ECON 331

ASSIGNMENT #7

```
> restart;
> with(linalg):
Warning, the protected names norm and trace have been redefined and unprotected
```

Production function:

```
> q:=26*z1+24*z2-7*z1^2-12*z1*z2-6*z2^2;
```

$$q := 26\,z1 + 24\,z2 - 7\,z1^{2} - 12\,z1\,z2 - 6\,z2^{2}$$

Profit function ():

```
> pi:=q-w1*z1-w2*z2;
```

$$\pi := 26\,z1 + 24\,z2 - 7\,z1^{2} - 12\,z1\,z2 - 6\,z2^{2} - w1\,z1 - w2\,z2$$

Factor demand functions:

```
> sols:=solve({diff(pi,z1),diff(pi,z2)},{z1,z2});
```

$$sols := \left\{ z1 = 1 + \frac{1}{2}\,w2 - \frac{1}{2}\,w1,\, z2 = 1 - \frac{7}{12}\,w2 + \frac{1}{2}\,w1 \right\}$$

Second-order conditions:

```
> H:=hessian(pi,[z1,z2]);
```

$$H := \begin{bmatrix} -14 & -12 \\ -12 & -12 \end{bmatrix}$$

```
> det(H);
```

$$24$$

Since and , the S.O.C. for a unique maximum are satisfied. This means that the profit function is strictly concave. This also implies that the production function is strictly concave since the cost function is linear.

Comparative Statics:

The production function in general functional form:

```
> q:=(z1,z2)->f(z1,z2):
```

The profit function in general functional form:

```
> pi:=(z1,z2)->f(z1,z2)-w1*z1-w2*z2:
```

The first-order conditions:

```
> D[1](pi)=0;
```

$$(z1, z2) \rightarrow D_1(f)(z1, z2) - w1 = 0$$

```
> D[2](pi)=0;
```

$$(z1, z2) \rightarrow D_2(f)(z1, z2) - w2 = 0$$

The first-order conditions imply that the marginal products of the factors are equal to their factor prices at the profit maximising levels of the factors, ie: , and .

The supply function is found by expressing the production function as a function of the profit maximising levels of the factors:

```
> maxq:=(w1,w2)->f(z1(w1,w2),z2(w1,w2));
```

$$maxq := (w1, w2) \rightarrow f(z1(w1, w2), z2(w1, w2))$$

The comparative static result we seek can be expressed in general functional form:

```
> dmaxq_dw1:=D[1](maxq);
```

$$dmaxq\_dw1 := (w1, w2) \rightarrow D_1(f)(z1(w1, w2), z2(w1, w2))\, D_1(z1)(w1, w2) + D_2(f)(z1(w1, w2), z2(w1, w2))\, D_1(z2)(w1, w2)$$

The first term in each product is the marginal product of the factor which we know is equal to the appropriate factor price at the profit maximum. The second term in each product is: , . These derivatives are evaluated at the profit maximising levels of the factors. Their values are:

```
> assign(sols);
> dz1_dw1:=diff(z1,w1);
> dz2_dw1:=diff(z2,w1);
```

$$dz1\_dw1 := \frac{-1}{2}$$

$$dz2\_dw1 := \frac{1}{2}$$

So is:

```
> dmaxq_dw1:=w1*dz1_dw1+w2*dz2_dw1;
```

$$dmaxq\_dw1 := -\frac{1}{2}\,w1 + \frac{1}{2}\,w2$$

```
>
```

Notice that the sign of this derivative is ambiguous, it depends on the relative magnitudes of $\mathbf{w_1}$ and $\mathbf{w_2}$. In particular, if $\mathbf{w_1} < \mathbf{w_2}$, an increase in will lead to an increase in the profit maximising level of output! If you are surprised by this result ask your TA to explain!