ECON 331 Final Exam Spring 1998 (a)
Name:_____

Instructions: The exam is worth 60 marks. All questions are equally weighted. You are to do all questions. Answer all questions in the exam booklets provided. Time allowed is 3 hours. **You must hand in the exam with your booklets.**

1. A competitive frim has the following production function, $q = 60K + 34L - 4KL - 6K^2 - 3L^2$. If p is the market price, w is the wage, and r is the rental rate of captital, then:

   (a) Write down the firm's profit function.

   (b) solve for the optimal K and L

   (c) Use the Hessian to check the second order conditions

2. A consumer with the following utility function $u = xy^2$ faces two constraints. The first is the normal budget constraint where he has $100 and the price of x and y are both $1. Also, the consumer has been issued 120 ration coupons by the government which he must use whenever he buys either x or y. It takes 2 coupons to buy an x and 1 coupon to buy a y (for example: to buy 3 units of x it would require $3 and 6 coupons.

   (a) Write down the lagrangian for this problem.

   (b) Applying the steps of Kuhn-Tucker, find the optimal x and y. Identify which constraints are binding. Draw a carefully labeled graph of your solution.

3. BC Hydro is setting up a power plant in a foreign country and it has to plan its capacity. The peak period demand for power is given by

$$p_1 = 200 - 0.5q_1$$

and the off-peak is given by

$$p_2 = 190 - 0.5q_2$$

The variable cost in each market is 10 per unit and capacity costs 20 per unit (and can be used in each market)

   (a) write down the lagrangian for this problem

   (b) Find the optimal outputs, prices and capacity for this problem.

   (c) How much of the capacity is paid for by each market?

4. A monopolist is currently operating in a market which has the demand curve $p = 200 - q$. He has no fixed costs and his marginal cost is constant at $40. A second firm is considering entering this market. The second firm's cost function is $c(q) = 400 + 60q$.

   (a) If the second firm enters, what would be the cournot duopoly quantities and profits?

   (b) What would be the the stackelberg quantities?

   (c) If the monopolist had decided to deter entry, what would have been the limit output and the monopolist's profits?

   (d) What would be a better strategy for the monopolist: deter entry or allow the market to become a duopoly?

5. Skippy has the utility function $u = x^2y$ and the budget constraint $B = p_x x + p_y y$.

   (a) Find the optimal x and y and verify that Skippy's utility is maximized.

   (b) Use the bordered hessian to derive the Slutsky equation. Identify and sign (if possible) both the income and substitution effects.

6. A macro economy can be characterized by the following set of equations:

$$S(Y, \tfrac{M}{P}) + T(Y) = I(r) + G_0$$
$$L(Y, r) = M/P$$

where P is the price level and M/P is referred to as "real balances". It is usually assumed that $S_{\frac{M}{P}} < 0$.

   (a) Find the Jacobian of this system and determine its sign.

   (b) Use Cramer's rule to find $\partial Y / \partial P$ and determine the sign of this partial derivative.