ECON 331

ASSIGNMENT #8

```
> restart;
```

a)

Skip's utility function:

```
> U:=x*(y+1):
```

The Lagrangian:

```
> L:=U+lambda*(B-Px*x-Py*y);
```

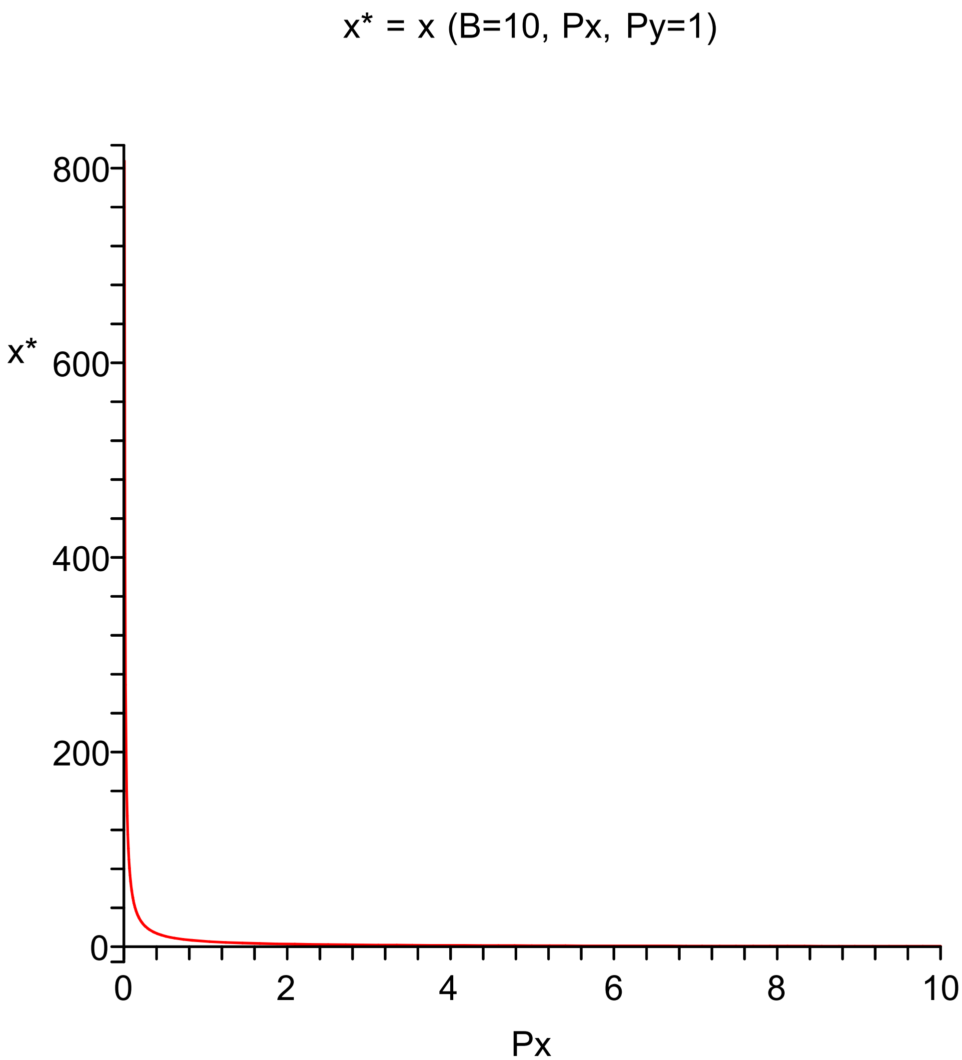
$$L := x\,(y+1) + \lambda\,(B - Px\,x - Py\,y)$$

Demand functions:

