ECONOMICS 331
Mathematical Economics
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Homework Assignment 8

Skip has the following utility function: $U(x, y) = x(y + 1)$, where x and y are quantities of two consumption goods whose prices are p_x and p_y respectively. Skip has a budget of B. Therefore the Skip's maximization problem is

$$x(y + 1) + \lambda(B - p_x x - p_y y)$$

a) From the first order conditions find expressions for the demand functions

$$x^* = x(p_x, p_y, B) \qquad y^* = y(p_x, p_y, B)$$

Carefully graph x^* and y^*. Graph Skip's indifference curves. What kind of good is y?

b) Verify that skip is at a maximum by checking the second order conditions.

c) By substituting x^* and y^* into the utility function find an expressions for the indirect utility function,

$$U = U(p_x, p_y, B)$$

d) By rearranging the indirect utility function, derive an expression for the expenditure function,

$$B^* = B(p_x, p_y, U_0)$$

Interpret this expression. Find $\partial \mathrm{B}/\partial \mathrm{p}_x$ and $\partial \mathrm{B}/\partial \mathrm{p}_y$.

Skip's maximization problem could be recast as the following minimization problem:

$$p_x x + p_y y \text{ s.t. } U_0 = x(y + 1)$$

e) Write down the lagrangian for this problem.

f) Find the values of x and y that solve this minimization problem and show that the values of x and y are equal to the partial derivatives of the expenditure function, $\partial \mathrm{B}/\partial \mathrm{p}_x$ and $\partial \mathrm{B}/\partial \mathrm{p}_y$ respectively. (Hint: use the indirect utility function)