

ECON 331 Final Exam

SATURDAY, APRIL 12, 1997

K. Wainwright

Name _____

Instructions.

- – Put your name on the EXAM and the Booklets you use for your answers.
- The Exam Sheet must also be handed in as proof you wrote the test.
- Time allowed is **2.5 HOURS. Total marks = 100**

Part One: Shorter problems, 10 marks each

1. Minimize $z = 3x^2 - xy + 2y^2 - 4x - 7y + 12$ subject to the inequality constraint $x + y \geq 15$. Is the constraint binding or non-binding?
2. Given the utility function $u = x^a y^b$, show that both u and the monotonic transformation, $v = \ln(x^a y^b)$ have the same *MRS*.
3. Given $y = 4xe^{3x}$
 - (a) Find the critical point.
 - (b) Check to see if the function is at a maximum or minimum.
4. A \$100 bar of gold is appreciating in value according to the formula

$$V = 100e^{\sqrt{t}}$$

How long should the gold bar be kept to maximize the present value if under continuous compounding (a) $r = 0.08$? (b) $r = 0.12$?

5. Suppose an industry has two firms which have identical cost functions of the form $C(q_i) = K + cq_i$ (where $i = 1, 2$ and $0 < c < 1$). The market demand function is $p = 1 - q$ (where $q = q_1 + q_2$).
 - (a) Find the Cournot duopoly price and quantities.
 - (b) Assuming firm 1 gets to choose output first, find the Stackelberg price and quantities.

Part Two: Longer Problems; Do 2 of 3 questions. 20 marks each.

6. Skippy lives on an island where she produces two goods, x and y , according to the production possibility frontier $200 \geq x^2 + y^2$, and she consumes all the goods herself. Her utility function is

$$u = x \cdot y^3$$

Skippy also faces an environmental constraint on her total output of both goods. The environmental constraint is given by $x + y \leq 20$

- (a) (10 marks) Write down the Kuhn Tucker first order conditions.
- (b) (10 marks) Find Skippy's optimal x and y . Identify which constraints are binding.
7. A simple form of the $IS - LM$ model is

$$Y = C(Y, \frac{M_0}{P}) + I(r) + G_0 \quad M_0/P = L(Y, r)$$

Note that the term, $\frac{M}{P}$ appears in the consumption function. This what is sometimes referred to as the *Real Balances Effect*.

- (a) (2 marks) Make a sensible assumption about the sign of $\partial C / \partial (\frac{M}{P})$? Justifying your assumption (only your first sentence will be read).
- (b) (9 marks) Setup and sign the Jacobian of this system.
- (c) (9 marks) Determine the comparative static results about how changes in M_0 affect Y . Use the normal economic assumptions about the derivatives of, $I(r)$ and $L(Y, r)$.
8. Myrtle has the following maximization problem

$$\text{Max} \quad u = x^{1/3} y^{2/3} \quad \text{subject to} \quad B = p_x x + p_y y$$

where x and y are quantities of two consumption goods whose prices are p_x and p_y respectively. Myrtle has a budget of B .

- (a) (4 marks) From the first order conditions find expressions for the demand functions $x^* = x(p_x, p_y, B)$, $y^* = y(p_x, p_y, B)$
- (b) (4 marks) Verify that Myrtle is at a maximum by checking the second order conditions.
- (c) (4 marks) Find an expression for the Expenditure Function, $B = B(p_x, p_y, U_0)$
Myrtle's problem could be recast as the following:

$$\text{Minimize} \quad p_x x + p_y y \quad \text{s.t.} \quad U_0 = x^{1/3} y^{2/3}$$

where U_0 is equivalent to the maximum utility obtained from the above problem

- (d) (4 marks) Find the values of x and y that solve this minimization problem.
- (e) (4 marks) Show that your solution to x from the minimization problem is equal to