Econ 431: *Mathematical Economics* — *Summer 2007*

Assignment 5 — **Kevin Wainwright**

1. Solve the differential equation
$$\frac{dy}{dt} = 0.1y - 1$$
with the initial condition $y(0) = 5$

2. Let $K(t)$ be the capital stock at time t. Let δ be the rate of depreciation ($\delta > 0$) and I_0 is a constant level of investment. Net investment can be described by the following differential equation
$$\frac{dK}{dt} = I_0 - \delta K$$
and the initial condition $K(0) = K_0$. Find the solution to $K(t)$. Does $K(t)$ converge to a steady-state?

3. **Dynamics of national debt accumulation.**
Let $D(t)$ represent the dollar value of debt and let $Y(t)$ be the value of GDP at time t. Suppose that debt is a constant proportion of GDP, denoted by $.\theta$ such that
$$\frac{dD}{dt} = \theta Y \qquad \theta > 0$$
Further assume that national income grows according to the following differential equation
$$\frac{dY}{dt} = \mu Y \qquad \mu > 0$$
Finally, assume that, at time $t = 0$, the initial values of debt and income are D_0 and Y_0. *Show the following:*

   (a) the solution to $Y(t)$ is
   $$Y(t) = Y_0 e^{\mu t}$$

   (b) the general solution to $D(t)$ is
   $$D(t) = \theta Y_0 \frac{e^{\mu t}}{\mu} + c_0$$
   where c_0 is an arbitrary constant of integration.

   (c) Using the initial condition, show that the specific solution for $D(t)$ is
   $$D(t) = D_0 + \frac{\theta}{\mu} Y_0 \left(e^{\mu t} - 1\right)$$

   (d) Let the interest payments on the debt be rD, where r is the rate of interest. The ratio
   $$z(t) = \frac{rD(t)}{Y(t)}$$
   is the share of national income used to service the interest on the national debt. By substituting in your previous solutions, show that
   $$\lim_{t \to \infty} z(t) = r\frac{\theta}{\mu}$$