ECONOMICS 431

MAPLE HOMEWORK

Assignment 1

(331 HW 7)

Suppose that the output q of a firm depends on the quantities of z_1 and z_2 that it employs as inputs. Its output level is determined by the production function

1. $$q = 26z_1 + 24z_2 - 7z_1^2 - 12z_1z_2 - 6z_2^2$$

2. Write down the firm's profit function when the price of q is \$1 and the factor prices are w_1 and w_2(per unit) respectively.

3. Find the levels of z_1^* and z_2^* which maximize the firm's profits. Note that these **are** the firm's **demand** curves for the two inputs.

4. Verify that your solution to **[2]** satisfies the second order conditions for a maximum.

5. What will be the effect of an increase in w_1 on the firm's use of each input and on its output q? [hint: You do not have to explicitly determine the firm's supply curve of output to determine $\partial q/\partial w_1$. Instead write out the total derivative of q and make use of the very simple expressions for $\partial q/\partial z_1$ and $\partial q/\partial z_2$ at the optimum that can be obtained from the first order conditions.]

6. Is the firm's production function strictly concave? Explain.

Assignment 2

(331 HW 8)
Skip has the following utility function: $U(x, y) = x(y + 1)$, where x and y are quantities of two consumption goods whose prices are p_x and p_y respectively. Skip has a budget of B. Therefore the Skip's maximization problem is

$$x(y + 1) + \lambda(B - p_x x - p_y y)$$

a) From the first order conditions find expressions for the demand functions

$$x^* = x(p_x, p_y, B) \qquad y^* = y(p_x, p_y, B)$$

Carefully graph x* and y*. Graph Skip's indifference curves. What kind of good is y?

b) Verify that skip is at a maximum by checking the second order conditions.

c) By substituting x* and y* into the utility function find an expressions for the indirect utility function,

$$U = U(p_x, p_y, B)$$

d) By rearranging the indirect utility function, derive an expression for the expenditure function,

$$B^* = B(p_x, p_y, U_0)$$

Interpret this expression. Find $\partial \mathbf{B}/\partial \mathbf{p}_x$ and $\partial \mathbf{B}/\partial \mathbf{p}_y$.

Skip's maximization problem could be recast as the following minimization problem:

$$p_x x + p_y y \text{ s.t. } U_0 = x(y + 1)$$

e) Write down the lagrangian for this problem.

f) Find the values of x and y that solve this minimization problem and show that the values of x and y are equal to the partial derivatives of the expenditure function, $\partial \mathbf{B}/\partial \mathbf{p}_x$ and $\partial \mathbf{B}/\partial \mathbf{p}_y$ respectively. (Hint: use the indirect utility function)

Assignment 3

[1] Consider the following duopoly market where the market demand curve is given by

$$p = 120 - (q_1 + q_2)$$

where q_1 and q_2 are the outputs of firm 1 and firm 2 respectively.

Firm $1's$ cost function is

$$TC(q_1) = 75 + 35q_1$$

and firm $2's$ cost function is

$$TC(q_2) = 100 + 40q_2$$

Find the equilibrium prices, quantities, and profits when:
a) When firm 1 is a monopolist using the limit output stratagy to keep firm 2 out of the market.
b) When firm 1 and firm 2 are cournot duopolists.
c) When firm 1 and firm 2 are duopolists but firm 1 chooses his output first, taking into account the fact that firm $2's$ choice of depends on firm $1's$ choice of output.
Show all your work.
Graph your results from a,b, and c.

Assignment 4 (331 HW 10)

[**1**] Suppose that there are two types of electricity (peak and off-peak).Half the day is peak and half the day is off-peak. To produce a unit of electricity per half-day requires a unit of turbine capacity costing 8 cents per day (interest charges on a permanent loan). The cost of a unit of capacity is the same whether it is used at peak times only or off-peak also. In addition to the costs of turbine capacity, it costs 6 cents in operating costs (labour and fuel) to produce 1 unit per half day.
Suppose the demand for electricity per half day during peak hours is

$$p = 22 - 10^{-5}q$$

and during off-peak hours is

$$p = 18 - 10^{-5}q$$

where **q** is units of electricity per half-day and **p** is price in cents.

a) Write down the Kuhn-Tucker conditions for profit maximization.
b) What are the profit maximizing peak and off-peak prices?
c) If a unit of capacity cost only 3 cents per day, what would the profit maximizing peak and off-peak prices be?