

ECON 431
ASSIGNMENT #2
(331 HW 8)

```
> restart;
```

a)

Skip's utility function:

```
> U:=x*(y+1) :
```

The Lagrangian:

```
> L:=U+lambda*(B-Px*x-Py*y) ;
```

Demand functions:

```
> sols:=solve({diff(L,x),diff(L,y),diff(L,lambda)},{x,y,lambda});
```

```
> assign(sols);
```

```
> with(plots):
```

```
> B:=10:
```

```
> Py:=1:
```

```
> plot(x,Px=0..10,labels=[`Px`,`x*`],title=`x* = x (B=10, Px, Py=1)`);
```

```
> unassign('Py'):
```

```
> Px:=1:
```

```
> plot(y,Py=0..10,labels=[`Py`,`y*`],title=`y* = y (B=10, Px=1, Py)`);
```

Check the nature of good Y:

```
> unassign('B'):
```

```
> Diff(y,B)=diff(y,B) ;
```

Skip's indifference curves:

```
> unassign('x','y','Px','lambda'):
```

```
> implicitplot({U=10,U=20,U=30},x=0..20,y=0..20,title=`Skip's Indifference  
Map`);
```

b)

Second-order conditions:

```
> with(linalg) :  
> H:=hessian(L, [lambda, x, y]) ;  
> det(H) ;
```

Since the determinant of the bordered Hessian is positive, the second-order sufficient condition for a constrained maximum is satisfied.

c)

The indirect utility function:

```
> assign(sols) ;  
> 'U'=simplify(U) ;
```

d)

The expenditure function:

```
> readlib(isolate) :  
> simplify(isolate(Uo=U, B)) ;  
> B:=rhs(") ;
```

The expenditure function reveals the minimum expenditure/income required to attain a specific level of utility, U_0 , at given prices, P_x and P_y .

The partial derivatives of the expenditure function with respect to the prices:

```
> dB_dPx:=diff(B, Px) ;  
> dB_dPy:=diff(B, Py) ;
```

e)

Skip's expenditure minimization problem:

```
> unassign('x', 'y') :  
> Z:=Px*x+Py*y+mu*(Uo-x*(y+1)) ;
```

f)

Solutions to the expenditure minimization problem:

```
> sols2:=simplify(solve({diff(Z, x), diff(Z, y), diff(Z, mu)}, {x, y, mu})) ;  
> assign(sols2) ;
```

Use the following commands to simplify 'sols2':

```
> x:=radsimp(convert(x, radical)) ;  
> y:=expand(radsimp(convert(y, radical))) ;
```

The solutions to the expenditure minimization problem are the 'compensated' (utility held constant) demand functions.

Verify that the compensated demand functions are equal to the partial derivatives of the expenditure function with respect to the respective prices. (Hint: substitute the indirect utility function for U_0):

> `subs (Uo=U, dB_dPx) ;`

> `subs (Uo=U, dB_dPy) ;`