Homework Assignment 9

1. Given the utility maximization problem

$$\text{Max } U = xy \text{ subject to } B = p_x x + p_y y$$

   (a) Derive an expression for the Slutsky equation for x and y when p_x changes. Identify the income and substitution effects. Can you sign the effects?

   (b) Using the Implicit function theorem (where the Hessian is also the Jacobian), find the comparative statics $\partial x/\partial B$ and $\partial x/\partial p_x$ by using Cramer's Rule. Compare your solutions from the implicit approach to the ones you get by differentiating the $x^*(p_x, p_y, B)$ function directly.

2. The following graph is found in most economics texts and it illustrates how an economy can get outside its PPF through free trade. Solving this problem is a straight forward application of Lagrange.
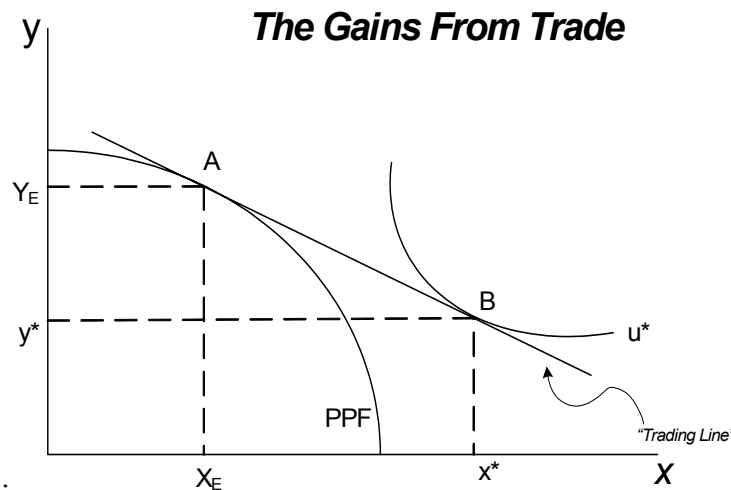

Figure 1:

Skippy lives on an island full of resources that lets her produce two goods: x and y. Her production possibility frontier (constraint) is given by

$$x^2 + y^2 = 200$$

Skippy works alone to maximize her utility which is given by $u = x^2 y$

   (a) Set up the Lagrangian for Skippy and solve for x^*, y^*. Use your solution to calculate u^*.

   (b) Carefully graph and label the PPF and Indifference Curve at the optimum.

   Now Skippy has the opportunity of to trade with Myrtle on a neighboring island.

Skippy can trade whatever x and y she has at prices p_x and p_y. To maximize her gains from trade, Skippy needs to follow a 2-step process.

1. First maximize her endowment position (maximize trading line) subject to her PPF, where the solution to this lagrange problem (X_E, Y_E) is her production decision and are her endowments she will use for trade.

2. Second, She maximizes utility subject to the constraint, $p_x X_E + p_y Y_E = p_x x^* + p_y y^*$. where x^*, y^* are the quantities she trades for that maximizes her utility.

(c) If $p_x = p_y = 1$, solve the Lagrangian that maximizes her endowments. Then use the endowment values and prices given in the trading constraint and maximize her utility by the Lagrangian method. Calculate her utility number.

(d) Find the values for the lagrange multipliers for each step. Give an economic intrepretation to each.

(e) CAREFULLY construct a graph containing her PPF, Trading Line, and indifference curve at the optimum. Be sure to include all information you have derived. Is her utility number greater than in part one?

(f) Now suppose the trading prices change such that $p_x = 0.5$. Using your original endowment values (X_E, Y_E) from before, calculate Skippy's new utility maximizing bundle. Draw the new trading line and indifference curve on your graph in (b) above.

(g) Since the trading prices have changed, Skippy needs to adjust her production decision. Re-do (c), (d) and (e) with the new prices.