (in floatinf point format) is:

```
> evalf(sols[2]);
```

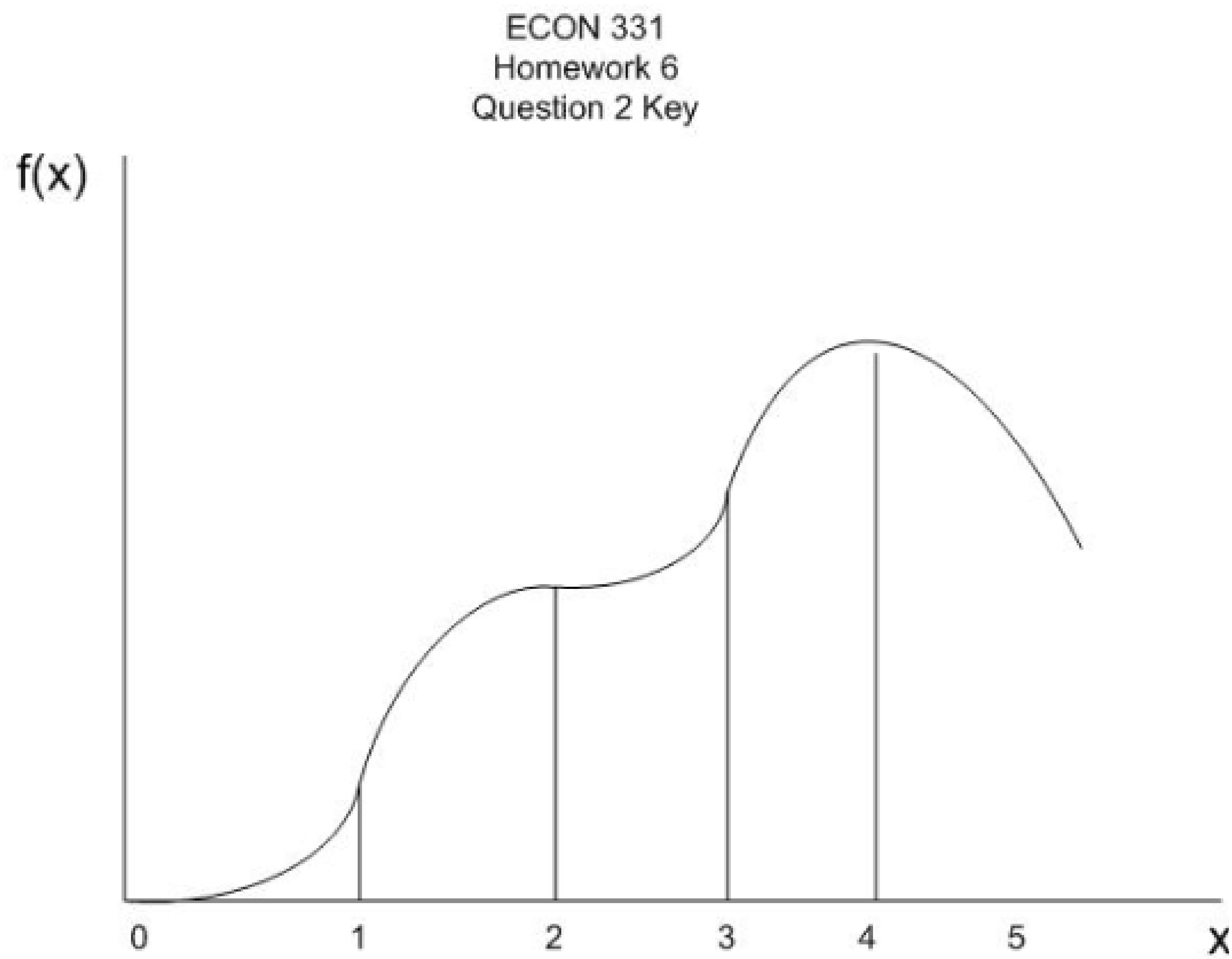
$$\{y = 6.561552813\}$$

A plot of the function, :

```
> plot(pi, y=-1.5..8.5);
```




```
> restart:
>
```



Question 3: Exponential and Logarithmic derivatives

```
> f:=ln(3*x^2+exp(2*x));
```

$$f := \ln(3\,x^2 + \mathbf{e}^{(2\,x)})$$

```
> f_x:=diff(f,x);
```

$$f\_x := \frac{6\,x + 2\,\mathbf{e}^{(2\,x)}}{3\,x^2 + \mathbf{e}^{(2\,x)}}$$

```
> restart:
```

Optimal Timing problem:

The growth function for trees is

```
> V:=265*exp(75-40/t);
```

$$V := 265\,\mathbf{e}^{\left(75 - \frac{40}{t}\right)}$$

The present value of V is

```
> V*exp(-r*t);
```

$$265\, e^{\left(75 - \frac{40}{t}\right)}\, e^{(-r\, t)}$$

```
> PV:=simplify(%);
>
```

$$PV := 265\, e^{\left(-\frac{-75\, t + 40 + r\, t^2}{t}\right)}$$

```
> dPV:=diff(PV,t);
```

$$dPV := 265 \left(-\frac{-75 + 2\, r\, t}{t} + \frac{-75\, t + 40 + r\, t^2}{t^2}\right) e^{\left(-\frac{-75\, t + 40 + r\, t^2}{t}\right)}$$

```
> r:=.1;
```

$$r := 0.1$$

Sub in the interest rate and solve for t

```
> solve(dPV=0,t);
```

$$20., -20.$$

The optimal time to harvest the trees is 20 years

Alternative method: monotonic transformation with natural log

```
> pv:=ln(PV);
```

$$pv := \ln\left(265\, e^{\left(-\frac{-75\, t + 40 + 0.1\, t^2}{t}\right)}\right)$$

```
> t_star:=solve(diff(pv,t)=0,t);
```

$$t\_star := -20., 20.$$

```
>
```

$$-\frac{0.1000000000\, (t^2 - 400.)}{t^2}$$

```
>
```