Econ 431: *Mathematical Economics* *Summer 2007*

Assignment 1 Kevin Wainwright

Instructions: The following assignment is a review of material from the second half of ECON 331. These questions focus on chapters 8 to 13 of the textbook.

1. Consider the following exponential problems:

   (a) If $f(x,y) = yxe^{y/x}$ show that Young's theorem, which is $f_{xy} = f_{yx}$, is true

   (b) (5 pt) Suppose that the value of a stand of trees increases according to the following function

   $$V(t) = 265e^{(75-40/t)}$$

   If the market rate of interest is r, derive a solution, $t^*(r)$, for when should the trees be harvested in order to maximize the present value of the stand.

2. The following are the demand and supply functions in a two commodity market model. P_1 and P_2 are the prices received by the producers of the two commodities. However, the government wishes to encourage the use of the second commodity so that consumers of this commodity receive a rebate or subsidy s per unit on their purchases of this commodity. Thus consumers pay P_1 and $\pi = P_2 - s$ for these two commodities (see also below).

   $$Q_1^d = D_1(P_1, \pi) \qquad Q_1^s = S_1(P_1) \qquad \pi = P_2 - s$$

   $$Q_2^d = D_2(P_1, \pi) \qquad Q_2^s = S_2(P_2)$$

   The demand and supply functions given above may be assumed to have continuous partial derivatives.

   (a) What are the normal economic assumptions about the derivatives of these demand and supply functions when the two commodities are substitutes?

   (b) The market clearing conditions for the markets above may also be assumed to have a solution for $s = 0$. Show then that they solve implicitly for P_1 and P_2 as functions of s about $s = 0$. You may assume the following inequality is true which says that the own price effects are stronger than the cross price effects.

   $$(\partial D_1/\partial P_1 - S_1')(\partial D_2/\partial \pi - S_2') - (\partial D_1/\partial \pi)(\partial D_2/\partial P_1) > 0$$

   (c) Show that the following two comparitive static results hold. The results are $\partial P_1/\partial s < 0$ and $\partial \pi/\partial s < 0$.

3. A simple form of the $IS - LM$ model is

   $$D = S(Y) - I(R) \qquad D = G(R) - T_0$$

   $$M_0/P = L(Y, R)$$

   Here D is the government deficit, $G(R)$ is total government expenditure, T_0 is total government tax revenue and P is the GNE deflator.

   (a) Make a sensible assumption about the sign of $G'(R)$. Justifying your assumption (only your first sentence will be read).

   (b) Determine the comparitive static result $\partial D/\partial T_0$. Use the normal economic assumptions about the derivatives of $S(Y), I(R)$ and $L(Y, R)$. You may also assume that

   $$G'(R) + I'(R) < 0.$$

4. A competitive firm sells its product at a price, p. The firm uses three inputs: z_1, z_2, z_3. whose respective input prices are w_1, w_2, and w_3. The firm's production functions is

$$q = 10z_1 + 10z_2 + 10z_3 + z_1z_2 - z_1^2 - z_2^2 - z_3^2$$

   (a) (3 pt) Write down the firm's profit function

   (b) (3 pt) Solve for the optimal z_1, z_2, z_3

   (c) (2 pt) Check the hessian to see if you do have a maximum.

   (d) (2 pt) Is the profit function strictly concave?

5. A consumer has the utility function $u = x^2 + y^2$ and the budget constraint $B = p_x x + p_y y$

   (a) Find x^*, y^*, and check second order conditions.

   (b) Suppose $B = 10$ and $p_y = 2$. Carefully graph the demand function for x^* with p_x on the vertical axis. Include as much detail as possible in your graph (*intercepts, curvature, critical points, etc.*).

   (c) In a budget constraint- indifference curve graph, carefully illustrate the equilibrium when $p_x = 1$ and $p_x = 3$. (assume $B = 10$ and $p_y = 2$). *You will be marked on accuracy and detail* (curvature, intercepts, etc.)

6. Skippy lives on an island where she produces two goods, *x and y,* according the the production possibility frontier $200 \geq x^2 + y^2$, and she consumes all the goods herself. Her utility function is $U(x, y) = 3 \ln x + \ln y$ .Skippy also faces and environmental constraint on her total output of both goods. The environmental constraint is given by $x + y \leq 18$

   (a) (5 pt) Write down the Kuhn Tucker first order conditions.

   (b) *(5 pt)* Find Skippy's optimal *x and y.* Identify which constaints are binding.

7. Myrtle has the following maximization problem

$$\text{Max} \quad u = x^{1/3}y^{2/3} \quad \text{subject to} \quad B = p_x x + p_y y$$

   where x and y are quantities of two consumption goods whose prices are p_x and p_y respectively. Myrtle has a budget of B.

   (a) (3 pt) Find an expression for the indirect utility function v

   (b) (3 pt) Write down and verify the result derived from the envelope theorem, known as *Roy's identity*

   Myrtle's utility maximization problem could be recast as the following:

$$Minimize \quad p_x x + p_y y \quad s.t. \quad U_0 = x^{1/3}y^{2/3}$$

   where U_0 is equivalent to the maximum utility obtained from the above problem

   (c) (3 points) Find the x^* and y^* that solve this minimization problem. Use these solutions to find an expression for the *Expenditure Function*, B^*

   (d) (3 points) Verify Shephard's lemma by deriving the compensated demand function for x from the expenditure function

8. Suppose $f(x_1, x_2)$ is homogeneous of degree 1.show that

$$f_{11}x_1 + f_{12}x_2 \equiv 0$$

   be sure to explain what steps, rules and theorems you used in your answer