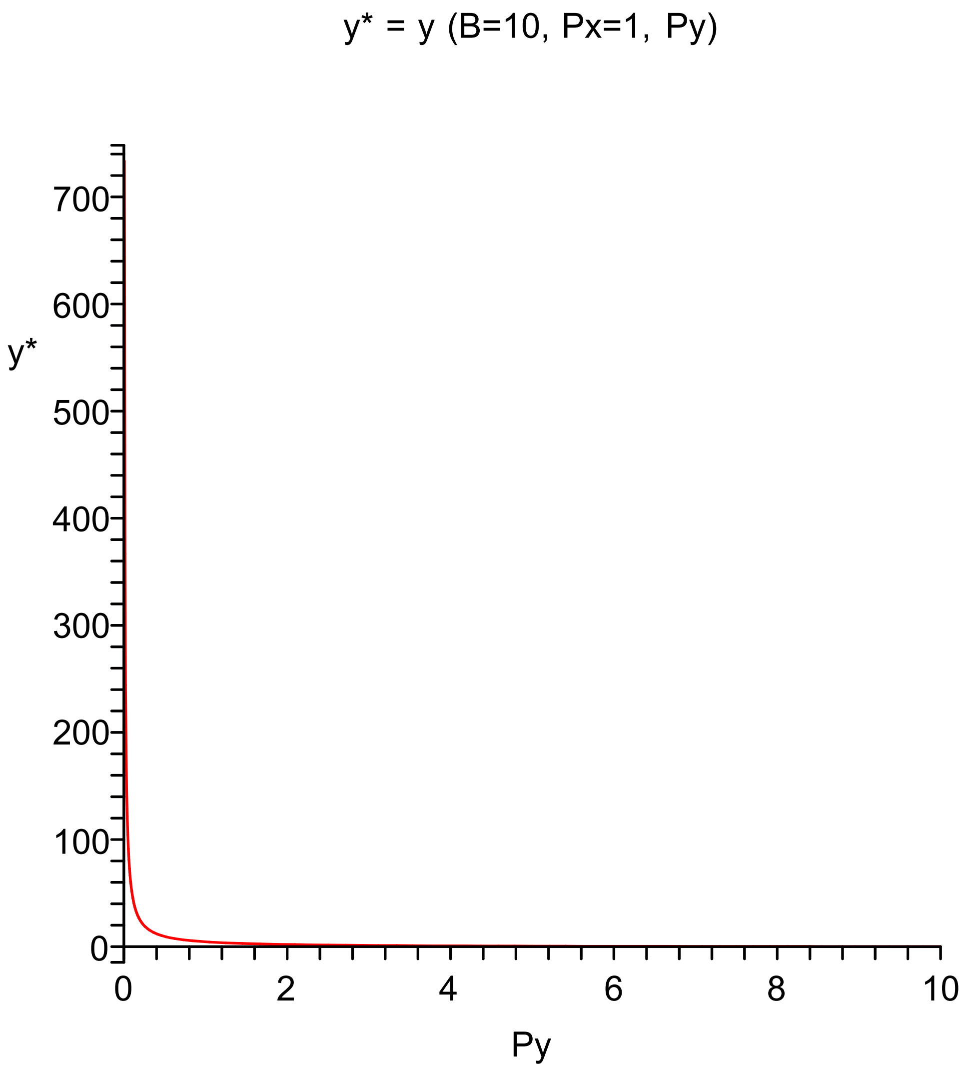
```
> sols:=solve({diff(L,x),diff(L,y),diff(L,lambda)},{x,y,lambda});
```

$$sols := \left\{ \lambda = \frac{B + Py}{2\,Px\,Py},\, y = \frac{-Py + B}{2\,Py},\, x = \frac{B + Py}{2\,Px} \right\}$$

```
> assign(sols);
> with(plots):
Warning, the name changecoords has been redefined
> B:=10:
> Py:=1:
> plot(x,Px=0..10,labels=[`Px`,`x*`],title=`x* = x (B=10, Px, Py=1)`);
```




```
> unassign('Py'):
> Px:=1:
> plot(y,Py=0..10,labels=[`Py`,`y*`],title=`y* = y (B=10, Px=1, Py)`);
```



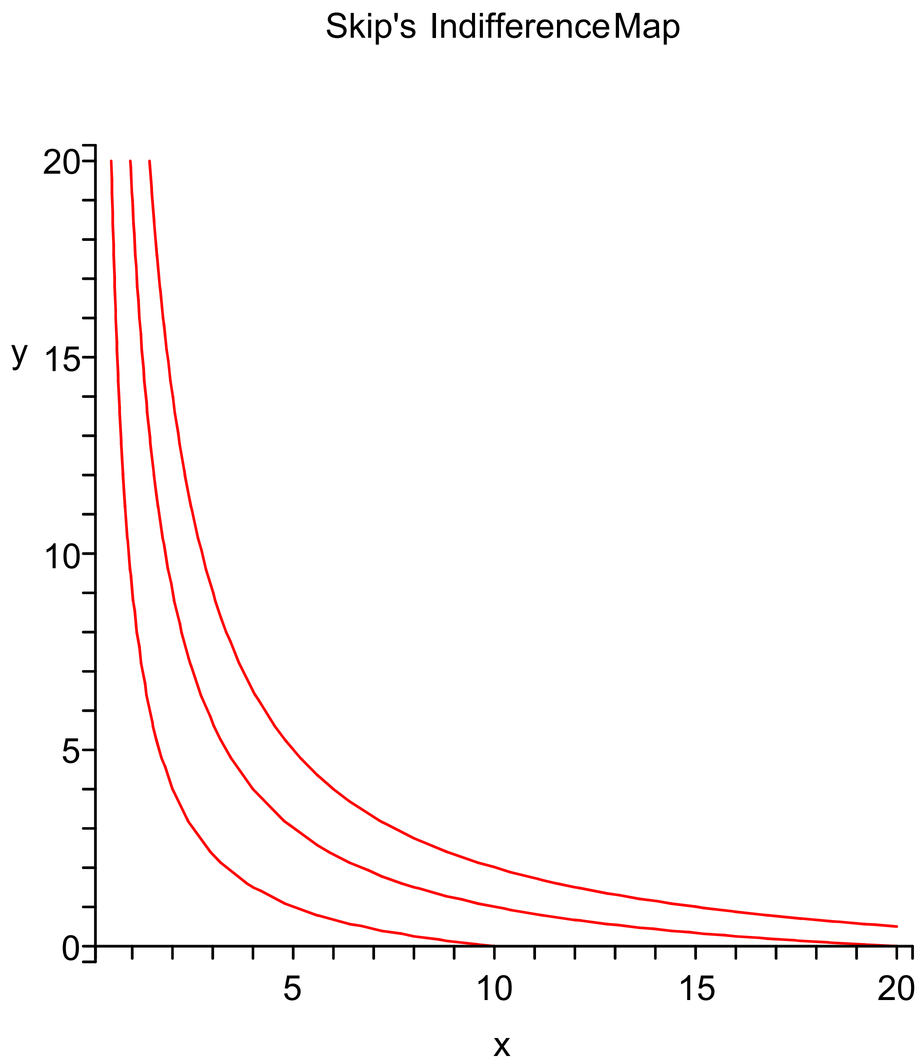
Check the nature of good Y:

```
> unassign('B'):
> Diff(y,B)=diff(y,B);
```

