ECON 431

ASSIGNMENT #3

(331 HW 9)

QUESTION 1

a)

```
> restart;
```

The limit-output model:

```
> p:=120-(q1+q2):
> c1:=75+35*q1:
> c2:=100+40*q2:
> pi1:=p*q1-c1;
> pi2:=p*q2-c2;
```

The best response of firm 2 to the ouput produced by firm 1:

```
> q2:=solve(diff(pi2,q2),q2);
```

Firm 1 chooses output (lim_q1) so that $\pi_2 = 0$:

```
> lim_q1:=solve(pi2,q1);
```

Choose the value for lim_q1 which maximises π_1:

```
> q1:=lim_q1[1]:
> q2:=0:
> 'pi1(lim_q1=60)'=pi1;
> 'p(lim_q1=60)'=p;
> q1:=lim_q1[2]:
> 'pi1(lim_q1=100)'=pi1;
```

So, lim_q1 = 60.

Set up functions for plot:

```
> p:=120-Q:
> q1:=lim_q1[1]:
```

Firm 2 faces the 'residual' demand (D2 in plot):

```
> q2:=Q-lim_q1[1];
> ac2:=c2/q2;
> mr2:=60-2*q2;
> mc2:=40:

> with(plots):
> Z:=plot({p,mr2,ac2,mc2},Q=60.1..100, P=0..70,title=`Limit Output
  Equilibrium`):
> U:=textplot([100,46,'AC2']):
> V:=textplot([100,40,'MC2']):
> W:=textplot([100,18,'D2']):
> J:=textplot([90,5,'MR2']):
> display([Z,U,V,W,J]);
```

b)

```
> restart;
```

The Cournot Duopoly:

```
> p:=120-(q1+q2):
> c2:=100+40*q2:
> c1:=75+35*q1:
> pi1:=p*q1-c1:
> pi2:=p*q2-c2:
```

The Cournot quantities:

```
> cournot_q:=solve({diff(pi1,q1),diff(pi2,q2)},{q1,q2});
```

Cournot price:

```
> cournot_p:=subs(cournot_q[1],cournot_q[2],p);
```

Cournot profits:

```
> profit[1]:=subs(cournot_q[1],cournot_q[2],pi1);
> profit[2]:=subs(cournot_q[1],cournot_q[2],pi2);
> with(plots):
> Z:=implicitplot({diff(pi1,q1),diff(pi2,q2),pi1=825,pi2=525},q1=0..100,q2=
  ..100,title=`Cournot Equilibrium`):
> U:=textplot([5,80,'R1']):
> V:=textplot([32,40,'Isoprofit_2']):
> W:=textplot([5,40,'R2']):
> J:=textplot([45,25,'Isoprofit_1']):
> display([Z,U,V,W,J]);
```

c)

The Stackelberg Model:

Firm 2 best response function:

```
> q2:=solve(diff(pi2,q2),q2);
```

Firm 1 maximises profit subject to the best response from firm 2:

Substitute the best response function into π_1:

```
> pi1;
```

Solve for the Stackelberg q1 from the first-order condition:

```
> q1:=solve(diff(pi1,q1),q1);
```

Stackelberg q2:

```
> q2;
```

Stackelberg profits:

```
> pi1;
> pi2;
```

Stackelberg price:

```
> p;
> restart;
> p:=120-(q1+q2):
> c1:=75+35*q1:
> c2:=100+40*q2:
> pi1:=p*q1-c1:
> pi2:=p*q2-c2:
> with(plots):
> Z:=implicitplot({pi1=1875/2,diff(pi2,q2)=0},q1=0..80,q2=0..40,title=`Stac
  elberg Equilibrium`):
> U:=textplot([25,30,'R2']):
> V:=textplot([20,10,'Isoprofit_1']):
> display([Z,U,V]);
```