SHORT ANSWER. Show all your work. In each case, check 2nd order conditions

1) Use the method of Lagrange multipliers to find the critical points of $f(x, y, z) = 2x + 4y - 4z$ subject to the constraint $x^2 + y^2 + z^2 = 9$. 1) _____

2) Use the method of Lagrange multipliers to determine the critical points of $f(x, y, z) = x^2 - 3y^2 - z^2 + 6$ subject to the constraint $5x - 3y + z = 21$. 2) _____

3) The Cobb–Douglas production function for a company is given by $P(k, l) = 163k^{1/5}l^{4/5}$ where P is the monthly production value when k is the number of units of capital and l is the number of units of labor. Suppose that capital costs \$105 per unit, labor costs \$70 per unit, and the total cost of capital and labor is limited to \$152,250. Use Lagrange Multiplier's to write the system of equations you would use to find the number of units of capital and labor that maximize production. 3) _____

4) The Cobb–Douglas production function for a company is given by $P(k, l) = 70k^{3/4}l^{1/4}$ where P is the monthly production value when k is the number of units of capital and l is the number of units of labor. Suppose that capital costs \$450 per unit, labor costs \$75 per unit, and the total cost of capital and labor is limited to \$60,000. Use Lagrange multipliers to write the system of equations you would use to find the number of units of capital and labor that maximize production. 4) _____

5) The Cobb–Douglas production function for a company is given by $P(k, l) = 20k^{2/3}l^{1/3}$ where P is the monthly production value when k is the number of units of capital and l *is* the number of units of labor. Suppose that capital costs \$150 per unit, labor costs \$225 per unit, and the total cost of capital and labor is limited to \$270,000. Use Lagrange multipliers to write the system of equations you would use to find the number of units of capital and labor that maximize production. 5) _____

6) Use the method of Lagrange multipliers to determine the critical points of $f(x, y) = 4x^2 + 2y^2 + 3$ subject to the constraint $x + 2y = 9$. 6) _____

7) Use the method of Lagrange multipliers to determine the critical points of $f(x, y) = x + 2y$ subject to the constraint $xy = 8$. 7) _____

8) The production function for a company's product is $P = 100L + 50k - L^2 - k^2$, where P is the output that results from L units of labor and k units of capital. The unit costs of labor and capital are 6 and 3, respectively. If the company wants the total cost of inputs to be 30, determine the greatest output possible subject to this budget constraint. 8) _____

9) To fill an order for 100 units of a product, a firm wishes to distribute the production between its two plants, Plant 1 and Plant 2. The total cost function is given by $c = f(q_1, q_2) = 0.5q_1^2 + 2q_1 + 32q_2 + 500$, where q_1 and q_2 are the number of units produced at Plants 1 and 2, respectively. How should the output be distributed in order to minimize costs? 9) _____

10) The Cobb–Douglas production function for a company is given by $P(k, l) = 65k^{0.3}l^{0.7}$ where P is the monthly production value when k is the number of units of capital and l is the number of units of labor. Suppose that capital costs \$60 per unit, labor costs \$140 per unit, and the total cost of capital and labor is limited to \$70,000. Use Lagrange multipliers to write the system of equations you would use to find the number of units of capital and labor that maximize production. 10) _____

11) Use the method of Lagrange multipliers to find the critical points of $f(x, y, z) = 4x + 2y - 4z$ subject to the constraint $x^2 + y^2 + z^2 = 1$. 11) _____

12) Use the method of Lagrange multipliers to determine the critical points of $f(x, y, z) = x^2 + 4y - z^2$ subject to the constraint $x + 2y - 4z = 3$. 12) _____

13) A firm has an order of 10,000 units of its product and has two plants at which to manufacture these units. Let q_1 be the number of units to be produced at the first plant and q_2 denote the number to be manufactured at the second plant. It is known that the cost function is given by $C = 48q_1^3 + 3q_2^3 + 25{,}000$. Use the method of Lagrange multipliers to determine how many units should be produced at each plant to minimize this cost function. 13) _____