$$\frac{\partial}{\partial B}\left(\frac{-Py+B}{2\,Py}\right)=\frac{1}{2\,Py}$$

Skip's indifference curves:

```
> unassign('x','y','Px','lambda'):
> implicitplot({U=10,U=20,U=30},x=0..20,y=0..20,title=`Skip's Indifference
  Map`);
```



b)

Second-order conditions:

```
> with(linalg):
> H:=hessian(L,[lambda,x,y]);
```

Warning, the protected names norm and trace have been redefined and unprotected

$$H := \begin{bmatrix} 0 & -Px & -Py \\ -Px & 0 & 1 \\ -Py & 1 & 0 \end{bmatrix}$$

```
> det(H);
```

Since the determinant of the bordered Hessian is positive, the second-order sufficient condtion for a constrained maximum is satisfied.

$$2\, Px\, Py$$

c)

The indirect utility function:

```
> assign(sols);
> 'U'=simplify(U);
```

$$U = \frac{(B + Py)^2}{4\, Px\, Py}$$

d)

The expenditure function:

```
> ### WARNING: persistent store makes one-argument readlib obsolete
  readlib(isolate):
> simplify(isolate(Uo=U,B));
```

$$B = 2\sqrt{Uo\, Px\, Py} - Py$$

```
> B:=rhs(%);
>
```

$$B := 2\sqrt{Uo\, Px\, Py} - Py$$

The expenditure function reveals the minimum expenditure/income required to attain a specific level of utility, , at given prices, Px and Py.

The partial derivatives of the expenditure function with respect to the prices:

```
> dB_dPx:=diff(B,Px);
```

$$dB\_dPx := \frac{Uo\, Py}{\sqrt{Uo\, Px\, Py}}$$

```
> dB_dPy:=diff(B,Py);
```

$$dB\_dPy := \frac{Uo\, Px}{\sqrt{Uo\, Px\, Py}} - 1$$

e)

Skip's expenditure minimization problem:

```
> unassign('x','y'):
> Z:=Px*x+Py*y+mu*(Uo-x*(y+1));
```

$$Z := Px\, x + Py\, y + \mu\, (Uo - x\, (y + 1))$$

f)

Solutions to the expenditure minimization problem:

```
> sols2:=simplify(solve({diff(Z,x),diff(Z,y),diff(Z,mu)},{x,y,mu}));
```

$$sols2 := \left\{ x = \mathrm{RootOf}(Px\, \_Z^2 - Uo\, Py),\ y = -\frac{-Uo + \mathrm{RootOf}(Px\, \_Z^2 - Uo\, Py)}{\mathrm{RootOf}(Px\, \_Z^2 - Uo\, Py)},\ \mu = \frac{Py}{\mathrm{RootOf}(Px\, \_Z^2 - Uo\, Py)} \right\}$$

```
> assign(sols2):
```

Use the following commands to simplify 'sols2':

```
> ### WARNING: note that `I` is no longer of type `radical`
  x:=radsimp(convert(x,radical));
```

$$x := \frac{\sqrt{Uo\ Px\ Py}}{Px}$$

```
> ### WARNING: note that `I` is no longer of type `radical`
  y:=expand(radsimp(convert(y,radical)));
```

$$y := \frac{Uo\ Px}{\sqrt{Uo\ Px\ Py}} - 1$$

The solutions to the expenditure minimization problem are the 'compensated' (utility held constant) demand functions.

Verify that the compensated demand functions are equal to the partial derivatives of the expenditure fuction with respect to the respective prices. (Hint: substitute the indirect utility function for):

```
> subs(Uo=U,dB_dPx);
```

$$\frac{Uo\ Py}{\sqrt{Uo\ Px\ Py}}$$

```
> subs(Uo=U,dB_dPy);
```

$$\frac{Uo\ Px}{\sqrt{Uo\ Px\ Py}} - 1$$

```
>
```