Econ 431: *Mathematical Economics* *Summer 2007*

Assignment 2 Kevin Wainwright

Instructions: The following is due Monday, June 4th and will be collected for grading.

1. Consider the following Kuhn-Tucker Problem:

$$Max \qquad U(x,y) = xy(9-x-y)$$

subject to

$$\begin{aligned} x+y &\leq 5 \\ x &\geq 0, y \geq 0 \end{aligned}$$

where the Lagrangian is

$$Z = xy(9-x-y) + \lambda(5-x-y)$$

   (a) Write out the Kuhn-Tucker conditions for this problem.

   (b) Assume that the constraint in binding, find x^*, y^* and λ^*. Do these solutions satisfy your K-T conditions? (be specific) Interpret the sign and magnitude of λ^*

   (c) Re-do the problem except now use the constraint: $x + y \leq 8$

   (d) Compare the value of λ^* for (b) and (c). What do they tell you?

2. *The following is based on a question from homework 1*:
Skippy lives on an island where she produces two goods, x and y, according the production possibility frontier $x^2 + y^2 \leq 180$, and she consumes all the goods herself. Skippy also faces and environmental constraint on her total output of both goods. The environmental constraint is given by $x + y \leq 18$. For each of the following utility functions:

   (a) $U(x,y) = 3\ln x + \ln y$.

   (b) $U(x,y) = xy^2$

   (c) $U(x,y) = xy$

      1. Write down the Kuhn Tucker first order conditions.
      2. Find Skippy's optimal x and y. Identify which constraints are binding.
      3. Carefully graph your results

3. Suppose that there are two types of electricity[1] (peak and off-peak).Half the day is peak and half the day is off-peak. To produce a unit of electricity per half-day requires a unit of turbine capacity costing 30 cents per day (interest charges on a permanent loan). The cost of a unit of capacity is the same whether it is used at peak times only or off-peak also. In addition to the costs of turbine capacity, it costs 20 cents in operating costs (labour and fuel) to produce 1 unit per half day. Suppose the demand for electricity per half day during peak hours is

$$p = 200 - 0.1q$$

and during off-peak hours is

$$p = 180 - 0.1q$$

where **q** is units of electricity per half-day and **p** is price in cents.

   (a) Write down the Kuhn-Tucker conditions for profit maximization.

   (b) What are the profit maximizing peak and off-peak prices?

   (c) If a unit of capacity cost only 10cents per day, what would the profit maximizing peak and off-peak prices be?

[1] This model comes directly from the textbook; Section 13.3 on Economic Applications